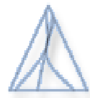 Economics Bulletin

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On the sustainable deficit of the government

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Abstract

In this note we derive the long-run public deficit to GDP ratio that is compatible with a sustainable public debt. Starting from the observation that a sustainable debt policy requires that the present value of public debt asymptotically converges to zero and that the debt to GDP ratio must be constant in the long-run, the deficit to GDP ratio is derived that meets these two conditions.

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1 Introduction

An implication of the Ricardo equivalence theorem is that an increase in public debt in the present must go along with higher taxes in the future. Bohn (1995, 1998) proposed a generalization of this theorem by stating that a rise in public debt must be accompanied by higher primary surpluses so that the level of the outstanding debt becomes a mean-reverting process. However, a postive reaction of the primary surplus to higher debt does not necessarily imply that the debt to GDP ratio remains constant in the long-run. Thus, one could come to the incorrect conclusion that a sustainable debt policy is compatible with a permanently rising debt to GDP ratio. That, however, is not possible since a steadily increasing debt to GDP ratio requires continuously rising primary surplus to GDP ratios. But, the latter is excluded because the primary surplus must be financed from GDP so that there is an upper bound for the primary surplus ratio. Consequently, the debt to GDP ratio must remain bounded in the long-run what applies in continuous time when the reaction of the primary surplus to GDP exceeds the difference between the interest rate and the GDP growth rate on average, a result that was first derived in Fincke and Greiner (2010).[1] A detailed and comprehensive analysis of the relationship between a sustainable debt policy and the primary surplus can be found in Greiner and Fincke (2015), ch. 2.1.

In the next section we derive the public deficit to GDP ratio that is compatible with a sustainable debt policy.

2 Sustainable public debt and the deficit ratio

The development of public debt over time in continuous time is described by the following differential equation:

$$\dot{B}(t) = r(t)B(t) - S(t), \tag{1}$$

with $B(t)$ public debt, $r(t) \in \mathbb{R}_+$ the interest rate, $S(t)$ the government surplus exclusive of interest payments on public debt. All variables are real and C^1 functions of time t and the dot over a variable stands for the derivative with respect to time d/dt.

Assume that the government sets the primary surplus to GDP ratio, $s(t) = S(t)/Y(t)$, so that it is a positive linear function of the debt to GDP ratio, $b(t) = B(t)/Y(t)$, and of a term that is independent of public debt, $\phi(t)$ (see Bohn, 1995, 1998). The primary surplus ratio, then, can be written as

$$s(t) = \psi(t)\, b(t) + \phi(t), \tag{2}$$

[1] In discrete time the reaction coefficient must exceed the interest rate growth rate differential divided by one plus the growth rate, cf. Blanchard et al. (2021).

where $\psi(t) \in \mathbb{R}_{++}$ is the coefficient determining how strong the primary surplus reacts to changes in the public debt ratio and that is time-varying.[2] The motivation for that assumption is that it makes the debt to GDP ratio a mean-reverting process excluding explosive debt paths.

The parameter $\phi(t) \in \mathbb{R} \setminus \{0\}$ is also time dependent and it is influenced by other economic variables, such as social spending or transitory government expenditures in general. For $\phi(t) > 0 \, (< 0)$ the level of the primary surplus rises (declines) as GDP increases. In the first case, i.e. for $\phi(t) > 0$, we speak of a procyclical fiscal policy and in the second case, i.e. for $\phi(t) < 0$, the fiscal policy is countercyclical.

Given (1) and (2) the debt to GDP ratio evolves according to

$$\dot{b}(t) = (r(t) - \psi(t))\, b(t) - \phi(t) - g(t)\, b(t), \tag{3}$$

with $g(t)$ the growth rate of GDP that is assumed to be positive on average, i.e. it may be zero or even negative at some points in time.

Before we proceed we first give the definition of a sustainable debt policy.

Definition 1 *Public debt is sustainable if and only if* $\lim_{t\to\infty} e^{-\int_0^t r(\mu)d\mu} B(t) = 0$ *holds.*

In lemma 1 a result from Greiner and Fincke (2015) is given that we need to proceed with our analysis.

Lemma 1 *A necessary and sufficient condition for the present value of public debt to converge to zero is* $\int_0^t \psi(\mu)d\mu = \infty$ *and for the debt to GDP ratio to remain bounded* $\int_0^t \psi(\mu)d\mu > \int_0^t (r(\mu) - g(\mu))\, d\mu > 0$ *for* $t \to \infty$*, respectively.*

Proof: See Greiner and Fincke, ch. 2.1. □

The first condition states that the reaction coefficient must be positive on average and the second states that on average the reaction coefficient must exceed the interest rate growth rate differential.[3]

Denoting by $d(t) \in \mathbb{R}_+$ the deficit to GDP ratio, i.e. $\dot{B}(t) = d(t)Y(t)$, proposition 1 gives the sustainable public deficit to GDP ratio (SPDR) in the long-run.

Proposition 1 *For the rule specified in equation* (2)*, the long-run sustainable public deficit to GDP ratio* d^s *is given by*

$$d^s = \frac{-\phi(t)\, g(t)}{\psi(t) - (r(t) - g(t))} \quad \text{for} \quad t \to \infty. \tag{4}$$

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[2] Any non-linear model can be approximated by a linear model with time-varying coefficients and the approximation is good if the parameter changes smoothly (cf. Granger, 2008).

[3] The interest rate growth rate differential is assumed to be positive on average. Otherwise, the budget constraint of the government would not pose a problem since the government could grow out of debt.

Proof: From (3) we obtain $\dot{b} = 0 \Leftrightarrow b^s = -\phi/(\psi - r + g)$. Further, the debt to GDP ratio evolves according to $\dot{b} = d - gb$ which implies $b^s = d^s/g$. Setting $b^s = b^s$ gives the result in the proposition. □

Before we proceed with the interpretation of proposition 1 we give the following corollary to that proposition.

Corollary 1 *Assume that the government sets the primary surplus according to the rule specified in equation (2). Then, in the long-run the government can be a debtor if and only if it runs a countercyclical fiscal policy.*

Proof: From the proof of proposition 1 we know that $b^s = -\phi/(\psi - r + g) = d^s/g$. For $g > 0$ and $\psi > r - g$ the debt ratio b^s is positive if and only if $\phi < 0$ holds, which characterizes a countercyclical fiscal policy. □

Corollary 1 shows that the government must run a countercyclical fiscal policy if it pursues a sustainable debt policy and if it is a debtor. A procyclical policy must go along with a negative government debt. That is, the government must be a creditor that lends to the private sector. In that case, the justification for the rule specified in equation (2) is that it prevents the private sector from playing a Ponzi game.

As regards the SPDR it is easily seen from (4) that a rise in the interest rate and in the GDP growth rate go along with a higher SPDR. The reason for the first result is that a higher interest rate implies a higher long-run debt to GDP ratio so that the deficit ratio is larger, too. It should be noted that the condition $\psi > r - g$ guarantees that the government pursues a sustainable debt policy. The second result that the SPDR increases with a higher GDP growth rate means that fast growing economies can run higher deficits. In particular, equation (4) illustrates that for stagnating economies the SPDR equals zero, implying that the governments in those economies must run balanced budgets.

With regard to the fiscal parameters it can be stated that both are negatively correlated with the SPDR. The stronger the primary surplus reacts to a rise in public debt, the smaller is the SPDR. This is due to the fact that a higher reaction coefficient implies a higher primary suplus so that the deficit of the government is smaller. The same holds for the autonomous term in the rule specified in equation (2). The higher the term ϕ is, the higher is the primary surplus leading to a smaller SPDR.

In a next step we assume that the primary surplus is set to its constant maximum value relative to GDP, i.e.

$$s(t) = \mu, \tag{5}$$

with $\mu \in \mathbb{R}_{++}$. Then, the debt to GDP ratio evolves according to

$$\dot{b}(t) = r(t)\, b(t) - \mu - g(t)\, b(t) \tag{6}$$

Before we proceed and derive the SPDR for this case, we state the following lemma demonstrating that in this case a sustainable debt policy is feasible only if the initial debt

to GDP ratio does not exceed a certain critical value.

Lemma 2 *If the government sets the primary surplus equal to a constant value, the debt policy is sustainable if the initial debt to GDP ratio $b(0)$ is smaller than or equal to the critical value given by $b_{crit} = \mu \int_0^\infty e^{-\int_0^\nu (r(\xi)-g(\xi))d\xi} d\nu$*

Proof: See Greiner and Fincke, ch. 2.1. □

Proposition 2 gives the SPDR for this case.

Proposition 2 *For a constant primary surplus to GDP ratio μ, the long-run sustainable public deficit to GDP ratio d^s is given by*

$$d^s = \frac{\mu\, g(t)}{r(t) - g(t)} \quad \text{for} \quad t \rightarrow \infty. \tag{7}$$

Proof: From (6) we obtain $\dot{b} = 0 \Leftrightarrow b^s = \mu/(r-g)$. Further, the debt to GDP ratio evolves according to $\dot{b} = d - gb$ which implies $b^s = d^s/g$. Setting $b^s = b^s$ gives the result in the proposition. □

Proposition 2 shows that the SPDR positively depends on the maximum value the primary surplus can take, μ, and positively on the GDP growth rate g. As regards the interest rate, it is easily seen that higher values of r imply a lower SPDR, in contrast to what was obtained in Proposition 1. The reason for this different result is that a higher interest rate implies higher interest payments of the government that can be sustained only with a lower debt to GDP ratio and, consequently, with a lower public deficit relative to GDP. The difference to the outcome of Proposition 1, where the SPDR is positively correlated with the interest rate, is that in this case the government raises the primary surplus when public debt rises, which is excluded in the situation underlying Proposition 2. Therefore, in the situation underlying Proposition 1 the rise in the primary surplus compensates the higher interest payments leading to a higher SPDR.

3 Conclusion

Starting from the assumption that the primary surplus is a rising function of the level of public debt and that the debt to GDP ratio must be constant in the long-run, we computed the sustainable public deficit to GDP ratio. It turned out that the latter positively depends on the interest rate and on the GDP growth rate and negatively on the reaction coefficient and on the autonomous term in the reaction function that is independent of public debt.

When the government sets the primary surplus to a constant value that is independent of public debt, the sustainable public deficit to GDP ratio negatively depends on the interest rate. The reason for this different outcome is that with a constant primary surplus

the higher interest payments can be sustained only if the debt ratio and the deficit ratio are smaller as the interest rate rises.

The second case may become relevant for economies with rising public debt to GDP ratios and a situation may arise such that the upper bound of the primary surplus relative to GDP becomes binding implying that a further increase of the primary surplus is excluded. In such a case the government cannot meet its obligations from public debt. Therefore, additional research seems to be necessary to obtain estimates of how high the maximum primary surplus relative to GDP could be.

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Declarations

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