Economics Bulletin

Volume 46, Issue 1

When lexicographically undominated strategies yield perfect equilibria

Ryoji Jinushi
Seikei University

Abstract

Perfect equilibrium, in which players share a common idea about future errors, is a widely used equilibrium concept for finite extensive-form games. Okada's (1991) lexicographic domination, by contrast, is based on each player's individual idea about future errors. We identify simple sufficient conditions under which perfect equilibria and lexicographically undominated strategy profiles are equivalent. For example, if each path includes at most two decision nodes, any lexicographically undominated strategy profile is perfect. When we consider a simple signaling game where the marginal impact of the receiver's action on the sender's payoff is type-invariant, if the receiver can select a special action called the Never-Best boundary action, both concepts coincide. Our results suggest that, under certain conditions, Selten's perfect equilibrium is consistent with weaker rationality assumptions than those underlying Selten's (1975) original definition.

Discussions with Siyang Xiong, Hiroki Nishimura and Tomoya Tajika contributed to this research. I thank Ryosuke Ishii, Daisuke Hirata, Takayuki Oishi, Akira Okada and the audiences of the Game Theory Workshop 2022 at Kyoto University and the seminar at Shimonoseki City University. This work was supported by JSPS KAKENHI Grant Number JP23K18789.
Citation: Ryoji Jinushi, (2026) "When lexicographically undominated strategies yield perfect equilibria", *Economics Bulletin*, Volume 46, Issue 1, pages 274-284
Contact: Ryoji Jinushi - rjinu001@ucr.edu.
Submitted: December 31, 2025. **Published:** March 30, 2026.

1. Introduction

Selten's (1975) *perfect equilibrium* is a popular equilibrium concept for finite extensive-form games, in which players share a common idea about future errors. Okada (1991) introduced a weaker concept, a *lexicographically undominated strategy profile*, in which each local strategy is not weakly dominated by any alternative local strategy in a neighborhood of the current profile. In other words, compared with perfect equilibrium, players are locally cautious about potentially different future errors across information sets. Okada (1991) proves that every perfect equilibrium is lexicographically undominated.

Identifying when individually cautious reasoning yields perfect equilibrium clarifies when perfect equilibrium is justified under weaker, purely individual assumptions. This also characterizes environments in which using perfect equilibrium is particularly compelling. As an early contribution, Okada's (1988) Theorem 3.7 shows that, in two-player normal-form games, every lexicographically undominated strategy profile is perfect.

In this paper, we investigate when individual cautious reasoning yields perfect equilibria.[1] For example, consider a simple game in which each path includes at most two decision nodes (hereafter a *short game*). If players avoid lexicographically dominated strategies, the resulting strategy profile is perfect (Theorem 1).[2]

When analyzing a signaling game in which the marginal impact of the receiver's action on the sender's payoff is type-invariant, if the receiver can select a special action called the *Never-Best boundary action*, lexicographically undominated strategy profiles are perfect (Theorem 2). In short, this special action is never played in equilibrium, but its outcome significantly impacts the sender's payoff. Thus, all best choices in equilibrium remain best under the perturbation in which the outcome makes others irrelevant. These conditions hold, for example, if the receiver determines the wage of senders and the sender's payoff is additive in the wage, as in Spence (1973), provided that the feasible set of sender wages includes a wage that is so high that it is not reasonable to select.[3]

In games with uncountably many actions, although the original definition of perfect equilibria is no longer applicable, lexicographically undominated strategy profiles can still be discussed without modifying their definition. Existing work has extended the definition of perfection to normal-form games with uncountably many strategies by proposing multiple adaptations of Selten's (1975) definition and comparing the resulting refinements.[4] Our results offer an alternative route for discussing "perfect equilibrium" in games with uncountably many actions via lexicographic domination.[5]

---

[1] We can find a similar discussion about normal-form perfectness in Okada (1988) and quasi-perfectness in Asheim and Perea (2005).

[2] Because short games can involve more than two players, Theorem 1 differs from Okada's (1988) Theorem 3.7.

[3] See the discussion in the Appendix.

[4] For example, Simon and Stinchcombe (1995), Carbonell-Nicolau (2011a,b,c) and Bajoori et al. (2013).

[5] In the Appendix, we demonstrate "perfect equilibrium" in short games with uncountably many actions via lexicographic domination.

2. Finite Extensive-Form Games

In this section, we first formulate a standard finite extensive-form game. Then, we introduce perfect equilibria in Selten (1975) and lexicographic domination in Okada (1991). Next, we show our main result that explains the relationship between lexicographic domination and perfect equilibrium in short games.

An n-player finite perfect-recall extensive-form game $\Gamma = (K, P, U, p, h)$ consists of the following five elements: the game tree K including the finite set of non-terminal nodes X , the player partition $P = (P_0, ..., P_n)$ on X, the information partition $U = (U_0, ..., U_n)$ on X, nature's probability distribution p, and a payoff function h. K consists of X including the origin Ø, terminal nodes in the finite set Z, and directed links between nodes. A_u represents the set of choices at each information set u. We call each sequence of nodes from the origin Ø to $z \in Z$ a play. We call each sequence of nodes from $x \in X$ to $x' \in X$ a path. In each play, a node from each information set can appear at most once. Player 0 (nature) follows an exogenous completely mixed probability distribution p_u over A_u at each information set $u \in U_0$ such that $p_u(a_u) > 0$ for all $a_u \in A_u$. The information set u is a singleton if $u \in U_0$. The payoff function h: $Z \to \mathbb{R}^n$ represents the players' payoff from $z \in Z$.

We denote a probability distribution b_{iu} over A_u as a local strategy for player $i \in I$ at an information set $u \in U_i$. b/b'_{iu} is a strategy profile following b'_{iu} at u and all other local strategies remain unchanged from b.

For each strategy profile b, the realization probability of each $x \in X$ and $z \in Z$, denoted by $\rho(x,b)$ and $\rho(z,b)$, is uniquely determined. The realization probability of an information set u is $\rho(u,b) = \sum_{x \in u} \rho(x,b)$. $\rho(z,b|x)$ is the conditional realization probability of $z \in Z$ after the players reached x. If $x \in X$ is not in the path from Ø to $z \in Z$, then $\rho(z,b|x) = 0$.

A local belief $\rho_u(x)$ represents the probability of each node $x \in u \in U_i$ that the player i believes at u. For any $i \in I$, a belief ρ is a function from $u \in U_i$ to a local belief ρ_u.

Definition 1 *A strategy profile $b \in B$ is completely mixed if $b_{iu}(a_u) > 0$ for all $a_u \in A_u$, for all $u \in U_i$ and for all $i \in I$.*

When $b \in B$ is completely mixed, the realization probability of $x \in u$ and the belief, $\rho_u(x) = \rho(x,b)/\rho(u,b)$, at each information set are uniquely determined. Such a belief is called a *consistent* belief with b. For each sequence of completely mixed strategy profiles $b^j \to b$, the unique sequence of beliefs exists and includes a convergent subsequence because the sequence is in a compact space (Bolzano-Weierstrass Theorem). Hereafter, we consider such a subsequence and skip this explanation.

The ex-ante expected payoff vector $H(b) = (H_1(b), ..., H_n(b))$ is

$$H(b) = \sum_{z \in Z} \rho(z,b)h(z). \tag{1}$$

The expected payoff vector at an information set u is

$$H(b,u|\rho) = \sum_{x \in u} \rho_u(x) \sum_{z \in Z} \rho(z,b|x)h(z). \tag{2}$$

When the consistent belief is unique,[6] we sometimes denote $H(b,u)$ instead of $H(b,u|\rho)$.

[6] For example, if b is a completely mixed strategy profile, the consistent belief is unique.

The following representation of perfect equilibrium is useful for comparison.[7]

Definition 2 *A strategy profile b is a perfect equilibrium if there exists a sequence $b^m \to b$ such that b^m is a completely mixed strategy profile for all $i \in I$, for all $u \in U_i$ and for all $b'_{iu} \in B_{iu}$,*

$$H_i(b^m/b_{iu}, u) \geqq H_i(b^m/b'_{iu}, u). \tag{3}$$

Hereafter, the set of all perfect equilibria $b \in B$ in Γ is denoted by $PE(\Gamma)$.

We refer to Okada (1991)'s *lexicographically undominated strategy profile*.[8]

Definition 3 (Definition 2.2 of Okada (1991)) *Fix $b \in B$. We say that b_{iu} lexicographically dominates b'_{iu} if and only if there exists a neighborhood $O \subset B$ of b such that*

$$H_i(b''/b_{iu}) \geqq H_i(b''/b'_{iu}) \tag{4}$$

for any strategy profile $b'' \in O$, and the inequality is strict for some $b'' \in O$.

A strategy profile b is called a lexicographically undominated strategy profile when each b_{iu} is not lexicographically dominated by any $b'_{iu} \in B_{iu}$ for any $i \in I$ and at any $u \in U_i$. Unlike perfect equilibria, a lexicographically undominated strategy profile does not require players to share a similar idea about future errors.

To analyze the relationship between perfect equilibria and lexicographically undominated strategy profiles, we utilize the following lemmas. For ease of exposition, we collect the proofs of all results in the Appendix.

Lemma 1 *If Γ has only two information sets, and if every path in Γ passes through both information sets, then for any $b, b'' \in B$, any $i \in I$, any $u \in U_i$ and any $\varepsilon \in (0,1)$,*

$$H((1-\varepsilon)b + \varepsilon b''/b_{iu}, u) = (1-\varepsilon)H(b/b_{iu}, u) + \varepsilon H(b''/b_{iu}, u). \tag{5}$$

Under certain conditions, each player's payoff is linear in the probabilities assigned to actions within each local strategy.[9] This is useful for understanding the substitutability among strategic choices. Although this linearity does not hold in many situations including short games, the following result is proven through this linearity.

Lemma 2 *When each path includes at most two decision nodes in Γ, if b_{iu} is not lexicographically dominated in b, there exists a sequence of completely mixed strategy profiles $b^k \to b$ such that for any $b'_{iu} \in B_{iu}$,*

$$H_i(b^k/b_{iu}, u) \geqq H_i(b^k/b'_{iu}, u). \tag{6}$$

From Lemmas 1 and 2, we obtain the following relationship in short games.

[7] Selten (1975) defines perfect equilibrium as a limit of Nash equilibria in a sequence of perturbed agent-normal-form games converging to the original agent-normal-form game. Our definition is equivalent to Selten's (1975) original definition.

[8] Okada (1991) calls strategy profiles strategy combinations, but we call them profiles here because this terminology is more commonly used today.

[9] We discuss this linearity in more general settings in our ongoing parallel project, Jinushi (2024).

Theorem 1 *When each path includes at most two decision nodes in Γ, a lexicographically undominated strategy profile is a perfect equilibrium (and vice versa[10]).*

In the Appendix, we present a simple application that illustrates the usefulness of this theorem in a setting with uncountably many actions.

3. Implication for Signaling Games

Signaling games are an important class of incomplete information games. We consider the following simple signaling game: Nature draws t from T according to $p_{\emptyset}$. One sender, denoted by 1, observes nature's action t and decides b_{1,u_t} on S at the information set u_t. One receiver, denoted by 2, observes the sender's signal s and decides b_{2,u_s} on A_r. T, S and A_r are finite sets.

Signaling games involve three decision nodes along each path and lie outside of Theorem 1. Then, under what conditions are lexicographically undominated strategy profiles equivalent to perfect equilibria in signaling games? One natural question is what happens when the receiver's action has a type-invariant marginal impact on sender's payoff. We formalize this requirement as follows:

Definition 4 (Type-Invariance of Receiver's Action on Sender's Marginal Payoffs) *For any pair of information sets after $t, t' \in T$ and for all strategy profiles $b, b' \in B$ satisfying*

$$b_{1,u_t}(s) = b_{1,u_{t'}}(s) \text{ for all } s \in S, \tag{7}$$

the following equality holds:

$$H_1(b, u_t) - H_1(b'/b_{1,u_t}, u_t) = H_1(b, u_{t'}) - H_1(b'/b_{1,u_{t'}}, u_{t'}) \tag{8}$$

where $H_1(b/b_{1,u_t}, u_t)$ denotes the sender's expected payoff at u_t when the sender at the information set after t follows b_{1,u_t} and players at all the other information sets follow local strategies in b.

To preview our main finding, the type-invariance condition alone is almost sufficient for the equivalence between perfect equilibria and lexicographically undominated strategy profiles. However, it fails to guarantee the equivalence only when, in equilibrium, the receiver utilizes an action that is best for senders at one information set and an action that is worst for senders at another.[11] Thus, if either the best or the worst action is never the receiver's best response, the type-invariance condition guarantees the equivalence. We formalize this condition as follows:

Definition 5 (Never-Best (NB) Action) *We call a receiver's action $a \in A_r$ a Never-Best action if, for any information set after $s \in S$ and for any completely mixed strategy profile $b \in B$, there exists $b' \in B$ such that $b'_{2,u_s}(a) = 0$ and*

$$H_2(b, u_s) < H_2(b/b'_{2,u_s}, u_s) \tag{9}$$

[10] Okada (1991) shows that every perfect equilibrium is lexicographically undominated.

[11] See the proof of Theorem 2 in the Appendix. In short, if either the best or worst action is not utilized, all senders can focus on future errors which put the same marginal impact on each signal.

where $H_2(b/b'_{2,u_s}, u_s)$ denotes the receiver's expected payoff at u_s when the receiver at the information set after s follows b'_{2,u_s} and players at all the other information sets follow local strategies in b.

Definition 6 (Boundary Action) *We call a receiver's action $a \in A_r$ a* boundary *action if there exists $t \in T$ such that, for any receiver's information set induced by signal $s \in S$ and whenever $b'_{2,u_s}(a) = 1$, we have either, for all completely mixed strategy profiles $b \in B$*

$$H_1(b, u_t) < H_1\left(b/b'_{2,u_s}, u_t\right), \tag{10}$$

or for all completely mixed strategy profiles $b \in B$,

$$H_1(b, u_t) > H_1\left(b/b'_{2,u_s}, u_t\right). \tag{11}$$

We say that a game satisfies the NB boundary-action condition if A_r includes a NB action that is a boundary action.

Formally restating our earlier intuition using these definitions, we obtain the following:

Theorem 2 *In a simple signaling game satisfying both the type-invariance condition and the NB boundary-action condition, lexicographically undominated strategy profiles are perfect.*[12]

In simple signaling games under the invariance condition, adding one extra receiver action that is dominated for the receiver is enough to apply Theorem 2. Put differently, the logic behind perfect equilibria and lexicographically undominated strategy profiles coincides when the receiver has a prohibitively costly action that generates an extreme payoff for the sender. We conjecture that the presence of such salient outside options can simplify strategic reasoning in more general settings.

A. Appendix

A.1 Proof of Lemma 1

Proof: For the player i such that $\emptyset \in U_i$ (if $i \neq 0$), H_i at $\emptyset$ from each choice depends on only the local strategy at the following node. At the following information set u, for player j such that $u \in U_j$, H_j from each choice is linear in ρ_u, and ρ_u is linear in $b_{i,\emptyset}$ (or $p_\emptyset$ if $i = 0$) because the realization probability of u is unchanged.

A.2 Proof of Lemma 2

For players $i, i' \in I$[13] and each pair $\emptyset \in U_i$ and $u \in U_{i'} \setminus \{\emptyset\}$, we consider a two-player agent-normal-form game $\Gamma'(\emptyset, u)$ such that a pure-strategy set is identical to a choice set $S_i = \{a_\emptyset \in A_\emptyset | \exists x \in u$ such that $a_\emptyset$ induces x in the game tree $K\}$ and $S_{i'} = A_u$ and the payoff from each outcome is identical.

[12] In the Appendix, we provide an example illustrating the importance of the NB boundary-action condition.

[13] If either player is nature (player 0), we can apply similar logic. This is because nature just follows p_u and does not change its behavior in the sequence. For simplicity, we skip the possibility.

Consider $b_{i',u}$ at u such that $\rho(u,b) > 0$. Then, there exist a player i's strategy σ_i such that $\sigma_i(s_i') = b_{i\emptyset}(s_i')/\sum_{s_i \in S_i} b_{i\emptyset}(s_i)$ for any $s_i' \in S_i$ and a player i''s strategy $\sigma_{i'} = b_{i'u}$. Since $b_{i'u}$ is not lexicographically dominated in the neighborhood of b in Γ, and $A_\emptyset \setminus S_i$ does not have any impact on $H_{i'}$ in u, the player i''s mixed strategy $\sigma_{i'}$ is not lexicographically dominated at σ. Thus, $\sigma_{i'}$ is not weakly dominated (hereafter dominated). In a two-player normal-form game, it is known that, if $\sigma_{i'}$ is not dominated, there exists a completely mixed $\hat{\sigma}_i$ making $\sigma_{i'}$ optimal (For example, see Appendix B of Pearce (1984)). Because of Lemma 1, for any $\varepsilon \in (0,1)$, $\sigma_{i'}$ is optimal for $(1-\varepsilon)\sigma_i + \varepsilon\hat{\sigma}_i$. Therefore, there exists a sequence of completely mixed strategy profiles $\sigma^k \to \sigma$ that makes player i''s strategy $\sigma_{i'}$ optimal. Any sequence $b^k_{i\emptyset}$ which satisfies $\sigma^k_i(s_i) = b^k_{i\emptyset}(s_i)/\sum_{s_i' \in S_i} b^k_{i\emptyset}(s_i')$ for all $s_i \in S_i$ satisfies the requirement in Lemma 2 for u such that $\rho(u,b) > 0$.

Second, consider an unreached information set u such that $\rho(u,b) = 0$. Since $b_{i'u}$ is not lexicographically dominated and $A_\emptyset \setminus S_i$ does not have any impact on $H_{i'}$ in u, the player i''s mixed strategy $\sigma_{i'}$ is not dominated in $\Gamma'(\emptyset,u)$. Then, there exists a completely mixed strategy $\hat{\sigma}$ which justifies $\sigma_{i'}$. Therefore, any sequence $b^k_{i\emptyset}$ which satisfies $\sigma_i(s_i') = b^k_{i\emptyset}(s_i')/\sum_{s_i \in S_i} b^k_{i\emptyset}(s_i)$ for all $s_i \in S_i$ satisfies the requirement in Lemma 2 for u such that $\rho(u,b) = 0$.

Third, for the optimality of $b_{i\emptyset}$, we consider a two-player normal-form game $\Gamma'(\emptyset)$ involving an agent of player i at $\emptyset$ and an incomplete dictator D who selects the combination of local strategy at any $u \in U \setminus \{\emptyset\}$. There exist at most two decision nodes in each path, and so there is a bijection from a mixed strategy of the dictator in $\Gamma'(\emptyset)$ to the combination of $b_{i'u}$ in Γ both of which give an identical outcome distribution against $b_{i\emptyset}$. σ_D is a mixed strategy that represents a combination of $b_{i'u}$ for all pair of $i' \in I$ and $u \in U_{i'} \setminus \{\emptyset\}$. Since $b_{i\emptyset}$ is lexicographically undominated in Γ, the player i's (unique) mixed local strategy in $\Gamma'(\emptyset)$ corresponding to $b_{i\emptyset}$ is lexicographically undominated, and so $b_{i\emptyset}$ is not dominated. From Pearce (1984) and Lemma 1, there exists a sequence of completely mixed strategy profiles $\sigma^k_D \to \sigma_D$ in which the player i's mixed strategy is optimal in $\Gamma'(\emptyset)$. Therefore, by using the bijection, there exists a sequence of completely mixed strategy profiles b^k that makes $b_{i\emptyset}$ optimal.

A.3 Proof of Theorem 1

Consider a lexicographically undominated strategy profile b.

In the following discussion, we assume that player 1 makes a decision at $\emptyset$.[14] In the following proof, $ac(x)$ denotes the unique action $a_\emptyset \in A_\emptyset$ that induces $x \in X$ in the game tree K. $ac(z)$ denotes the unique action $a_\emptyset \in A_\emptyset$ such that $a_\emptyset$ induces $z \in Z$ in the game tree K.

First, we focus on $\emptyset$. Since $b_{1,\emptyset}$ is lexicographically undominated, there exists a sequence of completely mixed strategy profiles $b^k \to b$ such that $H_1(b^k/b_{1,\emptyset}) \geqq H_1(b^k/b'_{1,\emptyset})$ for any $b'_{1,\emptyset} \in B_{1,\emptyset}$ (Lemma 2). Since $H_i(b^k/b'_{1,\emptyset})$ depends only on b^k_{iu} such that $u \in U \setminus \{\emptyset\}$ and does not depend on $b^k_{1,\emptyset}$, $b^k_{1,\emptyset}$ is optimal for any $b^k/b''_{1,\emptyset}$ where $b''_{1,\emptyset} \in B_{1,\emptyset}$. Throughout the rest of this proof, without further explanation, we consider a sequence of completely mixed b^k such that $H_1(b^k/b_{1,\emptyset}) \geqq H_1(b^k/b'_{1,\emptyset})$ for any $b'_{1,\emptyset} \in B_{1,\emptyset}$.

Second, we focus on $u \in U \setminus \{\emptyset\}$ and $i \in I$ such that $u \in U_i$. $H_i(b^k/b'_{iu}, u|\rho^k)$ depends on

[14] If the initial player is nature (player 0), we can apply similar logic. Nature's action is governed by the fixed distribution p_u and hence does not vary along the perturbation sequence. We therefore omit this case for simplicity.

ρ_u^k which is consistent with $b^k_{1,\emptyset}$. Since each b_{iu} is not lexicographically dominated, there exists a sequence (b^k, ρ^k) such that $b^k \to b$ and $\rho^k \in CO(b^k)$ which justifies b_{iu}. Then, there exists a sequence of the ratios $b^k_{1,\emptyset}(ac(x))/\rho(u, b^k)$ for each $x \in u$. $SR(k,u,x)$ denotes the ratio.

Consider a sequence of completely mixed strategy profiles and beliefs $(b'^k, \rho'^k) \to (b, \rho)$ where $\rho'^k \in CO(b'^k)$. Then, from the sequence, we can calculate a sequence of the non-zero realization probabilities for each $u \in U \setminus \{\emptyset\}$ which converges to $\rho(u,b)$. Denote the realization probability of u at the kth element in the sequence by $\alpha(k,u)$. Since each choice is connected to a unique node and a unique information set (if the following information sets exist), we can disjointly decide the ratio of each choice at Ø with which the sequence makes the realization probability converge to $\rho(x,b)$ and the ratio for the optimality is satisfied by the following way: For each $x \in u$, we decide $b^k_{1,\emptyset}(ac(x))$ such that $b^k_{1,\emptyset}(ac(x))/\alpha(k,u) = SR(k,u,x)$. Then, we get $b^k_{1,\emptyset} \to b_{1,\emptyset}$ and each b_{iu} is optimal to $b^k_{1,\emptyset}$. For any $i' \in I$ and $u \in U_{i'} \setminus \{\emptyset\}$, $b^k_{iu} = b'^k_{iu}$ and so $b_{i,\emptyset}$ is optimal in the sequence. For any choice at Ø connected to an outcome $z \in Z$, we set $b^k_{1,\emptyset}(ac(z)) = b'^k_{1,\emptyset}(ac(z))$. The constructed b^k satisfies $b^k \to b$, and each b_{iu} is optimal in the sequence, so b is perfect.

A.4 Implication for Games with Uncountably Many Actions

Unlike perfect equilibria based on completely mixed strategy profiles, we can apply lexicographic domination to games with uncountably many actions. By Theorem 1, when we consider a short game with uncountably many actions, if each local strategy is lexicographically undominated, we can interpret this strategy profile as a “perfect equilibrium”.

Consider a short game with three players. Player 1 selects a number $a_{u1} \in A_{u1} = [2,3] \cup [4,5]$, and then if $a_{u1} \leqq 3$, Player 2 selects a number $a_{u2} \in A_{u2} = [2,3]$, and otherwise Player 3 selects a number $a_{u3} \in A_{u3} = [4,5]$. Player 2 and 3 observe Player 1’s choice before making their decision.

When Player 1 and the subsequent player select a unique number, all players obtain 1. If either Player 1 or the next player selects a noninteger, and if the opponent selects an integer, the former obtains 0, and the other players obtain 1. Otherwise, all players obtain 0.

Since any integer lexicographically dominates any noninteger, each player can select only integers in each lexicographically undominated strategy profile. Player 1 can choose any local strategies selecting only integers. After Player 1’s integer, Player 2 or 3 selects the number Player 1 selected. In the other information sets, Player 2 and 3 can select any local strategy selecting only integers. This approach rejects uncountably many other subgame-perfect equilibria consisting of vulnerable nonintegers.

A.5 Proof of Theorem 2

For the sake of contradiction, suppose that a lexicographically undominated strategy profile b is not perfect. This implies that, by Definition 2, at least one b_{iu} is strictly inferior in each completely mixed strategy profile around b. Because b_{iu} is not lexicographically dominated, by considering agent-dictator games as in Appendix A.2, we can find a sequence of completely mixed strategy profiles converging to b under which b_{iu} is optimal. Thus, due to linearity, when we consider a combination consisting of such a sequence for each local strategy, our initial assumption implies that at least a conflict[15] arises among these sequences.

Such a conflict cannot occur between a sender's local strategy and a receiver's local strategy. This is because each player's payoff depends only on the opponents' local strategies. Therefore, we discuss the possibility of conflicts separately within the local strategies of each role.

Such a conflict cannot occur among receivers' local strategies. The receiver's expected payoff at each information set depends on the probability of a subset of senders' action, and there are no overlaps among the subsets. As we discussed in the initial paragraph of this proof, for each local strategy at each u, at least a sequence of completely mixed strategy profiles b^{k_u} in which the local strategy is optimal converges to b. We focus on a receiver's information set u and consider a sequence of completely mixed strategy profiles $b^{k'}$ converging to b. Because the sequence converges to b, for each $b^{k'}$, there exists b^{k_u} such that the realization probability of all $x \in u$ such that $\rho(x,b) = 0$ is lower than the probability in $b^{k'}$. Consider a new completely mixed strategy profile b' such that $\rho(x,b') = \rho(x,b^{k_u})$ for all $x \in u$ and the probability difference $\rho(x,b^{k'}) - \rho(x,b')$ is assigned to the sender's local strategy in such a way that the ratio of each choice is identical to the local strategy in b. Apart from this modification, b' is identical to $b^{k'}$. In b', the local strategy at u is optimal. In addition, b' is weakly closer to b than $b^{k'}$. We repeat this process for all u, and all receivers' local strategies in the information sets are optimal at the end. Thus, for any completely mixed strategy profile $b^{k'}$, we can construct another completely mixed strategy profile in which all receivers' local strategies are optimal while reducing the distance to b.

Under the invariance condition and the NB boundary-action condition, such a conflict cannot occur among sender's local strategies. When the set of lexicographically undominated choices at each sender's information set in b does not overlap, we can independently modify future errors after each sender's information set, similar to the receiver's case above.

Thus, only cases where the set of lexicographically undominated choices at each sender's information set in b shares a subset of signals are relevant. However, when the invariance condition and the NB boundary-action condition hold, by adjusting the perturbation on strategy profiles, we can make the changes in the expected payoff of local strategies identical among senders who share the lexicographically undominated signals. For example, if a game includes an NB boundary action, by assigning a relatively high probability to the action, we can make the change in the sender's payoff for each choice positive and identical. Any other effects of the receiver's choice on the sender's payoff are dominated by the possibility of the extreme action. The boundary action can play the similar role. Therefore, the initial assumption contradicts the invariance condition and the NB boundary-action condition in simple signaling games.

A.6 Signaling Games with Theorem 2

We examine when our results apply to simple signaling games. Following Spence (1973), suppose the receiver determines the wage of senders and the sender's payoff is additive in the wage.[16] Under these conditions, the type-invariance condition holds. Unlike Spence (1973), after the sender sends a signal, the receiver decides the wage. Assume $A_r \subseteq \mathbb{R}_+$ and $|A_r| \geqq 2$. In this setting, $\bar{a}$, which is the highest wage in A_r, is a boundary action.

[15] We say that one sequence conflicts with another if the two sequences are not identical. If there exists a sequence of completely mixed strategy profiles that makes every local strategy in b optimal, then no conflict arises in the present context.

[16] The sender's payoff is a function $\omega(t,s)$ plus the expected wage after sending s.

Whether an action is never-best or not depends on the receiver's payoff function. When wages are moderately higher, workers may make a greater effort, leading to increased production and thus benefiting the receiver. However, such a gain is bounded above in reality, so $\bar{a}$ is typically dominated by other choices.

The following simplified game tree[17], Game 1, is useful for understanding why we discuss an NB action. This game tree represents a simple signaling game where $T = \{H, L\}$, $A_r = \{\bar{a}, a', \underline{a}\}$ and $S = \{\bar{s}, s', \underline{s}\}$ where $\bar{a} > a' > \underline{a}$.

To understand the NB requirement, for simplicity, we suppose that the receiver is indifferent among all outcomes. When a type-t sender sends a signal s, if the receiver selects a, the sender obtains payoff $\omega(t,s) + a$ for any $t \in T$ and $s \in S$. We make the following assumptions:

$$\omega(t,\underline{s}) > \omega(t,s') > \omega(t,\bar{s}) \text{ for all } t \in T, \tag{12}$$

$$\omega(H,\bar{s}) + \bar{a} = \omega(H,s') + a' > \omega(H,\underline{s}) + \underline{a}, \tag{13}$$

and

$$\omega(L,\underline{s}) + \underline{a} = \omega(L,s') + a' > \omega(L,\bar{s}) + \bar{a}. \tag{14}$$

This game has the following lexicographically undominated strategy profile b: the type-H sender sends $\bar{s}$, the type-L sender sends s', and the receiver selects $\bar{a}$ after observing $\bar{s}$, a' after observing s', and $\underline{a}$ after observing $\underline{s}$. However, this profile b is not perfect, because either type of sender has an incentive to deviate from their prescribed strategy in any completely mixed strategy profile sufficiently close to b.

Declarations

Conflict of interest: The author certifies that the author has no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

Reference

Asheim, G.B. and Perea, A. (2005) Sequential and quasi-perfect rationalizability in extensive games. Games and Economic Behavior 53, 15–42.

Bajoori, E., Flesch, J. and Vermeulen, D. (2013) Perfect equilibrium in games with compact action spaces. Games and Economic Behavior 82, 490–502.

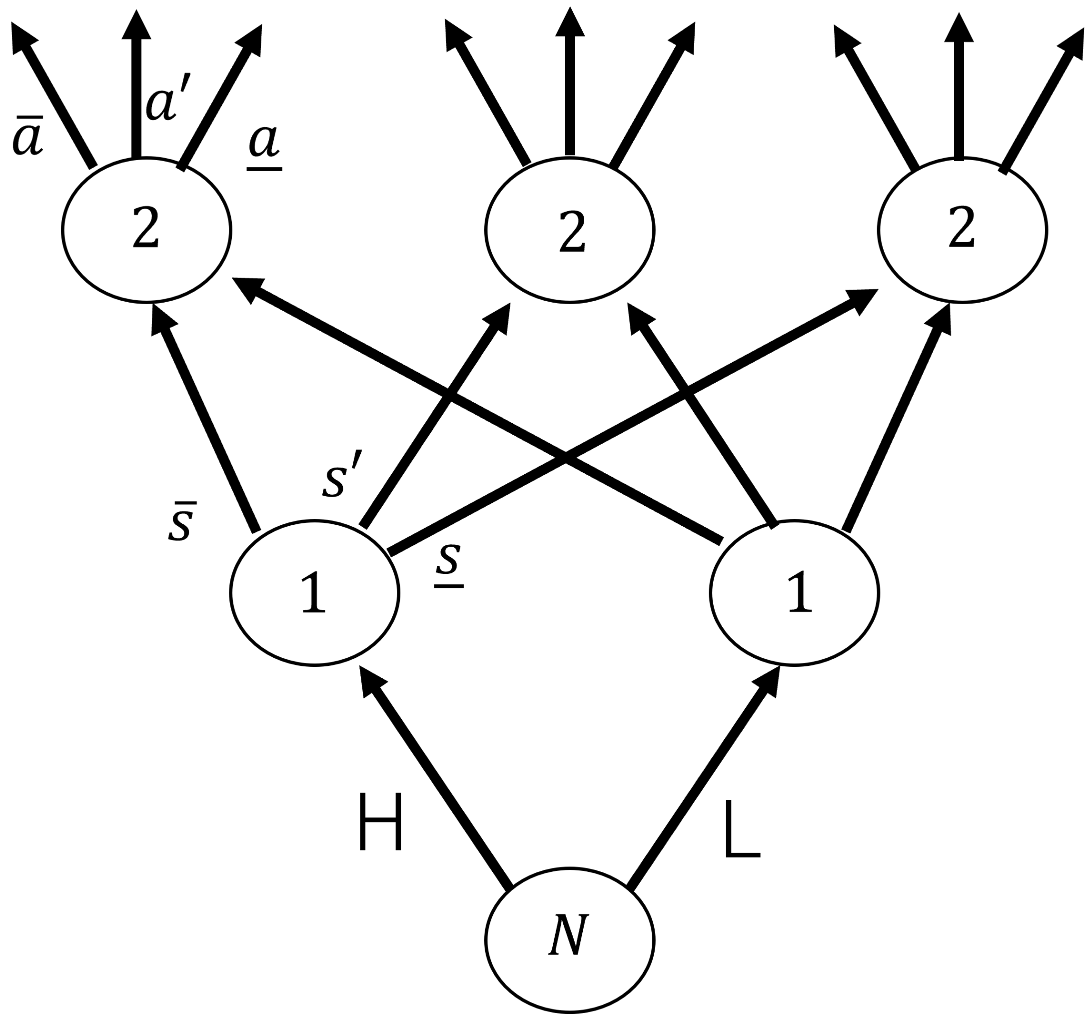
Carbonell-Nicolau, O. (2011a) The Existence of Perfect Equilibrium in Discontinuous Games. Games 2, 235–256.

Carbonell-Nicolau, O. (2011b) Perfect and limit admissible perfect equilibria in discontinuous games. Journal of Mathematical Economics 47, 531–540.

Carbonell-Nicolau, O. (2011c) On equilibrium refinement for discontinuous games. Int. Game Theory Rev. 13, 269–280.

Jinushi, R. (2024) Two Types of Perturbed Strategy Profiles. Mimeo.

[17] In Game 1, each node represents an information set. Under the standard representation, we need two nodes for each receiver's information set.



Game 1: Situations Where the NB Requirement Matters

Okada, A. (1991) Lexicographic domination in extensive games. International Journal of Game Theory 20, 95–108.

Okada, A. (1988) Perfect equilibrium points and lexicographic domination. International Journal of Game Theory 17, 225–239.

Pearce, D.G. (1984) Rationalizable Strategic Behavior and the Problem of Perfection. Econometrica 52, 1029-1050.

Selten, R. (1975) Reexamination of the perfectness concept for equilibrium points in extensive games. International Journal of Game Theory 4, 25–55.

Simon, L.K. and Stinchcombe, M.B. (1995) Equilibrium Refinement for Infinite Normal-Form Games. Econometrica 63, 1421–1443.

Spence, M. (1973) Job Market Signaling. The Quarterly Journal of Economics 87, 355–374.