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Valence, abstention, and electoral competition

David T Yi
Xavier University

Abstract

We present a simple two-candidate electoral competition model in which one candidate has a valence advantage and voters have option to abstain. We show that complete policy convergence and complete policy divergence as well as partial policy divergence arise as pure strategy Nash equilibria when certain conditions between valence advantage and abstention rates are satisfied. These results highlight the importance of understanding the interactions between the valence advantage and voter abstention in understanding candidates strategic behavior in electoral competition.

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Contact: David T Yi - yid@xavier.edu.
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1 Introduction

According to the Downsian model of two-party competition, office-motivated candidates tend to gravitate toward the policy preferences of the median voter.

However, real-world elections often deviate from this idealized framework. Voters are frequently polarized along political, social, and economic lines, and hardline voters may abstain from voting if neither candidate's policies align closely with their preferences. In fact, studies by McCarty, Poole, and Rosenthal (2016), Lee, Moretti, and Butler (2004), and Fowler and Hall (2016) reject the hypothesis of candidate convergence, which suggests that candidates in a two-party system gravitate toward the median voter's policy preferences. Moreover, non-policy factors, such as a candidate's competence, character, or other valence attributes, play a significant role in shaping voter decisions. Studies by Bernhardt and Ingberman (1985), Ansolabehere, Snyder, and Stewart (2001), Groseclose (2001), Aragones and Palfrey (2002), Stone and Simas (2010), Hummel (2010), Montagnes and Rogowski (2015), and Buisseret and Van Weeden (2022) highlight how valence factors can disrupt the median voter convergence predicted by the Downsian model.

In this paper, we examine how voter abstention and valence factors influence candidates' strategies and proposed policies in an election environment where voters are firmly divided along an ideological line. We derive conditions relating abstention rates and valence factors under which pure strategy Nash equilibria emerge, leading to complete policy divergence, complete policy convergence, or partial policy divergence in the strategies of competing candidates. While previous studies have examined valence and abstention separately in the context of electoral competition, these two factors have not been integrated into a single, cohesive game theoretic model. Our contribution lies in bringing together two previously distinct lines of research by developing a game theoretic model that explicitly incorporates both valence advantage and voter abstention rates. By analyzing how these two factors jointly shape candidate strategies and electoral outcomes, our model both supports and develops the conclusions of prior studies.

2 Model

In this electoral competition model, two candidates, A and B, compete for votes by proposing policy positions within a discrete policy space $\{l, m, r\}$, representing left, moderate, and right positions, respectively. The electorate comprises three voter groups L, M, and R each with an ideal preference for l,m, and r, respectively. Voters are assumed to be evenly distributed, with proportions α_L, α_M, and α_R such that $\alpha_L + \alpha_M + \alpha_R = 1$. Candidate A originates from group L (left-leaning), while Candidate B originates from group R (right-leaning) such that $A \in L$ and $B \in R$. Throughout the paper, we assume that Candidate A has a valence advantage v over Candidate B where $v \in (0, 1)$. This valence advantage captures factors such as competence or charisma. The winner of the election is determined by the total number of votes received, with voter abstention possible when candidates deviate from voters' ideal policy positions.

If Candidate $A \in L$ proposes policy l, all voters from voter group L fully support

Candidate A during the election campaign. However, if Candidate A opts for the moderate policy m, a fraction $p_L \in (0, 1)$ of group L abstains from voting. Similarly, if Candidate $B \in R$ proposes policy r, all voters in voter group R fully support Candidate B. However, if B opts for the policy m, a fraction $p_R \in (0, 1)$ of group R abstains from voting.

2.1 Payoffs for Candidates

In this section, we derive the candidates' payoffs. A candidate's payoff is based on the number of votes they receive, which depends on their chosen policy and their opponent's chosen policy.

2.1.1 Candidate A's payoffs

There are four payoffs to calculate for Candidate A, determined by the policy choices of both candidates: $A = \{l, m\}$ and $B = \{r, m\}$.

If Candidate A chooses policy l given that Candidate B chooses policy r, Candidate A receives the following number of votes:

$$V_A(l|B = r) = \alpha_L + \frac{1+v}{2} \cdot \alpha_M. \tag{1}$$

The term $(\frac{1+v}{2} \cdot \alpha_M)$ in equation (1) represents the effect of Candidate A's valence advantage in the election. While both candidates A and B adopt policies preferred by their respective base groups, more voters in group M (moderates) tend to support Candidate A due to this advantage. As $v \to 0$, $\frac{1+v}{2} \cdot \alpha_M \to \frac{1}{2} \cdot \alpha_M$, indicating that Candidates A and B evenly split the votes from the moderate voter group in the absence of valence advantage. Conversely, as $v \to 1$, $\frac{1+v}{2} \cdot \alpha_M \to \alpha_M$, meaning Candidate A captures almost all the votes from the moderate voter group when v approaches 1.

By a similar argument, Candidate A receives the number of votes described in equations (2), (3), and (4), based on the policies chosen by Candidate A and Candidate B:

$$V_A(m|B = r) = (1 - p_L) \cdot \alpha_L + \alpha_M \tag{2}$$

$$V_A(l|B = m) = \alpha_L \tag{3}$$

$$V_A(m|B = m) = (1 - p_L) \cdot \alpha_L + \frac{1+v}{2} \cdot \alpha_M \tag{4}$$

2.1.2 Candidate B's payoffs

Similar to Candidate A, Candidate B has four payoffs to calculate, determined by the policy choices of both candidates: $A = \{l, m\}$ and $B = \{r, m\}$.

If Candidate B chooses policy r given that Candidate A chooses policy l, Candidate B receives the following number of votes:

$$V_B(r|A = l) = \alpha_R + \frac{1-v}{2} \cdot \alpha_M. \tag{5}$$

Since both Candidate A and B are proposing policies preferred by their respective bases, Candidate B secures $(\frac{1-v}{2} \cdot \alpha_M)$ votes from the moderate voter group M, given Candidate A's valence advantage.

By a similar argument, Candidate B receives the number of votes described in equations (6), (7), and (8), based on the policies chosen by Candidate B and Candidate A:

$$V_B(m|A = l)) = (1 - p_R) \cdot \alpha_R + \alpha_M \tag{6}$$

$$V_B(r|A = m) = \alpha_R \tag{7}$$

$$V_B(m|A = m) = (1 - p_R) \cdot \alpha_R + \frac{1 - v}{2} \cdot \alpha_M \tag{8}$$

3 Results

The primary focus of this paper is to examine each candidate's strategic policy choices and identify the conditions under which Nash equilibrium outcomes arise in a simultaneous game setting. Table 1 below shows the normal-form representation of a two-candidate electoral competition game, with payoffs calculated as outlined in the previous section.

Table 1: Payoff Matrix

	$B : r$	$B : m$
$A : l$	$\alpha_L + \frac{1+v}{2} \cdot \alpha_M$, $\alpha_R + \frac{1-v}{2} \cdot \alpha_M$	α_L, $(1 - p_R) \cdot \alpha_R + \alpha_M$
$A : m$	$(1 - p_L) \cdot \alpha_L + \alpha_M$, α_R	$(1 - p_L) \cdot \alpha_L + \frac{1+v}{2} \cdot \alpha_M$, $(1 - p_R) \cdot \alpha_R + \frac{1-v}{2} \cdot \alpha_M$

In what follows, we examine how the valence advantage factor v, along with $p_L$and p_R, influences candidates' policy choices and the conditions under which each policy pair constitutes a Nash equilibrium outcome.

3.1 Candidate A's Best Response

3.1.1 When Candidate B chooses policy r

When Candidate B chooses policy r, policy l is a best response for Candidate A if and only if

$$V_A(l|B = r) \geq V_A(m|B = r).$$

Substituting the respective payoffs given by equations (1) and (2) into the inequality condition above, policy l is a best response if and only if

$$\alpha_L + \frac{1 + v}{2} \cdot \alpha_M \geq (1 - p_L) \cdot \alpha_L + \alpha_M.$$

Solving for p_L,

$$p_L \geq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_L}. \tag{9}$$

On the other hand, policy m is a best response to policy r if and only if

$$p_L \leq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_L}. \tag{10}$$

3.1.2 When Candidate B chooses policy m

When Candidate B chooses policy m, policy l is a best response for Candidate A if and only if

$$V_A(l|B=m) \geq V_A(m|B=m).$$

Substituting the respective payoffs given by equations (3) and (4) into the above inequality, policy l is a best response if and only if

$$\alpha_L - \frac{1+v}{2} \cdot \alpha_M \geq (1-p_L) \cdot \alpha_L.$$

Solving for p_L,

$$p_L \geq \frac{1+v}{2} \cdot \frac{\alpha_M}{\alpha_L}. \tag{11}$$

On the other hand, policy m is a best response to Candidate B's policy choice m if and only if

$$p_L \leq \frac{1+v}{2} \cdot \frac{\alpha_M}{\alpha_L}. \tag{12}$$

3.2 Candidate B's Best Response

3.2.1 When Candidate A chooses policy l

When Candidate A chooses policy l, policy r is a best response for Candidate B if and only if

$$V_B(r|A=l) \geq V_B(m|A=l).$$

Substituting the respective payoffs given by equations (5) and (6) into the above inequality, policy r is a best response if and only if

$$\alpha_R + \frac{1-v}{2} \cdot \alpha_M \geq (1-p_R) \cdot \alpha_R + \alpha_M.$$

Solving for p_R,

$$p_R \geq \frac{v+1}{2} \cdot \frac{\alpha_M}{\alpha_R}. \tag{13}$$

On the other hand, policy m is a best response to policy l if and only if

$$p_R \leq \frac{v+1}{2} \cdot \frac{\alpha_M}{\alpha_R}. \tag{14}$$

3.2.2 When Candidate A chooses policy m

When Candidate A chooses policy m, policy r is a best response for Candidate B if and only if

$$V_B(r|A = m) \geq V_B(m|A = m).$$

Substituting the respective payoffs given by equations (7) and (8) into the above inequality, policy r is a best response if and only if

$$\alpha_R \cdot p_R \geq \frac{1-v}{2} \cdot \alpha_M.$$

Solving for p_R,

$$p_R \geq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_R}. \tag{15}$$

On the other hand, policy m is a best response to Candidate A's policy choice m if and only if

$$p_R \leq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_R}. \tag{16}$$

3.3 Pure strategy Nash equilibrium conditions

In this section, we propose and examine the conditions under which the pure strategy Nash equilibrium exists. There are four potential unique pure strategy Nash equilibrium outcomes, as presented in Table 1: (l, r), (m, m), (m, r), and (l, m). The policy pair (l, r) represents a complete divergent outcome. The policy pair (m, m) represents a complete convergent outcome. The policy pairs (m, r) and (l, m) represent partial divergent outcomes.

Proposition 1. *The complete divergent policy pair (l, r) is a Nash equilibrium if and only if $p_L \geq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_L}$ and $p_R \geq \frac{v+1}{2} \cdot \frac{\alpha_M}{\alpha_R}$.*

Proof. The proposition follows from inequalities (9) and (13). □

Proposition 1 indicates that (1) if Candidate A has a significant valence advantage (v is high), the abstention rate for voter group R must be relatively high for Candidate B to choose policy position r, while Candidate A would choose policy position l even when the abstention rate for voter group L is relatively low. (2) if Candidate A has a small valence advantage (v is low), both Candidates A and B would stick to the policy positions preferred by their respective bases, provided the abstention rates for voter group L and R remain at modest levels. See Appendix for numerical examples.

When both Candidates A and B proposes policies preferred by their respective voter bases, Candidate A's valence advantage allows Candidate A to capture a larger portion of votes from the moderate voter group M. Therefore, Candidate B has an incentive to appeal to the moderate voters by proposing the moderate policy, *unless* the cost of deviating from their base is prohibitively high, as is the case here. In this environment, Candidate A is able to satisfy their voter base by proposing a policy preferred by their base while still capturing a larger share of moderate voters given the valence advantage.

Proposition 2. *The complete convergent policy pair* (m, m) *is a Nash equilibrium if and only if* $p_L \leq \frac{1+v}{2} \cdot \frac{\alpha_M}{\alpha_L}$ *and* $p_R \leq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_R}$.

Proof. The proposition follows from inequalities (12) and (16). □

Proposition 2 indicates that (1) if Candidate A has a significant valence advantage (v is high), the abstention rate for voter group R must be very low for Candidate B to deviate from their base and choose the moderate policy position m, while Candidate A would deviate from their base and choose moderate policy position m even when the abstention rate is relatively high. (2) if Candidate A has a small valence advantage (v is low), both Candidates A and B deviate from their respective bases and choose moderate policy position m, provided the abstention rates for voter group L and R are modestly low. See Appendix for numerical examples.

Given Candidate A's valence advantage, Candidate B receives fewer votes than Candidate A from the moderate voter group M when both candidates select policies favored by their respective bases. This creates an incentive for Candidate B to shift toward the moderate position, particularly when the cost of deviation for Candidate B is sufficiently low. Candidate A's optimal response to Candidate B choosing the moderate policy is to also select the moderate position, as long as Candidate A's cost of deviation is moderate. It is worth noting that valence advantage might be inversely correlated with the cost of deviation. If this correlation holds, Candidate A would capture a significantly larger share of moderate votes while still retaining substantial support from their own voter base.

Proposition 3. *The partial divergent policy pair* (m, r) *is a Nash equilibrium if and only if* $p_L \leq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_L}$ *and* $p_R \geq \frac{1-v}{2} \cdot \frac{\alpha_M}{\alpha_R}$.

Proof. The proposition follows from inequalities (10) and (15). □

Proposition 3 indicates that (1) if Candidate A has a significant valence advantage (v is high), the abstention rate for voter group L must be very low for Candidate A to deviate from their base and choose the moderate policy position m, while Candidate B would choose policy position r even if the abstention rate of voter group R is relatively low. (2) if Candidate A has a small valence advantage (v is low), Candidates A would choose policy m if voter group L's abstention rate is below a modest level, while Candidate B would choose policy r if the abstention rates for voter group R is greater than a modest level. See Appendix for numerical examples.

Given Candidate A's lower cost of deviation and valence advantage, Candidate A has an incentive to appeal to the moderates while still capturing a significant portion of their own base. Consequently, unless the cost of deviation for Candidate B is sufficiently lower, Candidate B is better off proposing a policy that secures all the votes from their own base.

Proposition 4. *The partial divergent policy pair* (l, m) *is a Nash equilibrium if and only if* $p_L \geq \frac{1+v}{2} \cdot \frac{\alpha_M}{\alpha_L}$ *and* $p_R \leq \frac{1+v}{2} \cdot \frac{\alpha_M}{\alpha_R}$.

Proof. The proposition follows from inequalities (11) and (14). □

Proposition 4 indicates that (1) if Candidate A has a significant valence advantage (v is high), the abstention rate for voter group L must be relatively high for Candidate A to choose policy position preferred by their own base, while Candidate B would choose the moderate policy position m as long as the abstention rate of voter group R is not excessively high. (2) if Candidate A has a small valence advantage (v is low), Candidate A would chooses policy l if the abstention rate for voter group L exceeds a modest level, while Candidate B would deviate from their base and choose the moderate policy m as long as the abstention rate for voter group R remains below a modest level. See Appendix for numerical examples.

When the cost of deviation for Candidate A is relatively higher than that for Candidate B, Candidate B's best strategy is to choose the moderate policy position, capturing votes from the moderates while still maintaining support from their own base. This outcome is feasible if Candidate A's loss from deviation (losing votes from their base) outweighs their gain from deviation (securing votes from the moderates). Such a scenario is likely to occur when the number of voters in group L is sufficiently larger than the number of voters in group M.

4 Conclusion

We presented a simple two-candidate electoral competition model in which one candidate has a valence advantage and voters have the option to abstain. Voters are firmly divided based on their ideological preferences along a single issue dimension. We showed that complete policy convergence, complete divergence, and partial policy divergence can arise as pure strategy Nash equilibria when certain conditions relating valence advantage and abstention rates are satisfied. These results highlight the importance of understanding the interaction between valence advantage and voter abstention in shaping candidates' strategic behavior in electoral competition.

For future work, it might be interesting to explore a setting where the abstention rates p_L and p_R are endogenous to the opposing candidate's policy choices. For example, p_L takes on different values depending on whether Candidate B chooses policy r or m, and similarly for p_R. This extension could capture voter responses, where turnout decisions are influenced not only by ideological distance but also by perceived threats or satisfaction with the opposition's platform. It would allow for richer dynamics and potentially new types of equilibria, especially in asymmetric valence settings.

Appendix

Examples for Complete Divergence: (l^*, r^*)

Let's suppose that $\alpha_L = \alpha_R = 0.4$, $\alpha_M = 0.2$, and $v = 0.6$ then p_R must be greater than 0.4 and p_L must be greater than 0.1 (See Proposition 1). The following normal form game demonstrates the complete divergence equilibrium under the condition specified above.

Example 1(a): $v = 0.6$, $p_L = 0.11$, $p_R = 0.41$

	$B : r$	$B : m$
$A : l$	$\mathbf{0.560, 0.440}$	$0.400, 0.436$
$A : m$	$0.556, 0.400$	$0.516, 0.276$

On the other hand, if Candidate A's valence advantage is fairly low, say $v = 0.1$, while $\alpha_L = \alpha_R = 0.4$ and $\alpha_M = 0.2$ then p_R and p_L must be greater than 0.275 and 0.225, respectively. The following normal form game demonstrates the complete divergence equilibrium under the condition specified above.

Example 1(b): $v = 0.1$, $p_L = 0.3$, $p_R = 0.3$

	$B : r$	$B : m$
$A : l$	$\mathbf{0.510, 0.490}$	$0.400, 0.480$
$A : m$	$0.480, 0.400$	$0.390, 0.370$

Examples for Complete Convergence: (m^*, m^*)

Let's suppose that $\alpha_L = \alpha_R = 0.4$, $\alpha_M = 0.2$, and $v = 0.6$ then p_R must be less than 0.1 and p_L must be less than 0.4 (See Proposition 2). The following normal form game demonstrates the complete convergence equilibrium under the condition specified above.

Example 2(a): $v = 0.6$, $p_L = 0.20$, $p_R = 0.09$

	$B : r$	$B : m$
$A : l$	$0.560, 0.440$	$0.400, 0.564$
$A : m$	$0.520, 0.400$	$\mathbf{0.480, 0.404}$

On the other hand, if Candidate A's valence advantage is fairly low, say $v = 0.1$, while

$\alpha_L = \alpha_R = 0.4$ and $\alpha_M = 0.2$ then p_R and p_L must be less than 0.225 and 0.275, respectively. The following normal form game demonstrates the complete convergence equilibrium under the condition specified above.

Example 2(b): $v = 0.1$, $p_L = 0.2$, $p_R = 0.2$

	$B : r$	$B : m$
$A : l$	$0.510, 0.490$	$0.400, 0.520$
$A : m$	$0.520, 0.400$	$\mathbf{0.430}, \mathbf{0.410}$

Examples for Partial Divergence: (m^*, r^*)

Let's suppose that $\alpha_L = \alpha_R = 0.4$, $\alpha_M = 0.2$, and $v = 0.6$ then p_L must be less than 0.1 and p_R must be greater than 0.1 (See Proposition 3). The following normal form game demonstrates the partial divergence equilibrium under the condition specified above.

Example 3(a): $v = 0.6$, $p_L = 0.09$, $p_R = 0.11$

	$B : r$	$B : m$
$A : l$	$0.560, 0.440$	$0.400, 0.556$
$A : m$	$\mathbf{0.564}, \mathbf{0.400}$	$0.524, 0.396$

On the other hand, if Candidate A's valence advantage is fairly low, say $v = 0.1$, while $\alpha_L = \alpha_R = 0.4$ and $\alpha_M = 0.2$ then p_R and p_L must be less than 0.225 and 0.275, respectively. The following normal form game demonstrates the partial divergence equilibrium under the condition specified above.

Example 3(b): $v = 0.1$, $p_L = 0.2$, $p_R = 0.25$

	$B : r$	$B : m$
$A : l$	$0.510, 0.490$	$0.400, 0.520$
$A : m$	$\mathbf{0.520}, \mathbf{0.400}$	$0.430, 0.390$

Examples for Partial Divergence: (l^*, m^*)

Let's suppose that $\alpha_L = \alpha_R = 0.4$, $\alpha_M = 0.2$, and $v = 0.6$ then p_L must be greater than 0.4 and p_R must be less than 0.4 (See Proposition 4). The following normal form game demonstrates the partial divergence equilibrium under the condition specified above.

Example 4(a): $v = 0.6$, $p_L = 0.41$, $p_R = 0.39$

	$B : r$	$B : m$
$A : l$	$0.560, 0.440$	$\mathbf{0.400}, \mathbf{0.444}$
$A : m$	$0.436, 0.400$	$0.396, 0.284$

On the other hand, if Candidate A's valence advantage is fairly low, say $v = 0.1$, while $\alpha_L = \alpha_R = 0.4$ and $\alpha_M = 0.2$ then p_L must be greater than 0.275 and p_R must be less than 0.275, respectively. The following normal form game demonstrates the partial divergence equilibrium under the condition specified above.

Example 4(b): $v = 0.1$, $p_L = 0.3$, $p_R = 0.25$

	$B : r$	$B : m$
$A : l$	$0.510, 0.490$	$\mathbf{0.400}, \mathbf{0.500}$
$A : m$	$0.480, 0.400$	$0.390, 0.390$

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