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Overlapping ownership in a unionized oligopoly

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Abstract

This paper analyzes the effects of overlapping ownership in a unionized Cournot duopoly with decreasing returns to labor. Firms partially internalize rivals' profits, and wage rates are set by firm-level unions that maximize total wage bills. An increase in overlapping ownership always increases producer surplus but lowers total union utility through a reduction in equilibrium wage rates. When products are close substitutes, both consumer surplus and social welfare exhibit a U-shaped relationship with respect to the degree of overlapping ownership. By contrast, when products are not close substitutes, overlapping ownership lowers consumer surplus and social welfare.

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1. Introduction

Overlapping ownership is widespread across industries, typically involving investment funds holding common stakes or firms engaging in cross-shareholding agreements (López and Vives, 2019). A growing body of research explores its implications. Reynolds and Snapp (1986) show that overlapping ownership can lower quantities and raise prices. Farrell and Shapiro (1990) demonstrate that it may enhance industry performance, even when it leads to higher prices. López and Vives (2019) find that overlapping ownership can increase welfare and consumer surplus when R&D spillovers are large. Bayona and López (2018) show that it can raise both consumer and total surplus even in the absence of spillovers. Sato and Matsumura (2020) show that in a circular market, overlapping ownership deters entry, with welfare effects that can be either positive or negative. Brito et al. (2020) find that with endogenous quality choice, it can raise consumer surplus while lowering total welfare. Cumbul (2021) examines overlapping ownership under information efficiencies and finds that it can improve expected consumer and total welfare. Li and Zhang (2021) show that when firms choose product positions, overlapping ownership amplifies anti-competitive effects. Li et al. (2023) show that in vertically differentiated markets, it can promote both consumer and social welfare. Can and Mukherjee (2025) find that overlapping ownership can raise consumer surplus and welfare under strategic tax policy, provided the marginal social cost of public funds is below one.

While the literature above focuses on product market outcomes, a smaller set of studies has begun to explore the labor market implications of overlapping ownership. Fanti (2016) examines how cross-ownership affects welfare in a unionized duopoly where firms operate under constant returns to labor. Azar and Vives (2021) show that in a general equilibrium setting, greater overlapping ownership reduces employment, real wage rates, and the labor share. Azar et al. (2022) provide the first empirical evidence that increased overlapping ownership in U.S. local labor markets causally reduces average employee earnings. Goshen and Levit (2022) argue that overlapping ownership depresses wage rates and employment by inducing firms to adopt strong governance, which reduces investment and creates a labor market monopsony. Steinbaum (2021) argues that overlapping ownership increases shareholder influence, prompting firms to reduce wage rates and employment.

This paper adds to the literature by examining the impact of overlapping ownership on both product and labor market outcomes in a unionized Cournot duopoly with decreasing returns to labor. Firm-level unions set wage rates to maximize total wage bills, and firms use labor as the sole input. Therefore, it also contributes to the literature on input pricing under downstream overlapping ownership (e.g., Chen et al. (2024); Li and Shuai (2022); Lømo (2024); Shuai et al. (2023)) by modeling unions as upstream input suppliers that set the input prices (the wage rates) when

downstream firms partially internalize rivals' profits. The analysis reveals a trade-off: overlapping ownership increases producer surplus but lowers total union utility by reducing wage rates. It further shows that consumer surplus and social welfare exhibit a U-shaped relationship with respect to overlapping ownership when products are close substitutes: initial increases in overlapping ownership reduce welfare, but sufficiently high levels reverse this effect. The paper thus contributes to the literature by linking overlapping ownership to labor market outcomes and highlighting cases where it improves overall welfare.

The structure of the paper is as follows. Section 2 introduces the model. Section 3 analyzes the equilibrium. Section 4 discusses the main findings. Section 5 offers concluding remarks.

2. The Model

Building on Fanti and Meccheri (2011), we analyze a Cournot duopoly with differentiated products, decreasing returns to labor, and firm-specific unions. Each firm relies solely on labor as an input and operates under a production technology defined by

$$q_i = \sqrt{l_i}. \tag{1}$$

This implies that producing q_i units requires $l_i = q_i^2$ units of labor, resulting in quadratic labor costs. Firms face the following inverse demand function:

$$p_i = a - q_i - \gamma q_j, \tag{2}$$

where $\gamma \in [0,1]$ captures the extent of product differentiation. When $\gamma = 1$, the products are perfect substitutes; when $\gamma = 0$, they are completely independent. Wage rates are determined by firm-level monopolistic unions that maximize the total wage bill:

$$V_i = w_i q_i^2. \tag{3}$$

Instead of assuming that firms maximize only their own profits, we follow López and Vives (2019) and allow each firm to partially internalize the rival's profit. Firm i maximizes a linear combination of its own profit and a share $\lambda \in [0,1]$ of the rival's profit:

$$\psi_i = \pi_i + \lambda \pi_j, \tag{4}$$

where

$$\pi_i = p_i q_i - w_i q_i^2. \tag{5}$$

The parameter λ measures the extent of profit internalization. The case $\lambda = 0$ corresponds to firms that maximize profits independently, while $\lambda = 1$ reflects full internalization of rival profits, as in a cartel or complete merger.

The game proceeds in two stages. In the first stage, unions simultaneously set wage rates w_i to maximize V_i, taking the rival's wage rate as given. In the second stage, firms choose quantities non-cooperatively, given wage rates, and maximize ψ_i. The equilibrium is obtained via backward induction.

Producer surplus is given by $PS = \pi_1 + \pi_2$. The representative consumer derives utility from consuming both products, captured by the quasi-linear function $U = a(q_1 + q_2) - \frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{2}$. Consumer surplus is defined as $CS = U - p_1 q_1 - p_2 q_2$. On the labor side, total union utility is $UU = V_1 + V_2$. Social welfare is the aggregate of consumer surplus, producer surplus, and total union utility: $SW = CS + PS + UU$.

3. Equilibrium Analysis

This section derives the subgame perfect equilibrium of the two-stage game using backward induction. In the second stage, taking wage rates as given, each firm chooses its quantity to maximize its objective function, which includes a weighted share of the rival's profit. The resulting best response function of firm i, given the quantity of firm j, is

$$q_i(q_j) = \frac{a - \gamma(1+\lambda)\, q_j}{2(1+w_i)}. \tag{6}$$

Substituting the rival's best response into this expression yields firm i's equilibrium quantity as a function of both wage rates:

$$q_i(w_i, w_j) = \frac{a\left(2 + 2w_j - \gamma(1+\lambda)\right)}{4w_j + 4w_i(1+w_j) + (2-\gamma-\gamma\lambda)(2+\gamma+\gamma\lambda)}. \tag{7}$$

In the first stage, unions simultaneously choose wage rates to maximize their respective total wage bills. The best response function of union i, given the rival's wage rate, is

$$w_i(w_j) = \frac{4 + 4w_j - \gamma^2(1+\lambda)^2}{4(1+w_j)}. \tag{8}$$

Assuming a symmetric equilibrium, where $w_i = w_j = w$, the equilibrium wage rate is

$$w^* = \frac{\sqrt{4 - \gamma^2(1+\lambda)^2}}{2}. \tag{9}$$

The second-order conditions for the optimization problems of both firms and unions hold. Given the equilibrium wage rate, the symmetric equilibrium values for quantity, price, employment, profit, producer surplus, consumer surplus, total union utility, and social welfare are derived. These expressions are reported in the Appendix.

4. Results

This section presents the main comparative statics results concerning the effects of overlapping ownership on equilibrium outcomes. We focus on how changes in the overlapping ownership parameter λ influence each firm's quantity, price, employment, and wage rate, as well as the welfare components, conditional on the value of the product differentiation parameter γ.

Proposition 1. *The equilibrium quantity of each firm, employment at each firm, and consumer surplus increase, while the price of each firm decreases as overlapping ownership increases, if and only if* $\frac{1}{\sqrt{2}} < \gamma$ *and* $\sqrt{\frac{2}{\gamma^2}} - 1 < \lambda$.

In equilibrium, each firm's quantity increases with overlapping ownership if and only if the product differentiation parameter γ and the ownership parameter λ are sufficiently high. Because employment at each firm and consumer surplus both rise with quantity, and the price of each firm declines as quantity expands, it follows that employment at each firm and consumer surplus increase, while the price of each firm decreases, in response to overlapping ownership under the same condition. When overlapping ownership rises, firms internalize rivals' profits more, which softens competition and would, by itself, reduce quantity and raise the price of each firm. Simultaneously, each union optimally cuts the wage rate, lowering marginal costs and pushing each firm's quantity up and each firm's price down. Quantity, employment, and consumer surplus increase, and each firm's price falls only if this wage-reduction effect dominates the softer-competition effect, which occurs exactly when $\frac{1}{\sqrt{2}} < \gamma$ and $\sqrt{\frac{2}{\gamma^2}} - 1 < \lambda$. Otherwise, the softer-competition effect dominates; thus, overlapping ownership reduces quantity, employment, and consumer surplus, and raises the price of each firm.

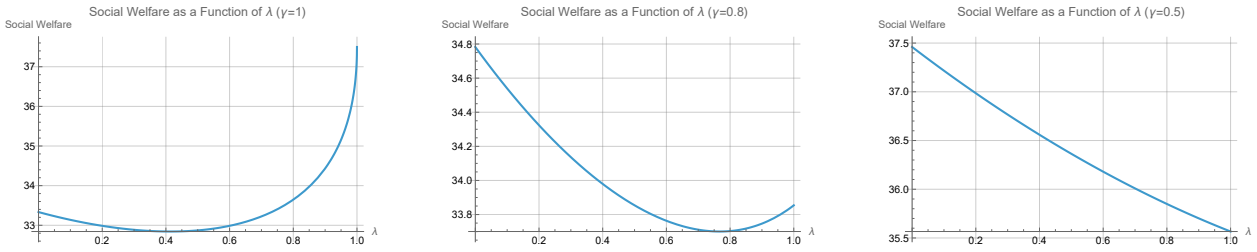
Proposition 2. *Equilibrium producer surplus increases, while the wage rate of each union and total union utility decrease, as overlapping ownership rises.*

In equilibrium, each firm's profit increases as overlapping ownership rises, leading to a higher producer surplus. At the same time, the wage rate of each union declines. Total union utility depends on two components: employment and the wage rate. When the product differentiation parameter γ or the ownership parameter λ is not sufficiently high, both employment and the wage rate fall, resulting in a decrease in the utility of each union and, as a result, a reduction in total

union utility. When γ and λ are sufficiently high, employment at each firm increases while the wage rate of each union continues to decline. The negative effect of falling wage rates dominates the positive effect of higher employment, resulting in a decrease in the utility of each union and, as a result, a decrease in total union utility. As overlapping ownership increases, firms internalize each other's profits more, which softens competition and raises profits. A higher λ makes firms internalize rivals' profits more; thus, for any given wage rate, they choose lower quantities. This shifts each union's effective labor demand downward. Since each union maximizes the wage bill, it responds by cutting its wage rate to limit the induced contraction in employment; hence, wage rates fall monotonically with overlapping ownership. Wage-rate reductions lower marginal costs and further boost profits; since producer surplus is the sum of profits, it increases in λ. Moreover, even if employment at each firm eventually rises for large enough γ and λ, the wage-rate reduction effect is strong enough to dominate, making total union utility fall.

Proposition 3. *Equilibrium social welfare increases with overlapping ownership if and only if* $\frac{1}{\sqrt{2}} < \gamma$ *and* $\sqrt{\frac{2}{\gamma^2}} - 1 < \lambda$.

Social welfare is the sum of consumer surplus, producer surplus, and total union utility. Consumer surplus increases with overlapping ownership only when products are sufficiently close substitutes and the degree of overlapping ownership is high. Under the same conditions, the sum of producer surplus and total union utility also increases, as the gain in producer surplus outweighs the loss in total union utility. Since consumer surplus also rises in this region, social welfare increases as well. In contrast, when products are not close substitutes or overlapping ownership is low, the decline in total union utility dominates the increase in producer surplus; the combined total of the two falls, and consumer surplus also decreases, leading to an overall reduction in social welfare.



(a) $\gamma = 1$ (b) $\gamma = 0.8$ (c) $\gamma = 0.5$

Figure 1: Social welfare as a function of λ for different values of γ, with $a = 10$.

Proposition 3 implies that, when products are close substitutes, both consumer surplus and social welfare follow a U-shaped relationship with respect to the degree of overlapping ownership. To illustrate this, we present numerical simulations generated using *Wolfram Mathematica 14.2*.

In all figures, we set $a = 10$. Figure 1a shows the case $\gamma = 1$; Figure 1b corresponds to $\gamma = 0.8$; and Figure 1c to $\gamma = 0.5$. In the last case, the condition $\frac{1}{\sqrt{2}} < \gamma$ is not satisfied, and overlapping ownership reduces social welfare.

In Figures 1a and 1b, where the condition on γ holds, we observe a U-shaped relationship. When products are homogeneous ($\gamma = 1$), social welfare is maximized at $\lambda = 1$. In contrast, when $\gamma = 0.8$, the welfare-maximizing level of overlapping ownership occurs at $\lambda = 0$.

It is also possible to extend the model to complementary products (i.e., $\gamma \in [-1, 0)$, where $\gamma = -1$ implies perfect complements). When products are complements, overlapping ownership is generally believed to be beneficial. In this case, we find that greater overlapping ownership reduces prices and increases each firm's quantity, employment, and profit. It also raises total producer surplus, total consumer surplus, total union utility, and social welfare. However, the wage rate of each firm still declines, and the increase in total union utility is driven by higher employment.

5. Conclusion

This paper analyzes the effects of overlapping ownership in a differentiated Cournot duopoly with wage-bill-maximizing unions and decreasing returns to labor. The results show that overlapping ownership always increases producer surplus while reducing equilibrium wage rates and total union utility. Its impact on consumer surplus and social welfare depends on the extent of overlapping ownership and the degree of product substitutability. In particular, both consumer surplus and social welfare increase with overlapping ownership only when products are close substitutes and the level of overlapping ownership is sufficiently high. This implies that when product differentiation is limited, both consumer surplus and social welfare exhibit a U-shaped relationship with respect to the degree of overlapping ownership. At low levels of profit internalization, an increase in overlapping ownership leads to reduced quantities, higher prices, and lower welfare. However, beyond a critical threshold, a further increase in overlapping ownership reverses these effects.

Intuitively, the gain in social welfare originates from increased quantities and lower prices when products are close substitutes. Although total union utility declines due to a reduction in wage rates, the resulting increase in producer and consumer surplus dominates, suggesting that pro-competitive effects can outweigh labor-side losses. This mechanism highlights how overlapping ownership may lead to net welfare gains despite adverse labor market outcomes.

Future research may explore the effects under Bertrand competition. In Cournot competition, given first-period wage rates, a firm's second-stage choice depends on its own wage rate and the rival's quantity; thus, the rival's wage rate drops out of the best response. Under Bertrand competition with overlapping ownership, the rival's employment, and hence its wage bill, which is partially internalized, varies with both prices; thus, a firm's best-response price will depend on its

own wage rate, the rival's wage rate, and the rival's price. This distinction may alter the implications of overlapping ownership and is left for future research.

References

Azar, J., Y. Qiu, and A. Sojourner (2022) "Common Ownership in Labor Markets" *Upjohn Institute Working Papers*.

Azar, J. and X. Vives (2021) "General Equilibrium Oligopoly and Ownership Structure" *Econometrica* ***89***, 999–1048.

Bayona, A. and Á. L. López (2018) "Silent financial interests and product innovation" *Economics Letters* ***170***, 109–112.

Brito, D., R. Ribeiro, and H. Vasconcelos (2020) "Overlapping ownership, endogenous quality, and welfare" *Economics Letters* ***190***, 109074.

Can, G. and A. Mukherjee (2025) "Cross Ownership Under Strategic Tax Policy" *The B.E. Journal of Economic Analysis & Policy* ***25***, 115–125.

Chen, L., T. Matsumura, and C. Zeng (2024) "Welfare Consequence of Common Ownership in a Vertically Related Market" *The Journal of Industrial Economics* ***72***, 996–1004.

Cumbul, E. (2021) "Group Formation, Mergers, and Overlapping Ownership in Aggregative Games with Information Sharing" *SSRN Working Paper*.

Fanti, L. (2016) "Interlocking cross-ownership in a unionised duopoly: when social welfare benefits from "more collusion"" *Journal of Economics* ***119***, 47–63.

Fanti, L. and N. Meccheri (2011) "The Cournot-Bertrand profit differential in a differentiated duopoly with unions and labour decreasing returns" *Economics Bulletin* ***31***, 233–244.

Farrell, J. and C. Shapiro (1990) "Asset ownership and market structure in oligopoly" *The RAND Journal of Economics* ***21***, 275–292.

Goshen, Z. and D. Levit (2022) "Agents of Inequality: Common Ownership and the Decline of the American Worker" *Duke Law Journal* ***72***, 1–69.

Li, Y. and J. Shuai (2022) "Input price discrimination and horizontal shareholding" *Journal of Regulatory Economics* ***61***, 48–66.

Li, Y. and J. Zhang (2021) "Product positioning with overlapping ownership" *Economics Letters* ***208***, 110058.

Li, Y., J. Zhang, and Z. Zhou (2023) "Vertical differentiation with overlapping ownership" *Economics Letters* ***222***, 110947.

López, Á. L. and X. Vives (2019) "Overlapping Ownership, R&D Spillovers, and Antitrust Policy" *Journal of Political Economy* ***127***, 2394–2437.

Lømo, T. L. (2024) "Overlapping ownership and input prices" *International Journal of Industrial Organization* ***94***, 103067.

Reynolds, R. J. and B. R. Snapp (1986) "The competitive effects of partial equity interests and joint ventures" *International Journal of Industrial Organization* ***4***, 141–153.

Sato, S. and T. Matsumura (2020) "Free entry under common ownership" *Economics Letters* ***195***, 109489.

Shuai, J., M. Xia, and C. Zeng (2023) "Upstream market structure and downstream partial ownership" *Journal of Economics & Management Strategy* ***32***, 22–47.

Steinbaum, M. (2021) "Common Ownership and the Corporate Governance Channel for Employer Power in Labor Markets" *The Antitrust Bulletin* ***66***, 123–139.

Appendix

The symmetric equilibrium values for quantity, price, employment, profit, producer surplus, consumer surplus, total union utility, and social welfare are expressed as follows.

$$q_1^* = q_2^* = q^* = \frac{a}{2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}}, \tag{10}$$

$$p_1^* = p_2^* = p^* = \frac{a\left(1+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)}{2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}}, \tag{11}$$

$$l_1^* = l_2^* = l^* = \frac{a^2}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}, \tag{12}$$

$$\pi_1^* = \pi_2^* = \pi^* = \frac{a^2\left(2+2\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)}{2\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}, \tag{13}$$

$$PS^* = \pi_1^* + \pi_2^* = 2\pi^*, \quad CS^* = \frac{a^2(1+\gamma)}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}, \tag{14}$$

$$UU^* = V_1^* + V_2^* = \frac{a^2\sqrt{4-\gamma^2(1+\lambda)^2}}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}, \quad SW^* = CS^* + PS^* + UU^*. \tag{15}$$

Proof of Proposition 1. We compute the partial derivatives of equilibrium quantity, employment, consumer surplus, and price with respect to λ. Each of these expressions is increasing (or decreasing for price) in λ if and only if the stated conditions on γ and λ hold.

$$\frac{\partial q^*}{\partial \lambda} = \frac{a\left(\frac{\gamma^2(1+\lambda)}{\sqrt{4-\gamma^2(1+\lambda)^2}} - \gamma\right)}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}, \tag{16}$$

$$\frac{\partial l^*}{\partial \lambda} = \frac{2a^2\left(\frac{\gamma^2(1+\lambda)}{\sqrt{4-\gamma^2(1+\lambda)^2}} - \gamma\right)}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^3}, \tag{17}$$

$$\frac{\partial CS^*}{\partial \lambda} = \frac{2a^2(1+\gamma)\left(\frac{\gamma^2(1+\lambda)}{\sqrt{4-\gamma^2(1+\lambda)^2}} - \gamma\right)}{\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^3}, \tag{18}$$

$$\frac{\partial p^*}{\partial \lambda} = -\frac{a\gamma(1+\gamma)\left(\gamma(1+\lambda)-\sqrt{4-\gamma^2(1+\lambda)^2}\right)}{\sqrt{4-\gamma^2(1+\lambda)^2}\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^2}. \tag{19}$$

□

Proof of Proposition 2. We derive the partial derivatives of producer surplus, wage rate, and total union utility with respect to λ. The partial derivative of producer surplus with respect to λ is positive, while the partial derivatives of wage rate and total union utility with respect to λ are negative, since the numerators and denominators in all expressions are positive.

$$\frac{\partial PS^*}{\partial \lambda} = \frac{a^2\gamma^2\left(2+2\lambda+\gamma(1+\lambda)(3\lambda-1)+(3-\lambda)\sqrt{4-\gamma^2(1+\lambda)^2}\right)}{\sqrt{4-\gamma^2(1+\lambda)^2}\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^3} > 0, \tag{20}$$

$$\frac{\partial w^*}{\partial \lambda} = -\frac{\gamma^2(1+\lambda)}{2\sqrt{4-\gamma^2(1+\lambda)^2}} < 0, \tag{21}$$

$$\frac{\partial UU^*}{\partial \lambda} = -\frac{a^2\gamma\left(8-\gamma(1+\lambda)\left(\gamma(1+\lambda)+\sqrt{4-\gamma^2(1+\lambda)^2}-2\right)\right)}{\sqrt{4-\gamma^2(1+\lambda)^2}\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^3} < 0. \tag{22}$$

□

Proof of Proposition 3.

$$\frac{\partial SW^*}{\partial \lambda} = \frac{2a^2\gamma\left(\gamma+\gamma\lambda-\sqrt{4-\gamma^2(1+\lambda)^2}\right)\left(1+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)}{\sqrt{4-\gamma^2(1+\lambda)^2}\left(2+\gamma+\gamma\lambda+\sqrt{4-\gamma^2(1+\lambda)^2}\right)^3}. \tag{23}$$

The partial derivative of social welfare with respect to λ is positive if and only if the same conditions identified in Proposition 1 are satisfied. □