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Strategic debt in a mixed duopoly: The limited liability effect

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Abstract

We study the impact of the private firm's debt on the equilibrium of a mixed duopoly by focusing on the effect of limited liability. The debt, combined with the limited liability clause, encourages the private firm to take into account only those states of the nature where demand is high. Debt therefore drives the private firm to increase its production. In response, the public firm reduces its production. Total production is increasing, causing the equilibrium price to fall and the consumer surplus to rise. The social welfare increases thanks to a more efficient allocation of total production between the two firms.

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1. Introduction

We analyze the level of debt chosen by a private firm in a mixed duopoly and its impact on its production strategy, as well as on the social welfare.

Interactions between the financial structure of firms and their product market strategies have been the subject of numerous studies.[1] These studies focused on the case where all firms are private. There are, however, many industries in which public or partly public firms are present, particularly in Europe and Asia.[2] It therefore seems interesting to extend the existing literature on the interactions between financing and production strategies to industries where private firms compete with public firms. This study focuses on the effect of the private firm's debt on its production strategy due to its limited liability.[3]

We study an industry where two firms, one public and the other private, compete in quantities with homogeneous goods.[4] Firms must choose their level of production before observing the level of demand.[5] Before the competition stage in quantities, the private firm has the possibility of incurring debt. This debt is purely strategic. The funds raised are immediately distributed to shareholders in the form of an exceptional dividend. If the private firm incurs enough debt to go bankrupt when demand is low, it modifies the weightings it assigns to the different possible states of demand when choosing production. This encourages it to increase its production. As the quantities chosen by the two firms are strategic substitutes, the public firm reacts by reducing its production. Total production is increasing, causing the equilibrium price to fall and the consumer surplus to rise. The use of strategic debt causes an increase in the profit of the private firm.[6] It also increases the social surplus. There are two sources of inefficiency in the equilibrium of the mixed duopoly in the absence of debt.[7] The first is the marginal cost of the private firm is lower than the expected equilibrium price. It is therefore socially desirable to increase the output of the private firm. The second is that the marginal cost of the public firm is higher than that of the private firm. It is therefore desirable to transfer part of the production from the public firm to the private firm. The effects of the debt help to reduce these two distortions for the level of debt chosen by the private firm in equilibrium.[8]

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[1] Faure Grimaud (1998), Cestone (1999) and Jacques (2022, chapter 15) provide surveys of this literature.

[2] Examples include aeronautics, car production, banks, air transport, electricity generation, etc.

[3] The effects of bankruptcy costs are analysed in another study (Jacques 2021).

[4] See Delbono and De Fraja (1990) and Jacques (2022, chapter 13) for surveys on the mixed oligopolies.

[5] Jacques (2021) assumes, on the other hand, that observation of demand is possible before choosing the quantities produced. The economic mechanisms present in the two models are therefore quite different.

[6] This result differs from the results obtained by Brander and Lewis (1986) in a private duopoly. In Brander and Lewis (1986), the use of debt allows a firm to increase its expectation of profit for a given debt of the other firm, but this is at the expense of the competing firm. As the two firms choose to go into debt, in equilibrium, profits are lower than without debt.

[7] See Delbono and De Fraja (1989).

[8] The result differs from that obtained by Jacques (2021) where there are values of the parameters for which the social surplus can decrease when the private firm goes into debt for strategic reason.

2. Model

We study a duopoly in which two firms compete in quantities with homogeneous goods. One of the firms (firm 1) is state-owned and has the objective of maximizing the social welfare. The other firm (firm 2) is private and maximizes its profit. The cost functions of the two firms are identical and equal to $c(q_i) = q_i^2$.

The level of demand is uncertain. The inverse demand function is linear: $p = \max(0, \alpha - q_1 - q_2)$. α is a random variable that is uniformly distributed over the interval $[\underline{\alpha}, \overline{\alpha}]$, with $\underline{\alpha} \geq \frac{1}{2}\overline{\alpha}$.[9] We note $E(\alpha)$ the expected value of α. Firms learn the true value of α only after they have chosen the quantities they wish to produce.

Before the firms compete in quantities, the private firm can go into debt in a perfectly competitive financial market whose interest rate is normalized to 0. This debt has only a strategic objective. The funds raised are immediately distributed to the shareholders of the private firm. The private firm must then repay an amount of D at the end of the Cournot competition stage. If the firm is unable to repay that amount, it goes bankrupt. The shareholders of the private firm benefit from a limited liability clause. To focus on the effects of limited liability, no bankruptcy costs are introduced into this model. We assume that firms, their creditors and consumers are risk neutral.

The timeline of the game is as follows: (1) The private firm chooses the level of its debt. (2) Firms choose their production levels. (3) The true value of α is observed, the equilibrium price is determined so as to equalize the supply and demand and the payoffs are distributed.

We note q_i the quantity produced by firm i, π_i the profit of firm i, p the equilibrium price, CS the consummers surplus, W the social welfare, VD_i the value of the debt of firm i and V_i the total value of firm i ($V_i = \pi_i + VD_i$).

3. Cournot competition

To identify the effects of debt on Cournot competition in a mixed duopoly, we calculate the equilibria obtained without and with debt.

[9] This assumption ensures that the profit of the private firm, before repayment of the debt, is positive, even if the demand is very low (i.e. $\alpha = \underline{\alpha}$) in the Cournot and in the Stackelberg equilibria.

3.1. Cournot competition without debt

The private firm seeks to maximize its expected profit:

$$E\left[\pi_2\left(q_1,q_2\right)\right] = \int_{\underline{\alpha}}^{\overline{\alpha}} \left[\left(\alpha - q_2 - q_1\right) q_2 - q_2^2\right] f\left(\alpha\right) d\alpha = \left[E(\alpha) - q_1 - 2q_2\right] q_2$$

Its best response to the quantity produced by the competing firm is given by:

$$\frac{\partial E\left[\pi_2\left(q_1,q_2\right)\right]}{\partial q_2} = 0 \Leftrightarrow q_2\left(q_1\right) = \frac{1}{4}\left[E(\alpha) - q_1\right]$$

The objective of the public firm is to maximize the expected social welfare, which is equal to the sum of expected firms' profits and expected consumer surplus:

$$E\left[W\left(q_1,q_2\right)\right] = \frac{1}{2}\left(q_1+q_2\right)^2 + \left[E(\alpha) - 2q_1 - q_2\right] q_1 + \left[E(\alpha) - q_1 - 2q_2\right] q_2$$

The best response of the state-owned firm to the quantity produced by the private firm is determined by:

$$\frac{\partial E\left[W\left(q_1,q_2\right)\right]}{\partial q_1} = 0 \Leftrightarrow q_1\left(q_2\right) = \frac{1}{3}\left[E(\alpha) - q_2\right]$$

We easily deduce the quantities of the Cournot equilibrium from the best replies of the firms:

$$\left\{ \begin{array}{l} q_1 = \frac{1}{3}\left[E(\alpha) - q_2\right] \\ q_2 = \frac{1}{4}\left[E(\alpha) - q_1\right] \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} q_2 = \frac{2}{11}E(\alpha) \\ q_1 = \frac{3}{11}E(\alpha) \end{array} \right\}$$

Although both firms have the same cost function, the state-owned firm produces more than the private firm in equilibrium, because it takes into account the impact of an increase in its production on the consumer surplus.

We compute the equilibrium price and the various payoffs according to the value of α:

$$p = \alpha - \tfrac{5}{11}E(\alpha) \quad ; \pi_1 = \left[\alpha - \tfrac{8}{11}E(\alpha)\right]\tfrac{3}{11}E(\alpha) \quad ; \pi_2 = \left[\alpha - \tfrac{7}{11}E(\alpha)\right]\tfrac{2}{11}E(\alpha)$$
$$CS = \tfrac{25}{242}[E(\alpha)]^2 \quad ; W = \tfrac{1}{11}\left[5\alpha - \tfrac{51}{22}E(\alpha)\right]$$

The assumption $\underline{\alpha} \geq \frac{1}{2}\overline{\alpha}$ ensures that the profit of the private firm is positive even if the demand is very low (i.e. $\alpha = \underline{\alpha}$).

It may be noted that the linear specification of the demand function has the effect that the surplus of consumers is independent of α.

From the previous expressions, we can calculate the expected values of payoffs:

$$E\left(\pi_1\right) = \frac{9}{121}[E(\alpha)]^2 \;\; ; \;\; E\left(\pi_2\right) = \frac{8}{121}[E(\alpha)]^2 \;\; ; \;\; E\left(SC\right) = \frac{25}{242}[E(\alpha)]^2 \;\; ; \;\; E\left(W\right) = \frac{59}{242}[E(\alpha)]^2$$

3.2. Cournot competition with debt

It is assumed that the private firm is in debt and must repay D at the end of the competition stage. In this subsection, we consider the value of D as given. We must distinguish three cases.

If D is low (lower than $\left[\underline{\alpha} - \frac{7}{11}E(\alpha)\right] \frac{2}{11}E(\alpha)$), the private firm is able to repay its debt even if the level of demand is low. The private firm continues to choose its level of production taking into account all states of demand. Its behavior is not affected by its debt. The equilibrium quantities are the same as in the absence of debt.

If D is very high,[10] the private firm is not able to repay its debt even if the demand is high. The private firm goes bankrupt and gets a zero profit in all states of the demand no matter how much it chooses to produce. The optimal behavior of this firm is then not defined. This case therefore poses a problem of resolution and does not offer much interest. So we choose to ignore it by imposing that the value of D is never higher than the profit of the private firm when $\alpha = \overline{\alpha}$.

The interesting case is when D lies between the two previous cases. The firm is not able to pay down its debt when the demand is low, but it can do so when the demand is high. We note $\widehat{\alpha}$ the bankruptcy threshold of the private firm. As the firm has limited liability, it is concerned only with the states of the nature in which the demand is high ($\alpha > \widehat{\alpha}$). It will therefore choose the quantity that maximizes its expected profit in these states of nature. Debt therefore has an impact on the best response function of the private firm. We focus on this case in the rest of this article.

The bankruptcy threshold is defined by:

$$(\widehat{\alpha} - 2q_2 - q_1)\, q_2 - D = 0 \Leftrightarrow \widehat{\alpha} = \frac{D}{q_2} + 2q_2 + q_1$$

Private firm seeks to maximize its expected profit:

$$\begin{aligned} E[\pi_2(q_1, q_2)] &= \int_{\widehat{\alpha}}^{\overline{\alpha}} [(\alpha - 2q_2 - q_1)\, q_2 - D]\, f(\alpha)\, d\alpha \\ &= \frac{\overline{\alpha} - \frac{D}{q_2} - 2q_2 - q_1}{\overline{\alpha} - \underline{\alpha}} \frac{1}{2} [(\overline{\alpha} - q_1 - 2q_2)\, q_2 - D] \end{aligned}$$

Its best reply to the quantity produced by the competing firm is given by:[11]

$$\begin{aligned} \frac{\partial E[\pi_2(q_1, q_2)]}{\partial q_2} &= 0 \Leftrightarrow \frac{1}{2} \frac{1}{\overline{\alpha} - \underline{\alpha}} \left(\overline{\alpha} - \frac{D}{q_2} - 2q_2 - q_1\right) \left(\overline{\alpha} + \frac{D}{q_2} - 6q_2 - q_1\right) = 0 \\ \Leftrightarrow \overline{\alpha} + \frac{D}{q_2} - 6q_2 - q_1 &= 0 \Leftrightarrow q_2 = \frac{(\overline{\alpha} - q_1) + \sqrt{(\overline{\alpha} - q_1)^2 + 24D}}{12} \end{aligned}$$

[10] We will specify the limit of this interval a little further.

[11] $\left(\overline{\alpha} - \frac{D}{q_2} - 2q_2 - q_1\right) = \overline{\alpha} - \widehat{\alpha} > 0$

We assume that the creditors are domestic investors. So we integrate their earnings expectations into the social surplus. With this assumption, the reaction function of the public firm does not change.

We calculate the selected quantities in equilibrium:

$$\left\{ \begin{array}{c} q_1 = \frac{1}{3}\left[E(\alpha) - q_2\right] \\ q_2 = \frac{(\overline{\alpha}-q_1)+\sqrt{(\overline{\alpha}-q_1)^2+24D}}{12} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} q_1 = \frac{35\underline{\alpha}+29\overline{\alpha}-\sqrt{(5\overline{\alpha}-\underline{\alpha})^2+816D}}{204} \\ q_2 = \frac{5\overline{\alpha}-\underline{\alpha}+\sqrt{(5\overline{\alpha}-\underline{\alpha})^2+816D}}{68} \end{array} \right\}$$

The quantity produced by the private firm is higher than without debt and increases with D. As quantities are strategic substitutes, the public firm reacts by reducing its own quantity.

We assumed that we were in the case where D has an intermediate value. This is the case if $\pi_2(\underline{\alpha}) < 0 < \pi_2(\overline{\alpha})$. The lower end of that range has already been specified. The upper end can now be determined. We must have:

$$D < \left(\overline{\alpha} - \frac{29\underline{\alpha} + 59\overline{\alpha} + 5\sqrt{(5\overline{\alpha} - \underline{\alpha})^2 + 816D}}{204} \right) \frac{5\overline{\alpha} - \underline{\alpha} + \sqrt{(5\overline{\alpha} - \underline{\alpha})^2 + 816D}}{68}$$

4. Choice of debt level

We have considered the effects of an exogenous level of debt. We are now making the level of debt endogenous. The private firm initially has zero debt. However, it can go into debt with outside investors in exchange for the promise to repay D at the end of the game. The funds raised are immediately distributed to the shareholders of the private firm. Since it has been assumed that the capital markets are competitive, the funds obtained are equal to the expected value of a debt of D. The private firm therefore chooses the value of D which maximizes $E(V_2) = E(\pi_2) + E(VD_2)$.

The total value of the private firm is equal to:

$$\begin{aligned} E(V_2) &= \int_{\widehat{\alpha}}^{\overline{\alpha}} \left[\left(\alpha - 2q_2 - q_1\right) q_2 - D\right] f(\alpha)\, d\alpha + \int_{\underline{\alpha}}^{\widehat{\alpha}} \left(\alpha - 2q_2 - q_1\right) q_2 f(\alpha)\, d\alpha + \frac{\overline{\alpha} - \widehat{\alpha}}{\overline{\alpha} - \underline{\alpha}} D \\ &= \int_{\underline{\alpha}}^{\overline{\alpha}} \left(\alpha - 2q_2 - q_1\right) q_2 f(\alpha)\, d\alpha = \left[E(\alpha) - 2q_2(D) - q_1(D)\right] q_2(D) \end{aligned}$$

By replacing the quantities with their expressions according to the level of the debt, we arrive at:

$$V_2(D) = \left(\frac{43\overline{\alpha} + 73\underline{\alpha} - 5\sqrt{(5\overline{\alpha} - \underline{\alpha})^2 + 816D}}{204} \right) \times \frac{5\overline{\alpha} - \underline{\alpha} + \sqrt{(5\overline{\alpha} - \underline{\alpha})^2 + 816D}}{68}$$

The private firm therefore chooses the following level of debt:

$$\frac{\partial V_2}{\partial D}(.) = 0 \Leftrightarrow D = \frac{(11\underline{\alpha} - 4\overline{\alpha})(\underline{\alpha} + \overline{\alpha})}{150} \Leftrightarrow D = \frac{11\underline{\alpha} - 4\overline{\alpha}}{75} E(\alpha)$$

The private firm chooses to go into debt to commit to producing more at the next stage.

5. Debt effects

The optimal level of debt is carried forward in the quantity formulas. In the perfect Nash equilibrium of the game, we have:

$$\begin{aligned} q_1 &= \frac{4}{15}E(\alpha) \quad ; \quad q_2 = \frac{1}{5}E(\alpha) \quad ; \quad E(p) = \frac{8}{15}E(\alpha) \quad ; \quad CS = \frac{1}{9}[E(\alpha)]^2 \\ E(\pi_1) &= \frac{16}{225}[E(\alpha)]^2 \quad ; \quad E(\pi_2) = \frac{1}{15}[E(\alpha)]^2 \quad ; \quad E(W) = \frac{56}{225}[E(\alpha)]^2 \end{aligned}$$

The quantities correspond to those of the Stackelberg equilibrium when the private firm is the leader (see Appendix). By increasing its debt, the private firm can commit to increase its production during the stage of competition in quantities. It chooses the level of debt that corresponds to the commitment to produce the quantity of a leader of Stackelberg.

The state-owned firm reduces its production in response, but by a lower amount. Indeed, quantities are strategic substitutes and the slope of the public firm's best response function is less than 1 (in absolute value). The total quantity goes up. The equilibrium price goes down and the consumer surplus raises.

The quantity produced by the private firm remains lower than that produced by the public firm. This result derives from the assumption that firms have the same convex cost function and from the first order conditions for maximizing their objective function. The public firm chooses the quantity which ensures the equality of the equilibrium price and its marginal cost. While the private firm chooses a quantity such that its marginal cost is lower than the equilibrium price. In equilibrium, the marginal cost of the private firm is therefore lower than that of the public firm. So the private firm produces less than the public firm.

The expectation of profit of the private firm increases while that of the public firm decreases. The former, however, remains below the latter (it is due to the fact that the output of the public firm is higher than that of the private firm).

Social welfare increases. In debt-free equilibrium, the marginal cost of the private firm is lower than the expected price. It is therefore socially desirable to increase the output of the private firm. In addition, in the equilibrium without debt, the private firm has a lower marginal cost than the state-owned firm. It is therefore possible to reduce total costs by moving part of the production from the public firm to the private firm. The indebtedness

of the private firm encourages it to focus only on the states of nature where demand is high. This encourages the private firm to increase its production and the state-owned firm to reduce its production. These movements increase the total production and reduce the production of the public company. These two movements contribute to an increase in the social surplus.

6. Conclusion

In this study, we looked at the effects of the debt of the private firm on the equilibrium of a mixed duopoly where firms compete in quantities. We focused on the effect of the private firm's limited liability. To isolate these effects, the effects of bankruptcy costs[12] have been neutralized by assuming they are zero.

If the debt of the private firm is sufficiently high, the private firm goes bankrupt when demand is low. Since the private firm is protected by a limited liability clause, it neglects the states of nature in which it goes bankrupt when it chooses its level of production. The over-weighting of the statements of the nature where the demand is high leads the firm to increase its production (compared to the case without debt). As quantities are strategic substitutes, the increase in the production of the private firm encourages the public firm to reduce its output level. Debt therefore has a strategic effect. It allows the private firm to encourage its public competitor to reduce its production. The strategic indebtedness of the private firm benefits consumers, as the total quantity increases and causes a fall in the price. The social welfare also increases because, firstly, the debt encourages the private firm to produce more, but in the absence of debt this firm produces too little (its marginal cost is lower than the expected equilibrium price) and, secondly, it causes a reallocation of production between the two firms. However, in the absence of debt, the marginal cost of the public firm is higher than that of the private firm in equilibrium. The allocation of production between the two firms is therefore inefficient in the absence of debt. Debt reduces this inefficiency.

In a mixed duopoly, the private firm wishes to commit to a high level of production before the phase of competition stage in quantities begins. The literature has identified several instruments to make this type of commitment credible, including producing before the public firm (Harris and Wiens 1980, Pal 1998, and Jacques 2004), over-investing in production capacity (Nishimori and Ogawa 2004, Lu and Poddar 2005, and Meunier, 2008), delegate the management of the firm to a manager by assigning it a different goal from maximizing profits (Barros 1995, White 2001 and 2002, and Ouattara 2013). In this note, we have shown that the use of debt is an additional means of committing to produce more.

[12]Reviewed in Jacques (2021).

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[13] Available online: https://sites.google.com/view/armeljacques/enseignements/économie-industrielle

Appendix: Stackelberg equilibrium

We calculate the equilibrium quantities assuming that the private firm is the leader.

The state-owned firm observes q_2 before choosing its production level q_1. The best reply of the public firm is:

$$q_1(q_2) = \frac{1}{3}[E(\alpha) - q_2]$$

The profit function of the private firm is equal to:

$$E[\pi_2(q_1, q_2)] = \int_{\underline{\alpha}}^{\overline{\alpha}} [\alpha - 2q_2 - q_1(q_2)] q_2 f(\alpha) d\alpha = \frac{1}{3}(\underline{\alpha} + \overline{\alpha} - 5q_2) q_2$$

In order to maximize its profit expectation, the private firm chooses to produce the quantity:

$$\frac{\partial E[\pi_2(q_1, q_2)]}{\partial q_2} = 0 \Leftrightarrow q_2 = \frac{1}{5}E(\alpha)$$

After observing this production, the public firm chooses to produce the quantity:

$$q_1 = \frac{4}{15}E(\alpha)$$