Economics Bulletin

Volume 45, Issue 4

The growth effects of capital income tax-funded pension program under endogenous retirement

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Abstract

This study aims to clarify the effects of capital income tax-funded pension reform on economic growth under endogenous retirement choice. By employing a two-period overlapping-generations (OLG) model with endogenous growth, the present study examines the effects of capital income tax on endogenous retirement and economic growth. When retirement decisions are endogenous, capital income tax affects economic growth through two opposing effects: a negative effect by lowering savings and a positive effect by decreasing the elderly labor supply. If elderly labor productivity is relatively low and within a certain range, an inverted U-shaped relationship between capital income tax and economic growth may exist.

I am grateful to Young-Jae Kim, Koichi Miyazaki, and Toshiki Tamai for their helpful comments and suggestions. I would also like to thank the conference and seminar participants at the 2022 Korean Economic and Business Association (KEBA), the Economic Seminar at Nagoya City University, the 31st Economics Workshop for Young Economists, the 2023 Japanese Economic Association Spring Meeting (JEA), and the 2023 Japan Association of Applied Economics Spring Meeting (JAAE). Editor`s Note: This paper was originally submitted under a different manuscript number on 02/26/2025 and accepted for publication on 11/19/2025. A system error required us to replace the original submission with a new manuscript with a later manuscript number.

Citation: Yaqi Wang, (2025) "The growth effects of capital income tax-funded pension program under endogenous retirement", *Economics Bulletin*, Volume 45, Issue 4, pages 2120-2128
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Submitted: January 08, 2026.   **Published:** December 30, 2025.

1. Introduction

With demographic aging, many countries that adopted the pay-as-you-go (PAYG) pension system face larger pension funding to support the retired beneficiaries. To solve this problem, three fifths of OECD countries increase the normal retirement age (the age of eligibility of pension schemes), which leads elderly people to decide to work longer (OECD, 2023). However, this solution does not fundamentally resolve the pension funding issue. How to finance the increasing pension spending is still a central concern of aging countries.

Recently, the debate about whether the funding sources of the PAYG pension system should be partially changed to capital income taxation become a contested issue (Kunze and Schuppert, 2010; Tyrowicz et al., 2024). Reflecting the increasing spontaneous labor participation of the elderly and the previous discussions, the present paper aims to examine the effects of capital income tax-funded pension reform on economic growth under endogenous retirement choice.

This study employs a two-period overlapping generations (OLG) model, which characterizes a reformed pension system funded by capital income tax and labor income tax. Growth is endogenously driven by the spillover effect in the spirit of Romer (1986), and the labor supply of the elderly is endogenous. Under this setting, the main findings of the present study are that capital income tax has two opposing effects on growth: one is a negative effect via a decrease in savings, and another is a positive effect via a decrease in elderly labor supply. When the elderly labor productivity is relatively small and within a certain range, an inverted U-shaped relationship between capital income tax and growth may exist. When the elderly labor productivity is relatively large, the negative effect dominates the positive effect; thereby, growth monotonically decreases with capital income tax.

The present study is related to the literature on capital income taxation and economic growth in the endogenous growth model. Whether capital income should be taxed in the long run has been discussed for a long time. On the one hand, earlier studies showed that capital income tax depresses long-run economic growth in the AK-type endogenous growth (Chamley, 1986; Judd, 1985). On the other hand, some studies successfully showed positive results that capital income tax may enhance economic growth (Uhlig and Yanagawa, 1996; Caballe, 1998; Haruyama and Itaya, 2006; Chen et al., 2017). However, the relationship between capital income tax and economic growth when using capital income tax to finance pension is omitted from these analyses.

One notable exception is Kunze and Schuppert (2010), who showed that this reform may stimulate economic growth by focusing on unemployment in an overlapping generations model with a wage bargaining process and a social security system combining pensions and unemployment benefits. Different from the mechanism assumed by Kunze and Schuppert (2010), the present study features the growth effect of a capital income tax-funded pension program through endogenous retirement choice from individuals instead of the unemployment rate determined from the wage bargaining process.

This work is also related to the literature on endogenous retirement choice and social security. In particular, Chen and Miyazaki (2020) pointed out a hump-shaped relationship between elderly agents' labor productivity and economic growth under endogenous retirement. However, they omitted social security and tax policies, whereas the present paper concentrates on the growth effects of capital income tax-funded social security.

The rest of this paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the balanced growth equilibrium and investigates the effects of capital income tax on endogenous retirement. Section 4 provides the main results, and Section 5 concludes.

2. Model

We consider a two-period OLG model, which is an extension of the seminal work of endogenous retirement choice in the OLG model from Hu (1979). We extend the model by introducing a capital income tax-funded pension system with endogenous growth. The population in period t is N_t and grows at a constant rate $n(> 1)$, that is, $N_{t+1} = nN_t$.

2.1 Individuals

Individuals have the following economic behavior during two periods and make all decisions in their young adulthood. In the first period, a young agent in period t supplies labor inelastically and pays the labor income tax, τ_t. Young individuals distribute their disposable income, $(1 - \tau_t)w_t$, between consumption, c_t, and savings, s_t. A young individual's budget constraint is

$$c_t + s_t = (1 - \tau_t)w_t. \tag{1}$$

In the second period, old individuals allocate one unit of time between working and retiring. They choose l_{t+1} unit of time for labor supply and retire at the rest of time, $1 - l_{t+1}$, to receive pension benefits, b_{t+1}, where $l_{t+1} \in (0,1)$. $\theta \in (0,1]$ is the labor productivity of the elderly relative to that of the young. Individuals receive the after-tax capital income, after-tax labor income, and pension benefits after retirement to consume. The budget constraint of an old individual in period $t + 1$ is as follows:

$$d_{t+1} = (1 - \tau_{t+1}^k)R_{t+1}s_t + (1 - \tau_{t+1})w_{t+1}\theta l_{t+1} + (1 - l_{t+1})b_{t+1}, \tag{2}$$

where d_{t+1} is consumption of the old, τ_{t+1}^k is capital income tax in period $t + 1$ and R_{t+1} is the gross interest rate in period $t + 1$.

An individual's lifetime utility function in period t is given by

$$U(c_t, d_{t+1}, l_{t+1}) = \ln c_t + \beta \ln d_{t+1} + \gamma \ln(1 - l_{t+1}), \tag{3}$$

where $\beta > 0$ and $\gamma > 0$ are preference parameters for the elderly consumption and leisure of the elderly, respectively. Solving the maximization problem, we obtain the consumption, elderly labor supply and saving functions as follows:

$$c_t = \frac{1}{1+\beta+\gamma}\left[(1 - \tau_{t+1})\frac{\theta w_{t+1}}{(1-\tau_{t+1}^k)R_{t+1}} + (1 - \tau_t)w_t\right], \tag{4}$$

$$s_t = \frac{1}{1+\beta+\gamma}\left[(\beta + \gamma)(1 - \tau_t)w_t - \frac{(1-\tau_{t+1})\theta w_{t+1}}{(1-\tau_{t+1}^k)R_{t+1}}\right], \tag{5}$$

$$l_{t+1} = 1 - \frac{\gamma}{1+\beta+\gamma} \frac{(1-\tau_{t+1})w_{t+1}\theta + (1-\tau_t)w_t R_{t+1}(1-\tau_{t+1}^k)}{(1-\tau_{t+1})w_{t+1}\theta - b_{t+1}}, \quad (6)$$

$$d_{t+1} = \frac{\beta}{1+\beta+\gamma}[(1-\tau_{t+1})\theta w_{t+1} + (1-\tau_t)w_t R_{t+1}(1-\tau_{t+1}^k)]. \quad (7)$$

2.2 Firms

A continuum of identical competitive firms exists. Each firm inputs capital and labor to produce homogeneous goods. The production function for each firm is given by

$$Y_t = AK_t^\alpha (M_t L_t)^{1-\alpha}, \quad (8)$$

where Y_t is aggregate output, $A(>0)$ is total factor productivity, and $\alpha \in (0,1)$ is capital share. K_t and L_t are capital and labor, respectively. M_t is labor-augmenting technology from the knowledge of workers. w_t is the wage rate, and R_t is the gross rate of return to capital. The first-order conditions for profit maximization are

$$R_t = \alpha A K_t^{\alpha-1}(M_t L_t)^{1-\alpha},$$
$$w_t = (1-\alpha)AK_t^\alpha (M_t)^{1-\alpha}(L_t)^{-\alpha},$$

respectively. We adopt a Romer-type spillover effect that generates an AK-type endogenous growth.[1] Labor-augmenting technology is defined as $M_t = K_t/L_t$ (Arrow, 1962; Romer, 1986; Grossman and Yanagawa, 1993), where labor-augmenting technology is given by aggregate knowledge per worker. Under the specification, the factor prices are given as follows:

$$w_t = (1-\alpha)A\frac{K_t}{L_t}, \quad (9)$$
$$R_t = \alpha A. \quad (10)$$

2.3 Government

The government finances pensions by taxing labor income, $w_t L_t$, and capital income, $R_t s_{t-1} N_{t-1}$. The budget constraint of the government in period t is

$$\tau_t w_t L_t + \tau_t^k R_t s_{t-1} N_{t-1} = N_{t-1}(1-l_t)b_t, \quad (11)$$

where $N_{t-1}(1-l_t)b_t$ is aggregate public pension expenditure.

The market-clearing conditions for labor and capital are

$$L_t = N_t + N_{t-1}\theta l_t, \quad (12)$$

and

$$K_{t+1} = N_t s_t. \quad (13)$$

Denoting $k_t \equiv K_t/L_t$, the capital market-clearing condition can be rewritten as

$$(n + \theta l_{t+1})k_{t+1} = s_t. \quad (14)$$

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[1] The AK-type endogenous growth setting has been used in models analyzing pensions and economic growth (Wigger, 1999; Kunze and Schuppert, 2010; Tabata, 2014; Miyazawa, 2021). Under this setting, previous studies showed that social security tax affects growth through fertility choices (Wigger, 1999), unemployment (Kunze and Schuppert, 2010), the reform from a defined benefit scheme to a defined-contribution scheme (Tabata, 2014), and grandparenting choices (Miyazawa, 2021). Different from the mechanisms shown by these works, the present study shows new insights by focusing on the mechanism through retirement choices.

3. Equilibrium analysis

Denote the output per capita by $y_t = Y_t/N_t$, and the gross growth rate of the capital–labor ratio by $g_t = k_{t+1}/k_t$. Derive gross growth rate of per capita output and we have

$$\frac{y_{t+1}}{y_t} = \frac{1+\theta l_{t+1}/n}{1+\theta l_t/n} g_t.$$

The gross growth rate of per capita output equals g_t in the balanced growth equilibrium (BGE) that elderly labor supply l is constant over time. Deriving the BGE conditions of the economy, the elderly labor supply and gross growth rate are given by

$$l_{t+1} = \frac{1}{[1-\alpha(1-\tau_{t+1}^k)]\theta}\left[Q - \frac{\gamma}{1+\beta+\gamma}\frac{(1-\tau_t)(1-\tau_{t+1}^k)\alpha A(1-\alpha)}{g_t}\right], \tag{15}$$

$$g_t = \frac{(\beta+\gamma)(1-\tau_t)(1-\alpha)A}{[(1+\beta+\gamma)(n+\theta l_{t+1})+\frac{(1-\tau_{t+1})\theta(1-\alpha)}{(1-\tau_{t+1}^k)\alpha}}. \tag{16}$$

where $Q \equiv \frac{(1+\beta)}{1+\beta+\gamma}(1-\tau_{t+1})(1-\alpha)\theta - (1-\alpha)n\tau_{t+1} - \alpha n\tau_{t+1}^k$.

Assumption 1. *Labor income tax and capital income tax are time-invariant in the long run, which is given by* $\tau_t = \tau_{t+1} = \tau$, $\tau_{t+1}^k = \tau_t^k = \tau^k$.

Proposition 1. *Suppose Assumption 1 holds. There exists a unique BGE with a constant elderly labor supply* l^* *and long-run gross growth rate* $g(>1)$ *given by*

$$l^* = \frac{\beta(1-\tau)(1-\alpha)\theta - n[(\beta+\gamma)(1-\alpha)\tau+\alpha\beta\tau^k+\alpha\gamma]}{[\beta+\gamma-\alpha\beta(1-\tau^k)]\theta}, \tag{17}$$

$$g = \frac{\alpha A(1-\tau^k)[\beta+\gamma-\alpha\beta(1-\tau^k)]}{\theta+\alpha(1-\tau^k)[n(1+\beta+\gamma)+\beta\theta]}. \tag{18}$$

To ensure that the elderly labor supply is positive, we assume $\theta > \bar{\theta}$, where $\bar{\theta} = \frac{(\beta+\gamma)(1-\alpha)n\tau+\alpha\beta n\tau^k+\alpha\gamma n}{\beta(1-\tau)(1-\alpha)}$. This condition means that an old agent decides to work when their labor productivity is higher than a certain level $\bar{\theta}$. To check if $\bar{\theta}$ is in a feasible range that is smaller than 1, I take a numerical example. Under the setting of $\alpha = 0.3, \beta = (0.99)^{30}$, $\gamma = 0.1, n = 1.2$, $\tau = 0.1$, $\tau^k = 0.1$, we have $\bar{\theta} \approx 0.29 < 1$. Thus, $\bar{\theta}$ lies within a feasible range. I also provide an intuition behind this condition. In general, $\bar{\theta}$ increases with a higher labor income tax rate, τ, a higher capital income tax rate, τ^k, and a higher population growth rate, n. Intuitively, higher income tax rates and faster population growth increase the burden on the working generation, which in turn raises the minimum level of labor productivity required for the elderly to work. Thus, in such an economy, the range of $\bar{\theta}$ within 0 to 1 becomes smaller, which implies a narrower region of labor productivity in which elderly labor supply remains positive.

Next, we consider the effects of elderly labor productivity on elderly labor supply. Differentiating Equation (17) with respect to θ yields

$$\frac{\partial l^*}{\partial \theta} = \frac{n[(\beta+\gamma)(1-\alpha)\tau+\alpha\beta\tau^k+\alpha\gamma]}{[\beta+\gamma-\alpha\beta(1-\tau^k)]\theta^2} > 0. \tag{19}$$

Lemma 1. *The elderly labor supply increases as the elderly labor productivity increases.*

Equation (19) implies that a rise in θ increases the labor supply of the elderly. A higher θ increases the labor income of the elderly, which leads to a decrease in elderly labor supply. At the same time, a higher price for leisure (retirement) motivates individuals to reduce leisure and increase labor supply. Finally, the second effect dominates the first effect. Thus, a rise in θ increases the elderly labor supply.

Then, we examine the effects of labor income and capital income taxes on equilibrium elderly labor supply in the BGE. Differentiating the elderly labor supply in Equation (17) with respect to τ, and τ^k, respectively, we have

$$\frac{\partial l_{t+1}^*}{\partial \tau} = -\frac{(1-\alpha)[\beta\theta+(\beta+\gamma)n]}{[\beta+\gamma-\alpha\beta(1-\tau^k)]\theta} < 0, \tag{20}$$

$$\frac{\partial l_{t+1}^*}{\partial \tau^k} = -\frac{\alpha\beta(1-\alpha)(1-\tau)[\beta\theta+(\beta+\gamma)n]}{[\beta+\gamma-\alpha\beta(1-\tau^k)]^2\theta} < 0. \tag{21}$$

Lemma 2. *Increases in labor income and capital income tax rates reduce the elderly labor supply.*

When capital income tax rates increase, individuals tend to reduce their savings and increase their consumption and leisure in old age.[2] Therefore, a rise in capital income tax rates increases leisure in old age and decreases elderly labor supply. At the same time, a higher capital income tax rate lowers after-tax capital income, which induces the elderly to continue to work. The first effect dominates the latter effect, implying an increase in capital income tax reduces elderly labor supply.

Next, we consider the effect of labor income tax on elderly labor supply. On the one hand, a rise in labor income tax rates lowers the price of leisure (retirement), which incentivizes elderly individuals to enjoy more leisure and retire earlier. On the other hand, the increased labor income tax rate decreases the after-tax labor income and savings, making the elderly choose to retire later. Finally, the first effect dominates the second effect. Hence, a rise in labor income tax decreases elderly labor supply.

4. Results

In the present economy, capital income tax affects economic growth through two effects: a direct effect by lowering savings and an indirect effect through a decrease in elderly labor supply. An increase in capital income tax rates directly lowers savings and discourages capital accumulation, implying a negative effect on growth. On the other hand, higher capital income tax rates stimulate early retirement (Lemma 2), and then a decline in elderly labor supply facilitates capital accumulation in the next period $k_{t+1} = s_t/(n+\theta l_{t+1})$, accelerating economic growth.

[2] This effect of capital income tax is also mentioned as a consumption effect in Chen et al. (2017).

Therefore, there is a negative effect through a decrease in savings and a positive effect through a decrease in elderly labor supply. The growth effect of capital income tax depends on these two opposing effects.

Differentiating the equilibrium gross growth rates in Equation (18) with respect to τ^k obtains

$$\frac{dg}{d\tau^k} \gtreqless 0 \text{ if } \tau^k \lesseqgtr \widehat{\tau^k}, \text{ where } \widehat{\tau^k} = 1 - \sqrt{\frac{\theta(\beta+\gamma)}{\alpha^2\beta[n(1+\beta+\gamma)+\beta\theta]}}.$$

Proposition 2. *There is a positive growth-maximizing capital income tax rate* $\widehat{\tau^k} \in (0,1)$ *in the BGE if* $\bar{\theta} < \theta < \frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2}$.

Since $\bar{\theta}$ is capital income tax dependent, to guarantee the elderly labor supply is positive when the growth-maximizing capital income tax rate is achieved, we assume $\widehat{\tau^k} < \tau^k$. When $\widehat{\tau^k} < \tau^k$, $\bar{\theta}\big|_{\tau^k=\widehat{\tau^k}} < \theta$ holds, and the economy remains positive elderly labor supply.

Proposition 2 implies an inverted U-shaped relationship between capital income tax and the growth rate of per capita output when elderly labor productivity $\theta \in (\bar{\theta}, \frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2})$. As we mentioned above, capital income tax has a negative effect on growth through a decrease in savings and a positive effect on growth through a decrease in elderly labor supply. The intuition behind this result is that when the capital income tax rate is low, its indirect effect on growth by reducing elderly labor supply is possibly larger than its direct effect by reducing savings; however, when the capital income tax rate is high, the direct saving reducing effect becomes larger than the indirect effect. In particular, the direct saving reduction effect (negative effect) more strongly works in the market, dominating the indirect effect under higher capital income tax rates.

Lemma 3. *The growth rate of per capita output decreases with the capital income tax rate when* $\theta \geq \frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2}$.

If θ is equal to or larger than $\frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2}$, the growth-maximizing capital income tax turns to zero. This means that the negative growth effect dominates the positive growth effect for all $\tau^k \in (0,1)$ when $\theta \geq \frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2}$. We already know that elderly labor supply increases with elderly labor productivity according to Lemma 1. Higher θ leads to higher l, which hinders capital accumulation $k_{t+1} = s_t/(n+\theta l_{t+1})$ and therefore weakens the elderly labor supply reducing effect (positive effect) of τ^k, finally resulting in a negative effect of capital income tax on growth.

Moreover, to confirm whether this lower bound level lies below 1, I assume parameters as $\alpha = 0.3, \beta = (0.99)^{30}$, $\gamma = 0.1, n = 1.2$, and obtain $\frac{\alpha^2\beta n(1+\beta+\gamma)}{\beta+\gamma-\alpha^2\beta^2} \approx 0.19 < 1$. Under the reasonable parameter value settings, the lower bound level of θ in Lemma 3 is in an appropriate range.

4.1 Discussions on the growth effects of labor income tax

Labor income tax rate affects growth through its effects on savings and elderly labor supply. However, these two opposing effects are completely cancelled out under Assumption 1 in the long run. To explain this neutrality, recall the Equation (16) and consider an economy in which Assumption 1 does not hold:

$$g_t = \frac{(\beta+\gamma)(1-\tau_t)(1-\alpha)A}{(1+\beta+\gamma)\left(n+\theta l^*_{t+1}(\tau_{t+1},\tau^k_{t+1})\right)+\frac{(1-\tau_{t+1})\theta(1-\alpha)}{\left(1-\tau^k_{t+1}\right)\alpha}} \equiv g\left(\tau_t,\tau_{t+1},\tau^k_{t+1}\right),$$

where

$$\frac{\partial g_t}{\partial \tau_t} < 0, \quad \frac{\partial g_t}{\partial \tau_{t+1}} > 0.$$

Based on Equation (16), the per capita output growth rate is negatively affected by τ_t and positively affected by τ_{t+1}. For the capital income taxation, the per capita output growth rate is affected only by τ^k_{t+1}. An increase in τ_t decreases s_t, which lowers k_{t+1}; at the same time, a higher τ_t decreases l_t and increase s_{t-1}, which increase k_t, reducing the long-run growth rate. On the other hand, an increase in τ_{t+1} decreases l_{t+1} and increases s_t, which increases k_{t+1} and does not affect k_t, leading to a higher long-run growth rate. If τ_t and τ_{t+1} simultaneously increase, the effects of τ on k_{t+1} and k_t are canceled out. For the capital income taxation, an increase in τ^k_{t+1} decreases s_t and l_{t+1}, thereby determines only k_{t+1}. Consequently, the growth effect of τ^k depends on these two opposing effects on k_{t+1} and the growth effect of τ is neutral.

5. Conclusion

This paper considers a reformed pension system financed by capital income tax and labor income tax. We investigate the effects of capital income tax and labor income tax on endogenous retirement and economic growth. We find that when retirement is endogenous, an inverted-U-shaped relationship between capital income tax and growth may exist if elderly labor productivity is relatively low.

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