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Inequality based on Lorenz dominance criteria. An application to Mayotte using nightlights

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Abstract

Using high-resolution spatial datasets for nightlights and population, we propose an alternative measure of income for the French overseas department of Mayotte in 2013 and 2019. From these two distributions, we examine changes in inequality between these periods by deriving both relative and absolute Lorenz curves. Standard criteria based on first- or second-degree relative Lorenz dominance are inconclusive, as the associated Lorenz curves intersect in both cases. By turning to the analysis of absolute Lorenz curves, we can discriminate between the two distributions, but only at the second-degree of Lorenz dominance. This finding suggests that, under the implied conception of distributive justice, Mayotte exhibited greater inequality in 2019 than in 2013.

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1 Introduction

The use of satellite night-light data is becoming increasingly popular, particularly in contexts where statistical data do not exist or are of poor quality [Gibson et al., 2020]). This is precisely the case for Mayotte, for which income data are unavailable over the medium term. This situation is highly detrimental since this small island faces major problems of poverty and inequality [Thibault, 2019, Merceron, 2020].To overcome this lack of individual income data, we propose to use two high-resolution spatial datasets, available over the 2012-2019 period, to assess the evolution of inequalities in Mayotte through Lorenz curves. More specifically, as Elvidge et al. [2012], traditional monetary income data are replaced by per capita light intensity within the population. In doing so, the purpose of this paper is to investigate how income inequalities evolve in the post-departmentalization context in Mayotte.

Following Atkinson [1970] and Kolm [1976] there is a wide agreement in the economic literature to use the Lorenz curve for measuring inequality. Furthermore, the relative first order Lorenz criterion could also be useful for identifying which of the two distributions is dominating. Even if this criterion is very powerful, its main drawback is that its implementation could lead to some impossibility results. Actually, when two Lorenz curves intersect the first relative Lorenz criterion is mute. This is precisely the case in our case study for Mayotte : when drawing Lorenz curves for two points in time, namely 2013 and 2019, they intersect. In this note, we show that it is possible to discriminate between the two distributions by using an alternative criterion, namely the absolute Lorenz [Moyes, 1987, 1992] - and by relying on higher order of dominance.

2 Background

2.1 Context

Located 8,000 km from mainland France in the Mozambique Channel, Mayotte is a small French island that has been part of France since 1841 and became an official department in 2011.[1] Currently, the island is positioned midway between developing and developed countries and faces significant gaps in many economic and social

[1] Fontaine and Hermet [2024] provides a comprehensive review of the historical evolution of institutional changes in Mayotte.

indicators compared to other French regions. The average GDP per capita, though growing, remains four times lower than that of mainland France. Furthermore, the island is characterized by strong demographic pressure, especially since its departmentalization in 2011. In particular, many residents of the Comoros take advantage of Mayotte's geographical proximity (approximately 70 km) to migrate irregularly. As a result, the proportion of foreigners has risen sharply, with half of the island's population now lacking French citizenship. At the same time, we observe no significant decline in poverty in Mayotte [Thibault, 2019]. For foreigners, their irregular administrative status does not allow them to access social benefits. In addition, their lack of qualifications prevents them from entering the formal labor market, and they often end up in the informal sector. For these reasons, the issue of inequality remains a central concern for this territory, especially in such an atypical economic and demographic context.

2.2 Data

In Mayotte, there are no income data collected or available in a comprehensive manner over a relatively long period. Indeed, the establishment of a statistical institute with the same capabilities as those of the four historic French overseas departments is an ongoing but unfinished process. Consequently, to estimate people's wealth, it is a requirement to use other data sources.

Night light Night light data are growing in popularity among economists, who initially relied on them to proxy economic activity at the local level [Sutton and Costanza, 2002, Henderson et al., 2012, Chen and Nordhaus, 2011, 2015]. The brightness of nightlights detected by satellites is especially useful in contexts where other data sources are either non-existent or presumed to be of poor quality.[2] For Mayotte, we are in the first case.

In this paper, we employ nightlight data obtained from second-generation satellites, which outperform the first generation in terms of quality and precision.[3] The nightlight data used in this study is available starting from 2012, which corresponds to

[2] For a comprehensive and detailed literature review on nightlight data, the interested reader can refer to Gibson et al. [2020].

[3] More specifically, we use night-time satellite images from the Day/Night Band (DNB) sensors of the Visible Infrared Imaging Radiometer Suite (VIIRS) aboard the Suomi National Polar-orbiting Partnership (S-NPP), rather than data from the Defense Meteorological Satellite Program.

the first complete year after the departmentalization of Mayotte. The final pixel resolution of nocturnal brightness is 0.004° (approximately 500 meters in horizontal resolution), implying that Mayotte is represented by approximately 1,500 pixels.

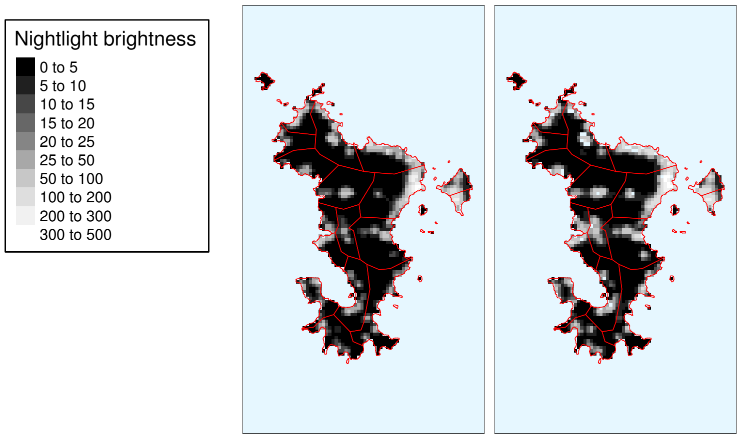

Figure 1: Nightlight brightness detected in Mayotte in 2013 (left) and 2019 (right).
Sources: VIIRS night light data and authors' own calculations.
Notes: Light intensity is measured in nano Watts per cm^2 per steradian (nWatts $cm^{-}2$ $sr^{-}1$).

Figure 1 shows the spatial distribution of nightlight in Mayotte in 2013 and 2019. Unlit pixel are depicted in black, while the most lit turn from grey to white depending on the radiance value. It is clear from Figure 1 that the most illuminated pixels correspond to the main urban areas of Mayotte, such as Mamoudzou (in the north-east of the island) and the area around Sada and Chiconi (in the west), which experienced a significant increase in light intensity over the period studied.

Population In addition to night-light data, data on the spatial distribution of the population in Mayotte are also used. These data, estimated and adjusted using a "top-down" method, come from the "*WorldPop*" research group [Stevens et al., 2015, Lloyd et al., 2019]. They are freely available at annual frequency, with a very high horizontal resolution of 1 km^2. Figure 2 shows the spatial distribution of the population in Mayotte between 2013 and 2019. As with nightlights, population changes are most significant in the north-east and west of the island.
In the rest of the paper, we combine these two spatial datasets - nightlights and population - to obtain a "light endowment per capita" for each of the 634 pixels that constitute Mayotte.

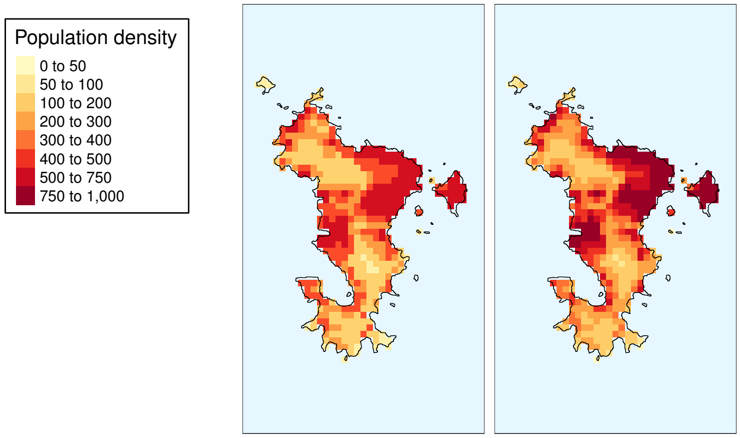

Figure 2: Spatial distribution of inhabitants in Mayotte in 2013 (left) and 2019 (right).
Sources: Population data from *WorldPop* and authors' own calculations.
Notes: Population is measured in people per km^2.

3 Relative Lorenz

For a given sample of the population $P = (1, 2, \ldots, i, \ldots, n)$, we consider a distribution of nightlights allocation per head $z = (z_1, z_2, \ldots, z_i, \ldots z_n)$ in ascending order, with $\forall i \in P$, $z_i \in [0, +\infty)$, and F a cumulative distribution function of z defined by $F(z) = \int_0^z f(t)dt$. For the population proportion $p \in [0, 1]$, we define the nightlights allocation of an individual at the p^{th} percentile in distribution F as $F^{-1}(p) = \inf\{t : F(t) \geq p\}$. The Lorenz curve of the distribution z is then defined by $L_z(p) = \frac{1}{\mu_F} \int_0^p F^{-1}(t)dt$, with μ_F the mean of nightlights of $F(z)$.

3.1 Relative Lorenz dominance of first degree

We say that a distribution z dominates another distribution w according to the 1^{st} order relative Lorenz criterion if its Lorenz curve L_z is nowhere below the Lorenz curve L_w. Figure 3 displays the corresponding relative Lorenz curves for 2013 and 2019. It appears that the two curves intersect approximately at the middle of the distribution. This indicates that individuals located at the bottom of the distribution of nightlight per capita capture a lower share of total income in 2019 than in 2013. In terms of inequality, since the two curves intersect, we cannot say that one Lorenz curve dominates the other. To deal with this situation, we use the relative Lorenz

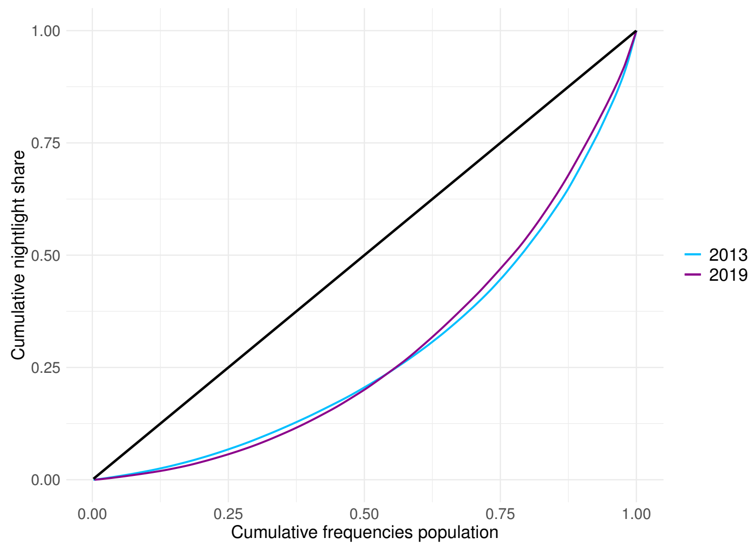

Figure 3: Relative Lorenz curves for nightlight per capita in Mayotte in 2013 and 2019 - Relative Lorenz dominance of first degree.
Sources: VIIRS nightlight data, population data from *WorldPop* and authors'own calculations.

dominance of second degree [4].

3.2 Relative Lorenz dominance of second degree

Following Aaberge [2009], a Lorenz curve L_z is said to second-degree dominate a Lorenz curve L_ω if :

$$\int_0^u L_z(t)\, dt \geq \int_0^u L_\omega(t)\, dt \;\; for\ all\ u \in [0;1] \tag{1}$$

and the inequality holds strictly for some $u \in]0,1[$.
The aggregated Lorenz curve can be viewed as a sum of weighted income shares, where the weights decrease linearly with increasing rank of the income receiver in the income distribution. A social decision-maker who prefers the second-degree dominating of two intersecting Lorenz curves therefore pays more attention to inequality in the lower than in the upper part of the income distribution.
Figure 4 displays the corresponding curves for 2013 and 2019. It appears that the two curves intersect again and are sometimes indistinguishable from each other. Such a pattern indicates that we cannot say that one curve dominates the other.

[4] Note that it is also possible to make tests of comparisons between dominance curves, see Davidson and Duclos [2000].

Using the relative Lorenz curve, we are unable to draw a definitive conclusion about whether inequality in Mayotte increased or decreased between 2013 and 2019.

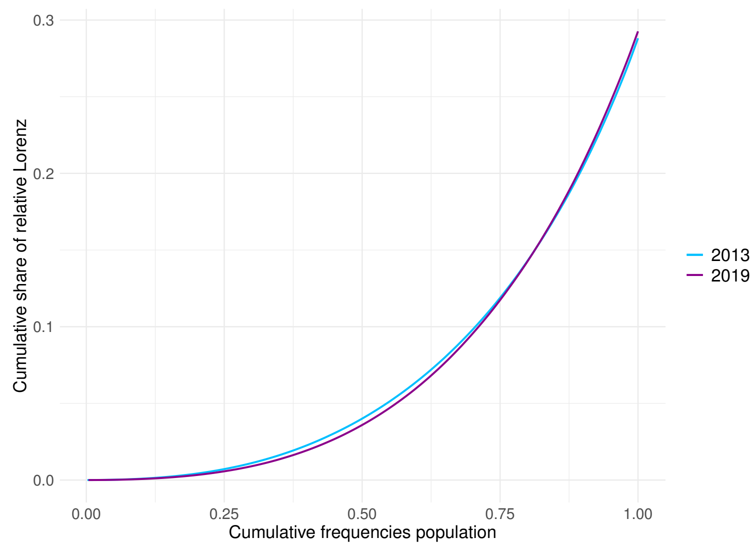

Figure 4: Relative Lorenz curves for nightlight per capita in Mayotte in 2013 and 2019 - Relative Lorenz dominance of second degree.
Sources: VIIRS nightlight data, population data from *WorldPop* and authors'own calculations.

4 Absolute Lorenz

Following Kolm [1976], we propose an alternative way comparing the distributions of nightlights per capita in Mayotte based on absolute inequality indices. Those indices are invariant to an addition of the same scalar to all individuals. In other words, absolute inequality index always considers improvement in terms of the absolute difference, whereas the relative inequality index considers it in terms of the relative difference. We use Moyes [1992] approach to define absolute Lorenz dominance criterion by means of the comparisons of the absolute Lorenz curves of the distributions considered. For a distribution of nightlights allocation per capita $z = (z_1, z_2, \ldots, z_i, \ldots z_n)$ in ascending order, let $\tilde{z}$ the distribution defined by $\tilde{z} = (z_1 - \mu_z, z_2 - \mu_z, \ldots, z_i - \mu_z, \ldots z_n - \mu_z)$ and $\tilde{F}$ a cumulative distribution function of $\tilde{z}$ defined by $\tilde{F}(\tilde{z}) = \int_0^{\tilde{z}} f(t)dt$. For the population proportion $p \in [0,1]$, we define the nightlights allocation of an individual at the p^{th} percentile in distribution $\tilde{F}$ as $\tilde{F}^{-1}(p) = \inf\{t : \tilde{F}(t) \geq p\}$. The absolute Lorenz curve of the distribution $\tilde{z}$ is then defined by $L_{\tilde{z}}(p) = \frac{1}{\mu_{\tilde{F}}} \int_0^p \tilde{F}^{-1}(t)dt$, with $\mu_{\tilde{F}}$ the mean of nightlights of $\tilde{F}(\tilde{z})$.

4.1 Absolute Lorenz dominance of first degree

We say that a distribution $\tilde{z}$ dominates another distribution $\tilde{w}$ according to the 1^{st} order absolute Lorenz criterion if its Lorenz curve $L_{\tilde{z}}$ is nowhere below the Lorenz curve $L_{\tilde{w}}$.[5] As pointed out by Moyes [1992], there is no theoretical reason or normative reason to prefer relative or absolute Lorenz dominance criterion. However, the capacity of discrimination of the absolute Lorenz dominance is more important than the relative one.[6] Absolute and relative inequality indices satisfy some invariance property. [7] Two completely different views of this property may be considered. On the one hand, adding or subtracting the same amount to all incomes does not change inequality (absolute indices satisfy this property). On the other hand proportional changes in incomes does not change inequality (relative indices satisfy this property).

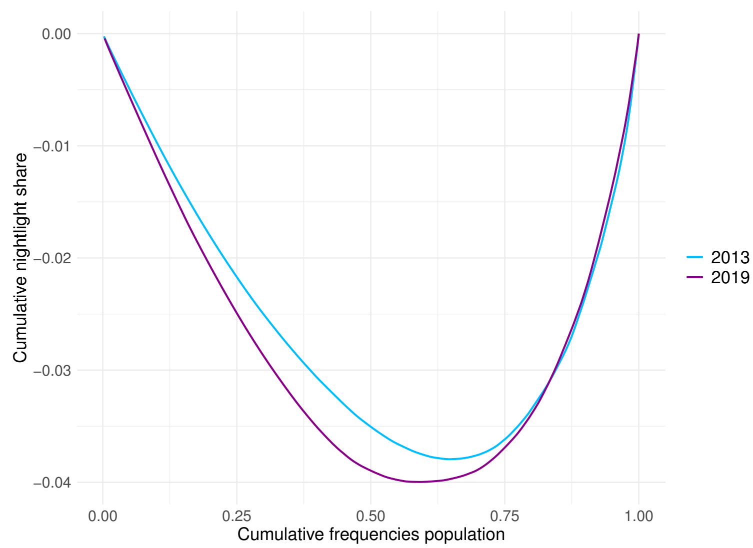

Figure 5: Absolute Lorenz curves for nightlight per capita in Mayotte in 2013 and 2019 - Absolute Lorenz dominance of first degree.
Sources: VIIRS nightlight data, population data from *WorldPop* and authors'own calculations.

Figure 5 shows the corresponding absolute Lorenz curves for 2013 and 2019. We observe that the 2013 curve is above the 2019 curve for a large portion of the distri-

[5]Another and perhaps simpler way to determine the absolute Lorenz curve of a distribution is to derive it from its relative Lorenz curve through the following relationship $LA(p;z) = \mu(z)[LR(p;z) - p]$

[6]Actually, considering 20 countries, Moyes [1992] shows that the relative Lorenz dominance criterion leads to 45% of conclusive results, while the absolute Lorenz dominance criteria leads to 90% conclusive results.

[7]See Villar [2017] for an exhaustive discussion on this issue.

bution. However, at the top of the distribution, the two curves overlap and intersect again. Even with an absolute criterion, we are unable to distinguish between the two distributions. Given the crossing point, one could say that there is less inequality in 2013 for 80 percent of the less well-off sample of the population considered.

4.2 Absolute Lorenz dominance of second degree

An absolute Lorenz curve $L_{\tilde{z}}$ is said to dominate a Lorenz curve $L_{\tilde{\omega}}$ via the absolute Lorenz dominance of second degree if :

$$\int_0^u L_{\tilde{z}}(t)\,dt \geq \int_0^u L_{\tilde{\omega}}(t)\,dt \quad for\ all\ u \in [0;1] \tag{2}$$

and the inequality holds strictly for $u \in]0,1[$.

Figure 6 displays the corresponding curves for 2013 and 2019. It appears that the two curves do not intersect anymore and indicates that 2013 presents less inequality than 2019. Consequently, an inequality-averse social planner who supports the

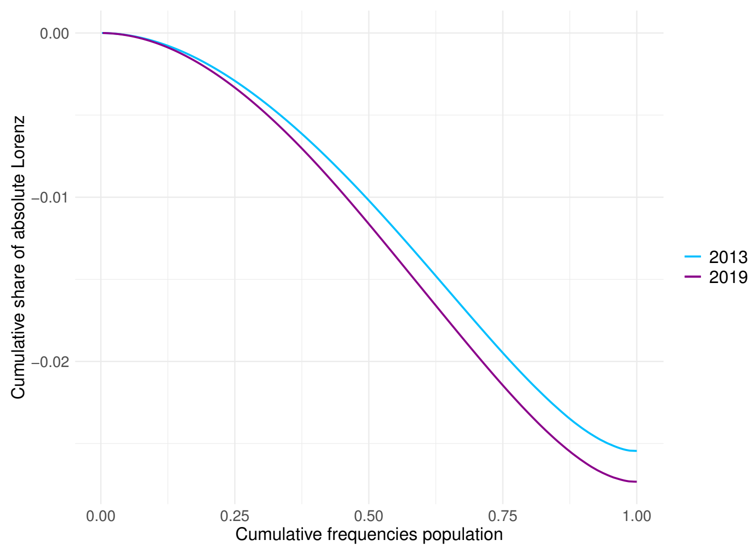

Figure 6: Absolute Lorenz curves for nightlight per capita in Mayotte in 2013 and 2019 - Absolute Lorenz dominance of second degre.
Sources: VIIRS nightlight data, population data from *WorldPop* and authors'own calculations.

criterion of second-degree absolute Lorenz dominance — assigning greater weight to changes occurring in the lower part of the absolute Lorenz curve than to those in the upper part — would conclude that inequality has increased in Mayotte between 2013 and 2019.

5 Conclusion

When Lorenz curves intersect, an unambiguous ranking cannot be achieved without introducing weaker ranking criteria than first-degree Lorenz dominance. To address this, we used an alternative dominance criterion that aggregates the Lorenz curve from below, specifically second-degree Lorenz dominance. This criterion places greater emphasis on transfers occurring in the lower part of the income distribution rather than in the upper part.

In light of our results, only the absolute concept of second-degree Lorenz dominance enables a conclusive decision regarding inequality between 2012 and 2019 in Mayotte. As suggested by Moyes [1987, 1992] for first-degree Lorenz dominance, it appears that, for second-degree dominance as well, the absolute concept provides a more discriminating measure of inequality than the relative concept. However, it is important to remember that these two approaches to comparing inequalities are grounded in different conceptions of distributive justice.

Our study highlights that an inequality-averse social planner, applying the criterion of second-degree absolute Lorenz dominance, would conclude that inequality has increased since Mayotte's departmentalization. Given Mayotte's unique geographic and demographic context, the poorest appear to be worse off. This situation presents additional challenges for policymakers.

References

Rolf Aaberge. Ranking intersecting lorenz curves. *Social Choice and Welfare*, 33 (2):235–259, 2009. ISSN 01761714, 1432217X. URL http://www.jstor.org/stable/41108006.

Anthony B Atkinson. On the measurement of inequality. *Journal of Economic Theory*, 2(3):244–263, 1970. ISSN 0022-0531. doi: https://doi.org/10.1016/0022-0531(70)90039-6. URL https://www.sciencedirect.com/science/article/pii/0022053170900396.

Xi Chen and William Nordhaus. A test of the new viirs lights data set: Population and economic output in africa. *Remote Sensing*, 7(4):4937–4947, 2015. ISSN 2072-4292. URL https://www.mdpi.com/2072-4292/7/4/4937.

Xi Chen and William D. Nordhaus. Using luminosity data as a proxy for economic statistics. *Proceedings of the National Academy of Sciences*, 108(21):8589–8594, 2011. ISSN 0027-8424. doi: 10.1073/pnas.1017031108. URL https://www.pnas.org/content/108/21/8589.

Russell Davidson and Jean-Yves Duclos. Statistical inference for stochastic dominance and for the measurement of poverty and inequality. *Econometrica*, 68(6): 1435–1464, 2000. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/3003995.

Christopher D. Elvidge, Kimberly E. Baugh, Sharolyn J. Anderson, Paul C. Sutton, and Tilottama Ghosh. The night light development index (nldi): a spatially explicit measure of human development from satellite data. *Social Geography*, 7: 23–35, 2012. URL https://api.semanticscholar.org/CorpusID:850944.

Idriss Fontaine and François Hermet. Changements institutionnels et développement économique à mayotte. *Revue économique*, 75(6):1161–1202, 2024.

John Gibson, Susan Olivia, and Geua Boe-Gibson. Night lights in economics: Sources and uses1. *Journal of Economic Surveys*, 34(5):955–980, 2020. doi: https://doi.org/10.1111/joes.12387. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/joes.12387.

J. Vernon Henderson, Adam Storeygard, and David N. Weil. Measuring economic growth from outer space. *American Economic Review*, 102(2):994–1028, April 2012. doi: 10.1257/aer.102.2.994. URL https://www.aeaweb.org/articles?id=10.1257/aer.102.2.994.

Serge-Christophe Kolm. Unequal inequalities. i. *Journal of Economic Theory*, 12(3):416–442, 1976. ISSN 0022-0531. doi: https://doi.org/10.1016/0022-0531(76)90037-5. URL https://www.sciencedirect.com/science/article/pii/0022053176900375.

Christopher T. Lloyd, Heather Chamberlain, David Kerr, Greg Yetman, Linda Pistolesi, Forrest R. Stevens, Andrea E. Gaughan, Jeremiah J. Nieves, Graeme Hornby, Kytt MacManus, Parmanand Sinha, Maksym Bondarenko, Alessandro Sorichetta, and Andrew J. Tatem. Global spatio-temporally harmonised datasets for producing high-resolution gridded population distribution datasets. *Big Earth Data*, 3(2): 108–139, 2019. doi: 10.1080/20964471.2019.1625151.

Sébastien Merceron. Revenus et pauvreté à mayotte en 2018. les inégalités de niveau de vie se sont creusées. Insee Analyse Mayotte 25, Institut National de la Statistique et des Etudes Economiques, 2020.

Patrick Moyes. A new concept of lorenz domination. *Economics Letters*, 23(2):203–207, 1987. ISSN 0165-1765. doi: https://doi.org/10.1016/0165-1765(87)90040-1. URL https://www.sciencedirect.com/science/article/pii/0165176587900401.

Patrick Moyes. Dominance, relative ou absolue, au sens de lorenz une comparaison internationale. *Revue économique*, 43(5):895–915, 1992. ISSN 00352764, 19506694. URL http://www.jstor.org/stable/3502481.

Forrest R. Stevens, Andrea E. Gaughan, Catherine Linard, and Andrew J. Tatem. Disaggregating census data for population mapping using random forests with remotely-sensed and ancillary data. *PLOS ONE*, 10(2):1–22, 02 2015. doi: 10.1371/journal.pone.0107042. URL https://doi.org/10.1371/journal.pone.0107042.

Paul C. Sutton and Robert Costanza. Global estimates of market and non-market values derived from nighttime satellite imagery, land cover, and ecosystem service valuation. *Ecological Economics*, 41(3):509–527, 2002. ISSN 0921-8009. doi: https://doi.org/10.1016/S0921-8009(02)00097-6. URL https://www.sciencedirect.com/science/article/pii/S0921800902000976.

Pierre Thibault. Des conditions de vie inegales entre villages. Insee Analyse Mayotte 25, Institut National de la Statistique et des Etudes Economiques, 2019.

Antonio Villar. *Lectures on inequality, poverty and welfare*, volume 685. Springer, 2017.