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Existence of a Condorcet winner when voters have other-regarding preferences

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Abstract

In standard political economy models, voters are "self-interested" i.e. care only about "own" utility. However, the emerging evidence indicates that voters often have "other-regarding preferences" (ORP), i.e., in deciding among alternative policies voters care about their payoffs relative to others. We extend a widely used general equilibrium framework in political economy to allow for voters with ORP, as in Fehr-Schmidt (1999). In line with the evidence, these preferences allow voters to exhibit "envy" and "altruism", in addition to the standard concern for "own utility". We give sufficient conditions for the existence of a Condorcet winner when voters have ORP. This could open the way for an incorporation of ORP in a variety of political economy models. Furthermore, as a corollary, we give more general conditions for the existence of a Condorcet winner when voters have purely selfish preferences.

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1. Introduction

The median voter theorem has been seminal to the development of modern political economy. The standard model relies on voters being self-interested. The main expository framework for this relies on Romer (1975), Roberts (1977) and Meltzer and Richard (1981) (or the RRMR framework) which is a simple general equilibrium model with endogenous labor supply.

Despite the standard assumption of self-interested voters, it seems very plausible that explanations for redistribution should be underpinned by the inherent human desire to care directly for the well being of others. In other words, in the domain of redistribution, it seems reasonable to postulate that individuals have *other regarding preferences*. An emerging empirical literature is strongly supportive of the role of other regarding preferences specifically in the domain of voting models; see, for instance, Ackert et al. (2007), Bolton and Ockenfels (2006) and Tyran and Sausgruber (2006). These papers establish that voters often choose policies that promote equity/fairness over purely selfish considerations. Bolton and Ockenfels (2006), for instance, examine the preference for equity versus efficiency in a voting game. Groups of three subjects are formed and are presented with two alternative policies: one that promotes equity while the other promotes efficiency. The final outcome is chosen by a majority vote. About twice as many experimental subjects preferred equity as compared to efficiency. Furthermore, even those willing to change the status-quo for efficiency are willing to pay, on average, less than half relative to those who wish to alter the status-quo for equity.

There are several models of fairness in the literature. We choose to use the Fehr-Schmidt (1999) (henceforth, FS) approach to fairness[1]. In this approach, voters care, not only about their own payoffs, but also about their payoffs relative to those of others. If their payoff is greater than the payoff of other voters then they suffer from *advantageous-inequity* (arising from, say, *altruism*) and if their payoff is lower than the payoff of other voters they suffer from *disadvantageous-inequity* (arising from, say, *envy*). Several reasons motivate our choice of the FS model.

1. The FS model is tractable and explains the experimental results arising from several games where the prediction of the standard game theory model with selfish agents

[1]Bolton and Ockenfels (2000) provide yet another approach of inequity averse economic agents but it cannot explain the outcome of the public good game with punishment, which is a fairly robust experimental finding. Charness and Rabin (2002) provide two successive versions of their model. In the first version, economic agents do not care directly about outcome differences or the role of 'intentions'. This model is unable to explain the results of the public good game with punishment. A second version of the model introduces the role of intentions. However, voting is anonymous and involves very large numbers of voters, hence, intentions, in all possibility have a minor, if any, role to play. For a survey of theoretical models of 'other regarding preferences', its neuroeconomic foundations, and the empirical results, see Fehr and Fischbacher (2002).

yields results that are not consistent with the experimental evidence. These games include the ultimatum game, the gift-exchange game, the dictator game as well as the public-good game with punishment[2].

2. The FS model focusses on the role of inequity aversion. However, a possible objection is that it ignores the role played by 'intentions' that have been shown to be important in experimental results (Falk et al. (2002)) and are treated explicitly in theoretical work (Rabin (1993), Falk and Fischbacher (2006)). However, experimental results on the importance of intentions come largely from bilateral interactions. Economy-wide voting, on the other hand, is impersonal and anonymous, thereby making it unlikely that intentions play any important role in this phenomenon.

3. Experimental results on voting lend support to the use of the FS model in such contexts. Tyran and Sausgruber (2006) explicitly test for the importance of the FS framework in the context of direct voting. They conclude that the FS model predicts much better than the standard selfish-voter model. In addition, the FS model provides, in their words, "strikingly accurate predictions for individual voting in all three income classes." The econometric results of Ackert et al. (2007), based on their experimental data, lend further support to the FS model in the context of redistributive taxation. The estimated coefficients of altruism and envy in the FS model are statistically significant and, as expected, negative in sign. Social preferences are found to influence participant's vote over alternative taxes. They find evidence that some participants are willing to reduce their own payoffs in order to support taxes that reduce advantageous or disadvantageous inequity. In the context of voting experiments, Bolton and Ockenfels (2006) conclude that "...while not everyone measures fairness the same way, the simple measures offered by [Bolton and Ockenfels (2000)] or FS provide a pretty good approximation to population behavior over a wide range of scenarios that economists care about."

Our first contribution is to show, quite generally, that a fair voter, despite having FS-preferences, chooses labour supply exactly like a selfish voter who does not have social preferences. However, when making a decision on the redistributive tax rate, the same fair voter uses social preferences to choose the tax rate in a manner that the selfish voter does not. In other words, in two separate domains, labour supply and redistributive choice, the

[2] In the first three of these games, experimental subjects offer more to the other party relative to the predictions of the Nash outcome with selfish preferences. In the public good game with punishment, the possibility of ex-post punishment dramatically reduces the extent of free riding in voluntary giving towards a public good. In the standard theory with selfish agents, bygones are bygones, so there is no ex-post incentive for the contributors to punish the free-riders. Foreseeing this outcome, free riders are not deterred, which is in disagreement with the evidence. Such behavior can be easily explained within the FS framework.

fair voter behaves *as if* he had selfish preferences in the first domain and social preferences in the second. We emphasize 'as if' because, of course, the voter has identical underlying FS-preferences in both domains.

An important question is the following. Does a Condorcet winner exist in a model with FS-preferences? The lack of a satisfactory answer is likely to hold up progress within the class of political economy models that seek to incorporate the important insights from the literature on 'other regarding preferences'. The current situation is analogous to the period of time before the median voter theorem was discovered by Duncan Black (1948a,b,c) (and later popularized by Anthony Downs, 1957) for the case of self-interested voters. However, once known, and popularized, the median voter theorem opened up the domain of modern political economy as we know it today.

It is worth noting that when the labor supply decision is endogenous, the median-voter theorem with selfish voters is known to hold only in special cases. Actual (and successful) applications largely use quasi-linear preferences with quadratic-disutility of labour effort. This forms the basis of Meltzer and Richard's (1981) celebrated result that the extent of redistribution varies directly with the ratio of the mean to median income. Piketty (1995) restricts preferences to the quasi-linear case with quadratic-disutility of labour. Benabou (2000) considers the additively-separable case with log-consumption and disutility of labor given by a constant-elasticity form.

Our second contribution is to prove the existence of a Condorcet winner when voters have FS-preferences in a RRMR framework when the own-utility function is quasi-linear, which is the most commonly used functional form in applications of the median voter theorems.

For classes of utility functions more general than the quasi-linear, the results depend on the specific functional form chosen and the specific parameter values. Our third contribution is to give sufficient conditions which, when satisfied, guarantee the existence of a Condorcet winner in such cases.

The plan of the paper is as follows. Section 2 formulates the model. Section 3 shows, quite generally, that apparently inconsistent behavior can be explained using FS-preferences. Section 4 introduces our main assumptions. Section 5 gives sufficient conditions for the existence of a Condorcet winner (there is further discussion of these in section 7). Section 6 establishes the existence of a Condorcet winner when own-utility is quasi-linear. Section 8 summarizes. Proofs are in the appendix.

2. Model

We consider a general equilibrium model as in Meltzer and Richard (1981). Let there be $n = 2m - 1$ voter-worker-consumers (henceforth, voters). Let the skill level of voter j be

s_j, $j = 1, 2, ..., n$, where

$$0 < s_i < s_j < 1, \text{ for } i < j, \tag{2.1}$$

Denote the skill vector by $\mathbf{s} = (s_1, s_2, ..., s_n)$ and the median skill level by s_m. Each voter has a fixed time endowment of one unit and supplies l_j units of labor and so enjoys $1 - l_j$ units of leisure, where

$$0 \leq l_j \leq 1. \tag{2.2}$$

Labour markets are competitive and each firm has access to a linear production technology such that production equals $s_j l_j$. Hence, the wage rate offered to each voter coincides with the marginal product, i.e., the skill level, s_j. Thus, the before-tax income of voter j is given by

$$y_j = s_j l_j. \tag{2.3}$$

Note that 'skill' here need not represent any intrinsic talent, just ability to translate labour effort into income[3]. Let the average before-tax income be

$$\overline{y} = \frac{1}{n} \sum_{j=1}^{n} y_j. \tag{2.4}$$

The government operates a linear progressive income tax that is characterized by a constant marginal tax rate, t, $t \in [0, 1]$, and a uniform transfer, b, to each voter that equals the average tax proceeds,

$$b = t\overline{y}. \tag{2.5}$$

The budget constraint of voter j is given by

$$0 \leq c_j \leq (1 - t)\, y_j + b. \tag{2.6}$$

In view of (2.3), the budget constraint (2.6) can be written as

$$0 \leq c_j \leq (1 - t)\, s_j l_j + b. \tag{2.7}$$

2.1. Preferences of voters

We define a voter's preferences in two stages. First, let voter j have a continuous *own-utility* function, $\widetilde{u}_j(c_j, 1 - l_j)$, defined over own-consumption, $c_j > 0$, and own-leisure, $1 - l_j$, $0 \leq l_j < 1$. We assume that own consumption is always desirable, so $\widetilde{u}_j(c_j, 1 - l_j)$ is a strictly increasing function of c_j.

[3]For example, a highly talented classical musician may be able to earn only a modest income, while a merely competent 'pop' musician may earn millions. In our model, the former would be classified as having a low s while the latter would be classified as having a high s. Similarly, in recent years, there has been a record level of skilled (in the ordinary sense of the word) migration into Britain from Eastern Europe. However, since they are predominantly accepting low pay work, they would be classified in our model as having low s.

Second, and for the reasons stated in the introduction, voters have *other-regarding preferences* as in Fehr-Schmidt (1999). Let $\mathbf{c}_{-j}$ and $\mathbf{l}_{-j}$ be the vectors of consumption and labour supplies, respectively, of voters other than voter j. Under Fehr-Schmidt preferences the *FS-utility* of voter j is

$$\begin{aligned}
&\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s}\right) = \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right) \\
&-\frac{\alpha_j}{n-1} \sum_{k \neq j} \max\left\{0, \widetilde{u}_k\left(c_k, 1-l_k; t, b, s_k\right) - \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right)\right\} \\
&-\frac{\beta_j}{n-1} \sum_{i \neq j} \max\left\{0, \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right) - \widetilde{u}_i\left(c_i, 1-l_i; t, b, s_i\right)\right\}, \qquad (2.8)
\end{aligned}$$

where

$$\text{for } \textit{selfish} \text{ voters } \alpha_j = \beta_j = 0, \qquad (2.9)$$

so $\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, 0, 0, \mathbf{s}\right) = \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right)$,

$$\text{for } \textit{fair} \text{ voters } 0 < \beta_j < 1 \text{ and/or } \alpha_j > 0, \qquad (2.10)$$

so $\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s}\right) \neq \widetilde{u}_j\left(c_j, 1-l_j; t, b, s_j\right)$.

Thus, $\widetilde{u}$ is also the utility function of a selfish voter, as in the standard textbook model. From (2.8), the fair voter cares about own payoff (first term), payoff relative to those where inequality is disadvantageous (second term) and payoff relative to those where inequality is advantageous (third term). The second and third terms which capture respectively, *envy* and *altruism*, are normalized by the term $n-1$. Notice that in FS-preferences, inequality is *self-centered*, i.e., the individual uses her own payoff as a reference point with which everyone else is compared to. Also, while the Fehr-Schmidt specification is directly in terms of monetary payoffs, it is also consistent with comparison of payoffs in utility terms. These and related issues are more fully discussed in Fehr and Schmidt (1999). From (2.10), β_j is bounded below by 0 and above by 1: $\beta_j > 1$ would imply that an individual could increase utility by simply destroying all his/her wealth; this is counterfactual.

Since $\widetilde{u}_j(c_j, 1-l_j)$ is a strictly increasing function of c_j, and since $\alpha_j \geq 0$ and $0 \leq \beta_j < 1$ it follows, from (2.8), that $\widetilde{U}_j\left(c_j, l_j; \mathbf{c}_{-j}, \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s}\right)$ is also a strictly increasing function of c_j. Hence, the budget constraint (2.7) holds with equality. Substituting $c_j = (1-t)\, s_j l_j + b$, from (2.7), into the utility function, $\widetilde{u}_j(c_j, 1-l_j)$, gives the following form for utility

$$u_j\left(l_j; t, b, s_j\right) = \widetilde{u}_j((1-t)\, s_j l_j + b, 1-l_j) \qquad (2.11)$$

Correspondingly, the FS-utilities take the form

$$\begin{aligned}
U_j\left(l_j; \mathbf{l}_{-j}, t, b, \alpha, \beta, \mathbf{s}\right) &= u_j\left(l_j; t, b, s_j\right) - \frac{\alpha_j}{n-1} \sum_{k \neq j} \max\left\{0, u_k\left(l_k; t, b, s_k\right) - u_j\left(l_j; t, b, s_j\right)\right\} \\
&\quad -\frac{\beta_j}{n-1} \sum_{i \neq j} \max\left\{0, u_j\left(l_j; t, b, s_j\right) - u_i\left(l_i; t, b, s_i\right)\right\}, \qquad (2.12)
\end{aligned}$$

Remark 1 : *(Weighted utilitarian preferences) First define the sets A_j and D_j as the set of voters with respect to whom voter j has respectively, advantageous and disadvantageous inequity. So*

$$A_j = \{i : i \neq j \text{ and } u_i(l_i; t, b, s_i) \leq u_j(l_j; t, b, s_j)\}, \tag{2.13}$$

$$D_j = \{k : k \neq j \text{ and } u_k(l_k; t, b, s_k) > u_j(l_j; t, b, s_j)\}. \tag{2.14}$$

Denote the respective cardinalities of these sets by $|A_j|$ and $|D_j|$. Then FS-utility (2.12) can be written in a way that is reminiscent of the weighted utilitarian form:

$$U_j(l_j; \mathbf{l}_{-j}, t, b, \mathbf{s}) = \omega_{jj} u_j(l_j; t, b, s_j) + \sum_{i \neq j}^{n} \omega_{ji} u_i(l_i; t, b, s_i), \tag{2.15}$$

$$\begin{array}{lll} i \in A_j & \Rightarrow & \omega_{ji} = \frac{\beta_j}{n-1} > 0, \\ i = j & \Rightarrow & \omega_{jj} = 1 - \frac{|A_j|\beta_j}{n-1} + \frac{|D_j|\alpha_j}{n-1} > 0, \\ k \in D_j & \Rightarrow & \omega_{jk} = -\frac{\alpha_j}{n-1} < 0. \end{array} \tag{2.16}$$

$$\sum_{i=1}^{n} \omega_{ji} = 1. \tag{2.17}$$

$$\textit{If voter } j \textit{ is selfish, then } \omega_{jj} = 1 \textit{ and } \omega_{ji} = 0 \ \ (i \neq j). \tag{2.18}$$

2.2. Sequence of moves

We consider a two-stage game. In the first stage, all voters vote directly and sincerely on the redistributive tax rate. Should a median voter equilibrium exist, then the tax rate preferred by the median voter is implemented. In the second stage, all voters make their labour supply decision, conditional on the tax rate chosen by the median voter in the first stage. On choosing their labour supplies in the second stage, the announced first period tax rate is implemented and transfers made according to (2.5).

In the second stage the voters play a one-shot Nash game: each voter, j, chooses his/her labour supply, l_j, given the vector, $\mathbf{l}_{-j}$, of labour supplies of the other voters, so as to maximize his/her FS-utility (2.12). In the first stage, each voter votes for his/her preferred tax rate, correctly anticipating the second stage play.

The solution is by backward induction. We first solve for the Nash equilibrium in labour supply decisions of voters conditional on the announced tax rates and transfers. The second stage decision is then fed into the first stage FS-utilities to arrive at the indirect utilities of voters, which are purely in terms of the tax rate. Voters then choose their most desired tax rates that maximize their indirect FS-utilities, with the proposal of the median voter being the one that is implemented.

3. Explaining (apparently) inconsistent behavior using Fehr-Schmidt preferences

Given the tax rate, t, and the transfer, b, both determined in the first stage, the voters play a one-shot Nash game (in the subgame determined by t and b). Each voter, j, chooses own labour supply, l_j, so as to maximize his/her FS-utility (2.12), given the labour supplies, $\mathbf{l}_{-j}$, of all other voters. Since in (2.15), $u_i\left(l_i; t, b, s_i\right)$, $i \neq j$, enter additively, and $\omega_{jj} > 0$, it follows that maximizing the FS-utility, $U_j\left(l_j; \mathbf{l}_{-j}, t, b, \alpha_j, \beta_j, \mathbf{s}\right)$, with respect to l_j, given $\mathbf{l}_{-j}$, t, b and $\mathbf{s}$, is equivalent to maximizing own-utility, $u_j\left(l_j; t, b, s_j\right)$, with respect to l_j, given t, b and s_j. We summarize this in the following proposition.

Proposition 1 *: In the second stage of the game, voter j, whether fair or selfish, chooses own labour supply, l_j, so as to maximize own-utility, $u_j\left(l_j; t, b, s_j\right)$, given t, b and s_j.*

Note that Proposition 1 would not hold if the u_i, $i \neq j$, entered non-additively into the FS-utility function. However, the empirical evidence strongly supports the adopted form for the FS-utility function.

Contrary to the assumption in standard economics, a large and emerging body of empirical evidence clearly suggest that individuals do not have a complete and consistent preference ordering over all states. We *do not* assume inconsistent preferences. Rather, a decision maker, despite having identical FS-preferences in two different domains, behaves *fairly* in one domain but *selfishly* in the other.

In particular, from Proposition 1, a fair voter, despite having social preferences, chooses labour supply exactly like a selfish voter who does not have social preferences. However, when making a decision on the redistributive tax rate, the same fair voter uses social preferences to choose the tax rate in a manner that the selfish voter does not. In other words, in two separate domains, labour supply and redistributive choice, the fair voter behaves *as if* he had selfish preferences in the first domain and social preferences in the second. We emphasize 'as if' because, of course, the voter has identical underlying social preferences in both domains.

This opens up yet another dimension to the literature on inconsistency of preferences. To the best of our knowledge, this point has not been recognized previously. For example, individuals, when making a private consumption decision might act so as to maximize their selfish interest. But in a separate role as part of the government, as a school governor or as a voter, could act so as to maximize some notion of public well being. An individual, when buying an air ticket, might also buy travel insurance, thus exhibiting risk averse behavior. But, the same individual, when he reaches his holiday destination, may visit a gambling casino and exhibit risk loving behavior there. Individuals might, for instance, send their own children to private schools (self interest) but could at the same time vote for more funding to government run schools in local or national elections (public interest). Thus, individuals can put on different hats in different situations. Further research might show that some, or all, of these might be explained using FS-preferences.

4. The main assumptions

In common with the literature, we assume that all voters have the same own-utility function, $\widetilde{u}(c, 1-l)$. We also assume that $\alpha_j = \alpha$ and $\beta_j = \beta$. Hence, voters differ only in that they are endowed with different skill levels, s_j. We assume that the utility function is twice continuously differentiable. We use the standard notation $\widetilde{u}_1 = \frac{\partial \widetilde{u}(c,L)}{\partial c}$ and $\widetilde{u}_2 = \frac{\partial \widetilde{u}(c,L)}{\partial L}$. We also assume that the utility function has the following, plausible, properties

$$\text{(a) } \widetilde{u}_1 > 0, \text{ (b) } l > 0 \Rightarrow \widetilde{u}_2(c, 1-l) > 0, \text{ (c) } \widetilde{u}_2(c, 1) = 0, \text{ (d) } \widetilde{u}_1(c, 0) \leq \widetilde{u}_2(c, 0), \quad (4.1)$$

$$\text{(a) } \widetilde{u}_{11} \leq 0, \text{ (b) } \widetilde{u}_{12} \geq 0, \text{ (c) } l > 0 \Rightarrow \widetilde{u}_{22}(c, 1-l) < 0, \text{ (d) } (\widetilde{u}_{12})^2 \leq \widetilde{u}_{11}\widetilde{u}_{22}. \quad (4.2)$$

From (4.1a), the marginal utility of consumption is positive, while (4.1b) implies that marginal utility of leisure is positive, unless $l = 0$ in which case (4.1c) says that the consumer is satiated with leisure. From (4.1d), when a consumer has no leisure, she always (weakly) prefers one extra unit of leisure to one extra unit of consumption. (4.2a) says that marginal utility of consumption is non-increasing. From (4.2b), consumption and leisure are complements while (4.2c) implies that the marginal utility of leisure is strictly declining unless, possibly, the consumer is satiated with leisure (in which case $\widetilde{u}_{22}(c, 1) = 0$). Conditions (4.1) and (4.2) guarantee that a maximum exists, that it is unique and that it is an interior point ($0 < l_i < 1$), unless $t = 1$ in which case the maximum will lie at $l_i = 0$. Conditions (4.2a,c,d) guarantee that $\widetilde{u}$ is concave.

We list, in Lemmas 1 and 2 below, some useful results.

Lemma 1 *(Properties of labour supply): (a) Given t, b and s_j, there is a unique labour supply for voter j, $l_j = l(t, b, s_j)$, that maximizes own-utility $u(l_j; t, b, s_j)$,*
(b) $t \in [0, 1) \Rightarrow 0 < l_j < 1$,
(c) $l_j = 0$ at $t = 1$,
(d) $t \in [0, 1] \Rightarrow \left[\frac{\partial u}{\partial l_j}(l_j; t, b, s_j)\right]_{l_j = l(t,b,s_j)} = 0$,
(e) $l(t, b, s_j)$ is continuously differentiable,
(f) $\frac{\partial l(t,b,s)}{\partial b} \leq 0$,
(g) for each $t \in [0, 1]$, the equation $b = \frac{1}{n} t \Sigma_{i=1}^{n} s_i l(t, b, s_i)$ has a unique solution $b(t, \mathbf{s}) \geq 0$; and $b(t, \mathbf{s})$ is continuously differentiable.

Substituting labour supply, given by Lemma 1(a), in $u(l_j; t, b, s_j)$ we get the indirect own-utility function of voter j:

$$v(t, b, s_j) = u(l(t, b, s_j); t, b, s_j). \quad (4.3)$$

Lemma 2 *(Properties of the indirect own-utility function, $v(t,b,s)$):*
(a) $\frac{\partial v}{\partial b} > 0$,
(bi) $\left[\frac{\partial v}{\partial s}\right]_{t=1} = 0$, *(bii)* $t \in [0,1) \Rightarrow \frac{\partial v}{\partial s} > 0$,
(ci) $\left[\frac{\partial v}{\partial t}\right]_{t=1} = 0$, *(cii)* $t \in [0,1) \Rightarrow \frac{\partial v}{\partial t} < 0$,
(di) $\frac{\partial^2 v}{\partial s \partial b} \leq 0$, *(dii)* $\left[\frac{\partial^2 v}{\partial s \partial b}\right]_{t=1} = 0$.

Substituting labour supply, $l(t,b,s_j)$, into (2.3) gives before-tax income:

$$y_j(t,b,s_j) = s_j l(t,b,s_j). \tag{4.4}$$

Substituting $b(t,\mathbf{s})$, given by Lemma 1 (g), into the indirect own-utility (4.3), gives

$$w_j(t,\mathbf{s}) = v(t,b(t,\mathbf{s}),s_j). \tag{4.5}$$

4.1. Existence of most desired tax rates (first stage)

Substituting labour supply, $l(t,b,s_j)$, into the utility function, $U_j(l_j;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s})$, and using (2.12) and (4.3), gives the indirect utility function, $V_j(t,b,\alpha,\beta,\mathbf{s})$, of voter j

$$\begin{aligned} V_j(t,b,\alpha,\beta,\mathbf{s}) &= v(t,b,s_j) - \frac{\alpha}{n-1}\sum_{k\neq j}\max\{0, v(t,b,s_k) - v(t,b,s_j)\} \\ &\quad - \frac{\beta}{n-1}\sum_{i\neq j}\max\{0, v(t,b,s_j) - v(t,b,s_i)\}, \end{aligned} \tag{4.6}$$

where v_j is defined in (4.3).

In the light of Lemmas 2(bi) and 2(bii), (4.6) becomes

$$V_j(t,b,\alpha,\beta,\mathbf{s}) = v(t,b,s_j) - \frac{\alpha}{n-1}\sum_{k>j}[v(t,b,s_k) - v(t,b,s_j)] - \frac{\beta}{n-1}\sum_{i<j}[v(t,b,s_j) - v(t,b,s_i)], \tag{4.7}$$

equivalently,

$$V_j(t,b,\alpha,\beta,\mathbf{s}) = \frac{\beta}{n-1}\sum_{i<j} v(t,b,s_i) + \left(1 - \frac{(j-1)\beta}{n-1} + \frac{(n-j)\alpha}{n-1}\right)v(t,b,s_j) - \frac{\alpha}{n-1}\sum_{k>j} v(t,b,s_k). \tag{4.8}$$

Let

$$W_j(t,\alpha,\beta,\mathbf{s}) = V_j(t,b(t,\mathbf{s}),\alpha,\beta,\mathbf{s}), \tag{4.9}$$

where $b(t,\mathbf{s})$ is given by Lemma 1(g). Then (4.5), (4.7), (4.8) and (4.9) give

$$W_j(t,\alpha,\beta,\mathbf{s}) = w_j(t,\mathbf{s}) - \frac{\alpha}{n-1}\sum_{k>j}[w_k(t,\mathbf{s}) - w_j(t,\mathbf{s})] - \frac{\beta}{n-1}\sum_{i<j}[w_j(t,\mathbf{s}) - w_i(t,\mathbf{s})], \tag{4.10}$$

where w_j is defined in (4.5). Equivalently,

$$W_j\left(t,\alpha,\beta,\mathbf{s}\right)=\frac{\beta}{n-1}\sum_{i<j}w_i\left(t,\mathbf{s}\right)+\left(1-\frac{\left(j-1\right)\beta}{n-1}+\frac{\left(n-j\right)\alpha}{n-1}\right)w_j\left(t,\mathbf{s}\right)-\frac{\alpha}{n-1}\sum_{k>j}w_k\left(t,\mathbf{s}\right). \tag{4.11}$$

Since $\widetilde{u}(c_i,1-l_i)$ is continuous by assumption, and since $l\left(t,b,s_i\right)$ and $b\left(t,\mathbf{s}\right)$ are continuous, by Lemma 1(e,g), it follows, from (2.11), (4.3), (4.5) and (4.10) or (4.11), that $W_j\left(t,\alpha,\beta,\mathbf{s}\right)$ is a continuous function of $t\in[0,1]$. Hence, $W_j\left(t,\alpha,\beta,\mathbf{s}\right)$ attains a maximum at some $t_j\in[0,1]$. This is the most desired tax rate for voter j. Thus, we have established:

Proposition 2 : *In the first stage of the game, for each voter j, there exists a most desired tax rate, $t_j\in[0,1]$, that maximizes his/her indirect FS-utility, $W_j\left(t,\alpha,\beta,\mathbf{s}\right)$, given $\alpha,\beta,\mathbf{s}$.*

Remark 2 : *In Remark 1, note that A_j and D_j, and hence also ω_{ji}, are functions of l_i, l_j, s_i, s_j, t and b: $\omega_{ji}=\omega_{ji}\left(l_i,l_j,s_i,s_j,t,b\right)$. However, for the special case $t<1$, $l_i=l\left(t,b,s_i\right)$ and $l_j=l\left(t,b,s_j\right)$ we get, using (4.3) and Lemma 2 (bii),*

$$\begin{aligned}A_j &= \left\{i:i\neq j \text{ and } u\left(l\left(t,b,s_i\right);t,b,s_i\right)\leq u\left(l\left(t,b,s_j\right);t,b,s_j\right)\right\}\\ &= \left\{i:v\left(t,b,s_i\right)<v\left(t,b,s_j\right)\right\}=\left\{1,2,...,j-1\right\},\end{aligned}$$

$$\begin{aligned}D_j &= \left\{i:i\neq j \text{ and } u\left(l\left(t,b,s_i\right);t,b,s_i\right)>u\left(l\left(t,b,s_j\right);t,b,s_j\right)\right\}\\ &= \left\{i:v\left(t,b,s_i\right)>v\left(t,b,s_j\right)\right\}=\left\{j+1,j+2,...,n\right\}.\end{aligned}$$

Hence, in this case, using (2.15), (4.3) and (4.5),

$$\begin{array}{lll} i<j & \Rightarrow & \omega_{ji}=\frac{\beta}{n-1}>0,\\ i=j & \Rightarrow & \omega_{jj}=1-\frac{(j-1)\beta}{n-1}+\frac{(n-j)\alpha}{n-1}>0,\\ k>j & \Rightarrow & \omega_{jk}=-\frac{\alpha}{n-1}<0.\end{array} \tag{4.12}$$

$$V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)=\sum_{i=1}^{n}\omega_{ji}v\left(t,b,s_i\right), \tag{4.13}$$

$$W_j\left(t,\alpha,\beta,\mathbf{s}\right)=\sum_{i=1}^{n}\omega_{ji}w_i\left(t,\mathbf{s}\right). \tag{4.14}$$

Using (4.12), we can see that (4.13) and (4.14) are in agreement with (4.8) and (4.11), as is to be expected. Also, note that the model with fair voters is similar in structure to the one with selfish voters in that a weighted social welfare function is maximized, where the weight placed by voter j on the i^{th} voter's indirect utility is ω_{ji}. Finding a tax rate to maximize (4.14) is a completely standard problem in public economics.

5. Existence of a Condorcet winner

We shall show that a majority chooses the tax rate, t_m, that is optimal for the median-skill voter, in the sense that, for each $j \neq m$, a majority prefers t_m over t_j. We do this by using the *single-crossing property* of Milgrom and Shannon (1994) and Gans and Smart (1996).

Definition 1 : *(Gans and Smart, 1996) The 'single-crossing' property holds if for tax rates t, T and voters j, J,*

$$t < T,\ j < J,\ W_j(t, \alpha, \beta, \mathbf{s}) > W_j(T, \alpha, \beta, \mathbf{s}) \Rightarrow W_J(t, \alpha, \beta, \mathbf{s}) > W_J(T, \alpha, \beta, \mathbf{s}).$$[4]

Lemma 3 : *(Gans and Smart, 1996) The 'single-crossing' property holds if $-\frac{\partial V_j}{\partial t} / \frac{\partial V_j}{\partial b}$ is an increasing function of j (where V_j is defined in (4.7)).*

Lemma 4 : *(Gans and Smart, 1996) If the 'single-crossing' property holds, then the median-voter is decisive, i.e., a majority chooses the tax rate that is optimal for the median-voter.*

The proofs of Lemmas 3, 4 can be found in Gans and Smart (1996).

We now introduce two further assumptions, A1 and A2, followed by the main result of this section.

A1: $t \in [0, 1) \Rightarrow \frac{\partial^2 v}{\partial s \partial t}(t, b, s) < 0$.

Recall, from Lemma 2(cii), that $t \in [0, 1) \Rightarrow \frac{\partial v}{\partial t} < 0$. Hence, $\frac{\partial}{\partial s}\left(t \frac{\partial v}{\partial t}\right) = t \frac{\partial^2 v}{\partial s \partial t} < 0$ can be interpreted as saying that an extra 1% on the redistributive tax rate hurts a poor person less than a rich person. Thus Assumption A1 roughly says that redistributive taxes hurt the poor less than the rich. This is the basic foundation of the modern welfare state, and as we show in section 6 below, is satisfied in the important case of quasi-linear preferences, which is widely used in the literature.

A2: $\frac{\partial V_j}{\partial t}(t, b, \alpha, \beta, \mathbf{s}) \leq 0$.

Since $\frac{\partial v}{\partial t} < 0$, for $t < 1$ (Lemma 2(cii)), an increase in tax (benefit, b, remaining fixed), is undesirable for a selfish-voter which is, of course, entirely reasonable. Assumption A2 extends this to fair-voters as well. It implies that envy is not so great as to make a fair-voter prefer an increase in tax, even if it has no gain for any one in terms of an increase

[4] Here we use "$<$" to denote the usual ordering of real numbers. In the more general setting of Gans and Smart (1996), "$<$" is used to denote several (possibly different) abstract orderings. In particular, a literal translation of Gants and Smart (1996) gives: $T < t$, $j < J$, $W_j(t, \alpha, \beta, \mathbf{s}) > W_j(T, \alpha, \beta, \mathbf{s}) \Rightarrow W_J(t, \alpha, \beta, \mathbf{s}) > W_J(T, \alpha, \beta, \mathbf{s})$, where "$j < J$ " has the usual meaning "j is less than J " but "$T < t$ " means "t is less than T ".

in the benefits, b (b is held fixed in computing $\frac{\partial V_j}{\partial t}$ in A2). Thus, the conditions in terms of Fehr-Schmidt preferences bear an intuitive interpretation.

In section 7, we will show that the class of utility functions that satisfy assumptions A1 and A2 is very large. Since an individual's behavior is determined by his/her Fehr-Schmidt preferences, these may be taken as the primitives for an individual who possesses them and the conditions levied on such preferences are indeed the relevant conditions on the primitives. Nevertheless, and also in section 7, we discuss the interpretation of assumptions A1, A2 in terms of a voter's own-utility $\widetilde{u}_j(c_j, 1 - l_j)$. We will find that there are no obvious insights to be gained from this.

Proposition 3 : *Under assumptions A1 and A2 a majority prefers the tax rate that is optimal for the median-skill voter.*

Corollary 1 : *Under assumption A1, if voters are selfish ($\alpha = \beta = 0$), then a majority prefers the tax rate that is optimal for the median-skill voter. Assumption A2 is satisfied in this case.*

As noted in the introduction, when labor supply is endogenous, the median-voter theorem with selfish voters is known to hold only in special cases. Corollary 1 establishes the existence of a Condorcet winner for a more general class of utility functions as compared to the existing literature.

6. Quasi-linear preferences

We assume here that the own-utility function is quasi-linear. This is the most commonly used functional form in various applications of the median voter theorems.

$$\widetilde{u}(c, 1 - l) = c - g(l), \tag{6.1}$$

where $g(l)$ is twice continuously differentiable and

$$g'(0) = 0,\ g'(1) \geq 1, \tag{6.2}$$

$$l > 0 \Rightarrow g'(l) > 0,\ g''(l) > 0. \tag{6.3}$$

Example 1 : *Let $g(l) = \frac{\epsilon}{1+\epsilon} l^{\frac{1+\epsilon}{\epsilon}}$, where the elasticity of labour supply, ϵ, is a positive constant. Then (6.2), (6.3) are satisfied.*

The special case

$$\widetilde{u}(c, 1 - l) = c - \frac{1}{2} l^2, \tag{6.4}$$

has particular significance in the literature. Meltzer and Richard (1981) use (6.4) to derive the celebrated result that the extent of redistribution varies directly with the ratio of the mean to median income. Piketty (1995) restricts preferences to the quasi-linear case with disutility of labour given by the quadratic form, (6.4).

Substitute, from the budget constraint, $c = (1-t)\,sl + b$, into (6.1) to get

$$u\,(l; t, b, s) = (1-t)\,sl + b - g\,(l)\,. \tag{6.5}$$

We list, in Lemmas 5, 6 and 7 below, some useful results.

Lemma 5 *For quasi-linear utility:* $\frac{\partial u}{\partial l} = (1-t)\,s - g'\,(l)$, $\frac{\partial u}{\partial t} = -sl$, $\frac{\partial u}{\partial b} = 1$, $\frac{\partial u}{\partial s} = (1-t)\,l$, $\frac{\partial^2 u}{\partial l^2} = -g''\,(l)$, $\frac{\partial^2 u}{\partial b \partial l} = 0$, $\frac{\partial^2 u}{\partial s \partial l} = 1-t$, $\frac{\partial^2 u}{\partial t \partial l} = -s$, $\frac{\partial^2 u}{\partial s \partial t} = -l$.

Lemma 6 *For quasi-linear utility:* (a) $\frac{\partial l(t,b,s)}{\partial b} = 0$, *(b)* $\frac{\partial l}{\partial s}\,(t,s) = \frac{1-t}{g''(l(t,s))}$.

From Lemma 6(a), we see that $l\,(t,b,s)$ is independent of b. Hence, from now on, we write $l\,(t,s)$ instead of $l\,(t,b,s)$ (and as we have already done in part (b) of Lemma 6).

Lemma 7 *For quasi-linear utility:* (a) $\frac{\partial v(t,b,s)}{\partial b} = 1$,
(bi) $\left[\frac{\partial^2 v(t,b,s)}{\partial s \partial t}\right]_{t=1} = 0$, *(bii)* $t \in [0,1) \Rightarrow \frac{\partial^2 v}{\partial s \partial t} < 0$.

Proposition 4 : *If utility is quasi-linear, then a majority prefers the tax rate that is optimal for the median-skill voter. In particular, assumption A1 holds and assumption A2 is not needed in this case.*

7. Further discussion of assumptions A1 and A2

In this section we address two questions. First, do A1 and A2 hold for any utility functions other than the quasi-linear case (section 6, above)? Second, if this class in non-empty, how can it be characterized?[5]

We shall argue that the class of utility functions for which A1 holds ($\frac{\partial^2 v}{\partial s \partial t} < 0$) is very broad indeed. Even a small subclass of this (characterized by Lemma 10, below) is extremely large and varied (due to the fact that, in Lemma 10, F and G are *arbitrary* functions). Due to this very varied nature of the utility functions that satisfy A1, we have not found a simple way to characterize them (other than, of course, $\frac{\partial^2 v}{\partial s \partial t} < 0$).

Lemma 9, below, shows that A2 places an upper bound on α. The value of this upper bound will depend on the specific utility function used and the values of its parameters. Whether this upper bound is adequately permissive, or is too tight, is ultimately an empirical question, which we find hard to answer a-priori.

[5]We are grateful to the two referees, the associate editor and the editor for raising these issues.

To rewrite A1 in terms of direct own-utility, use (2.11), (4.3), the envelope theorem and the chain rule, to get

$$\frac{\partial^2 v}{\partial t \partial s} = -l\widetilde{u}_1 - (1-t)\, sl^2 \widetilde{u}_{11} + \left[s\widetilde{u}_1 + (1-t)\, s^2 l \widetilde{u}_{11} - sl\widetilde{u}_{12}\right] \frac{(1-t)\,\widetilde{u}_1 + (1-t)^2\, ls\widetilde{u}_{11} - (1-t)\, l\widetilde{u}_{12}}{(1-t)^2\, s^2 \widetilde{u}_{11} - 2\,(1-t)\, s\widetilde{u}_{12} + \widetilde{u}_{22}}. \tag{7.1}$$

From (7.1) we see that $\frac{\partial^2 v}{\partial t \partial s}$ can be negative, zero or positive, depending on the particular direct own-utility function $\widetilde{u}\,(c, 1-l)$ under consideration. This remains the case even for separable case ($\widetilde{u}_{12} = 0$).

To rewrite A2 in terms of direct own-utility, first introduce the abbreviations

$$l_i = l\,(t, b, s_i)\,, \tag{7.2}$$

$$\widetilde{u}_1\,(i) = \widetilde{u}_1((1-t)\, s_i l\,(t, b, s_i) + b, 1 - l\,(t, b, s_i)), \tag{7.3}$$

then, from (4.7) and the envelope theorem, we get

$$\frac{\partial V_j}{\partial t} = -s_j l_j \widetilde{u}_1(j) - \frac{\alpha}{n-1} \sum_{k>j} \left[s_j l_j \widetilde{u}_1(j) - s_k l_k \widetilde{u}_1(k)\right] - \frac{\beta}{n-1} \sum_{i<j} \left[s_i l_i \widetilde{u}_1(i) - s_j l_j \widetilde{u}_1(j)\right]. \tag{7.4}$$

From (7.4) it appears that $\frac{\partial V_j}{\partial t}$ can be negative, zero or positive, depending on the values of α and β and the particular direct own-utility function under consideration.

To shed further light on assumptions A1 and A2, we first give three lemmas

Lemma 8 *:*

$$\frac{\partial^2 v}{\partial s \partial t} = -\left\{\frac{s}{l}\frac{\partial l}{\partial s} + \left[1 + \frac{s}{(\partial v/\partial b)}\frac{\partial}{\partial s}\,(\partial v/\partial b)\right]\right\} l \frac{\partial v}{\partial b}. \tag{7.5}$$

Lemma 9 *:*

$$\frac{\partial V_j}{\partial t} \leq 0 \Leftrightarrow \alpha \leq \frac{\left[n - j + (j-1)\,(1-\beta)\right] \frac{\partial v}{\partial t}\,(t, b, s_j) + \beta \sum_{i<j} \frac{\partial v}{\partial t}\,(t, b, s_i)}{\sum_{k>j} \left[\frac{\partial v}{\partial t}\,(t, b, s_k) - \frac{\partial v}{\partial t}\,(t, b, s_j)\right]}. \tag{7.6}$$

Lemma 10 *: The partial differential equation*

$$\frac{s}{\partial v/\partial b}\frac{\partial}{\partial s}\left(\frac{\partial v}{\partial b}\right) = -1, \tag{7.7}$$

has the solution

$$v\,(t, b, s) = s^{-1}\,(1-t)^{-1}\, F\,(b) + G\,(s\,(1-t))\,, \tag{7.8}$$

where F and G are arbitrary functions of b and $s\,(1-t)$, respectively.

7.1. Assumption A2

First, consider assumption A2. Consider the right hand side of the inequality in (7.6). Since $0 < \beta < 1$, and since $\partial v/\partial t < 0$ (recall Lemma 2(cii)), it follows the numerator in (7.6) is negative. If A1 holds, then the denominator in (7.6) is also negative. Hence the right hand side of (7.6) is positive. Thus A2 hold if α is either zero or strictly positive but sufficiently small. Positive values for α are found in the ultimatum game, the gift-exchange game, the dictator game and the public-good game with punishment. In all of these games, the willingness to reduce 'own' utility in order to punish a defector, or a free rider, appears to come from a human desire to redress an injustice, rather than from pure envy. However, if, in voting over taxes, voters have concern for those less well off ($\beta > 0$) but do not envy those better off (in the sense that a voter will not vote for a tax that hurts the better off but benefits no one), so that $\alpha = 0$, then A2 will certainly hold.

7.2. Assumption A1

We now turn to assumption A1. Let $t \in [0, 1)$. From Lemmas 1(b) and 2(a), we get that $l\frac{\partial v}{\partial b} > 0$. Recall that the skill level, s, is also the real before-tax wage rate. Hence, $\frac{s}{l}\frac{\partial l}{\partial s}$ is the elasticity of labour supply calculated before the distortionary effects of labour income taxes. A large body of evidence suggests that $\frac{s}{l}\frac{\partial l}{\partial s} > 0$.[6] We now turn to the most problematic term in (7.5), namely, $\frac{s}{(\partial v/\partial b)}\frac{\partial}{\partial s}(\partial v/\partial b)$. From Lemma 2(d) we know that $\partial^2 v/\partial b \partial s \leq 0$ and, hence, $\frac{s}{(\partial v/\partial b)}\frac{\partial}{\partial s}(\partial v/\partial b) \leq 0$, i.e., benefits are less important for higher wage earners. However, if $\partial v/\partial b$ varies sufficiently slowly with s, so that

$$\frac{s}{(\partial v/\partial b)}\frac{\partial}{\partial s}(\partial v/\partial b) \geq -1, \tag{7.9}$$

then, from (7.5), we would get $\frac{\partial^2 v}{\partial s \partial t} < 0$ and A1 would hold. In general, this will depend, of course, on both the functional form and the values taken by the parameters. Thus, the validity of A1, like that of A2, is ultimately an empirical question.

To find a utility function (other than quasi-linear) that satisfy A1, set

$$\frac{s}{(\partial v/\partial b)}\frac{\partial}{\partial s}(\partial v/\partial b) = -1. \tag{7.10}$$

The set of value functions that satisfy (7.10) is given by Lemma 10. Since F and G are *arbitrary* functions of b and $s(1-t)$, respectively, this set has the same cardinality as that of the set of all real valued functions and, hence, is a very large set indeed. One member of this set is given in Example 2, below.

[6] A large number of studies suggest positive labour supply elasticities (see, for example, Pencavel (1986) and Killingworth and Heckman (1986)). Negative labour supply elasticities may be due to estimating misspecified models (see Camerer and Loewenstein (2004), Chapter 1, 'Labor Economics', pp33-34).

Example 2 *: Let*

$$F(b) = b \text{ and } G(s(1-t)) = \frac{1}{\epsilon} s^{\epsilon} (1-t)^{\epsilon}. \tag{7.11}$$

Substituting from (7.11) into (7.8) gives the value function

$$v(t,b,s) = \frac{s^{\epsilon}(1-t)^{\epsilon}}{\epsilon} + \frac{b}{s(1-t)}, \tag{7.12}$$

This is a valid value function if, and only if, it is non-decreasing in each of b and $s(1-t)$. This will be the case if, and only if, $s^{\epsilon+1}(1-t)^{\epsilon+1} \geq b$. Substitute $b = c - (1-t)sl$ (from the budget constraint) into (7.12), then minimize with respect to $p = s(1-t)$, given c and l, to get the corresponding direct utility function

$$\widetilde{u}(c, 1-l) = \frac{1+\epsilon}{\epsilon} c^{\frac{\epsilon}{1+\epsilon}} - l. \tag{7.13}$$

The corresponding demand for labour can be found from (7.12) using Roy's identity or by maximizing (7.13) subject to the budget constraint $c = (1-t)sl + b$; and is given by

$$l(t,b,s) = (1-t)^{\epsilon} s^{\epsilon} - \frac{b}{s(1-t)}. \tag{7.14}$$

The corresponding demand for consumption is then

$$c(t,b,s) = (1-t)^{1+\epsilon} s^{1+\epsilon}.$$

Note that in Example 2, $1 + \frac{s}{(\partial v/\partial b)} \frac{\partial}{\partial s}(\partial v/\partial b) = 0$ by construction. It is clear that $\partial l/\partial s > 0$. Hence, form (7.5), $\frac{\partial^2 v}{\partial s \partial t} < 0$ and, consequently, A1 is satisfied. However, it must be remembered that this is just one simple example from a much larger class of utility functions that satisfy A1.

8. Summary

We replaced the self-interested voters in the Romer-Roberts-Meltzer-Richard framework with voters who have a preference for fairness, as in Fehr-Schmidt (1999). First, we showed, quite generally, that under Fehr-Schmidt preference, voters will care for others when voting over redistribution, but choose labour supply so as to maximize own utility. Second, we showed that a Condorcet winner exists when own-utility is quasi-linear and when voters have Fehr-Schmidt preferences. For classes of own-utility functions more general than the quasi-linear, the results will depend on the specific functional form and the specific parameter values. For the latter case, we gave sufficient conditions for the existence of a Condorcet winner. We believe that our contribution can open the way for further applications of behavioral concerns for fairness in political economy models.

9. Appendix: Proofs

Proof of Proposition 1: Let A_j, D_j and ω_{ji} be as in Remark 1. Consider voter j. In the second stage, voter j maximizes his/her own FS-utility $U_j\left(l_j;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$, recall (2.12), given the vector, $\mathbf{l}_{-j}$, of labour supplies of all other voters (recall subsection 2.2). Hence $u_i\left(l_i;t,b,s_i\right), i\neq j$, are fixed numbers. Since $u_j\left(l_j;t,b,s_j\right)$ is continuous in l_j, and since $\max\{0,x\}$ is continuous in x, it follows that $U_j\left(l_j;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$, as given by (2.12), is a continuous function of $l_j\in[0,1]$. Suppose that $U_j\left(l_j;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$ attains a maximum at $l_j^*\in[0,1]$. From (2.15), it follows that

$$U_j\left(l_j^*;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)=\omega_{jj}u_j\left(l_j^*;t,b,s_j\right)+\sum_{i\in A_j}\omega_{ji}u_i\left(l_i;t,b,s_i\right)+\sum_{k\in D_j}\omega_{jk}u_k\left(l_k;t,b,s_k\right).$$

We shall argue that l_j^* must maximize own-utility, $u_j\left(l_j;t,b,s_j\right)$. Suppose l_j^* does not maximize own-utility $u_j\left(l_j;t,b,s_j\right)$. Then we can find an l_j^{**}, sufficiently close to l_j^* (by the continuity of u_j), so that $u_j\left(l_j^{**};t,b,s_j\right)>u_j\left(l_j^*;t,b,s_j\right)$ and the set D_j is unchanged (since D_j is open). Hence, the set A_j is also unchanged (since A_j is the complement of D_j). Then

$$U_j\left(l_j^{**};\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)=\omega_{jj}u_j\left(l_j^{**};t,b,s_j\right)+\sum_{i\in A_j}\omega_{ji}u_i\left(l_i;t,b,s_i\right)+\sum_{k\in D_j}\omega_{jk}u_k\left(l_k;t,b,s_k\right).$$

Hence, $U_j\left(l_j^{**};\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)>U_j\left(l_j^*;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$, which cannot be, since l_j^* maximizes $U_j\left(l_j^*;\mathbf{l}_{-j},t,b,\alpha,\beta,\mathbf{s}\right)$. ■

Proof of Lemma 1: Given t,b and s_i, $u\left(l_i;t,b,s_i\right)$ is a continuous function of l_i on the non-empty compact set $[0,1]$. Hence, a maximum exists. Since $\widetilde{u}$ is twice continuously differentiable, so is u and, from (2.11), we get

$$\frac{\partial u}{\partial l_i} = (1-t)\,s_i\widetilde{u}_1\left((1-t)\,s_il_i+b,1-l_i\right)-\widetilde{u}_2\left((1-t)\,s_il_i+b,1-l_i\right), \quad (9.1)$$

$$\frac{\partial^2 u}{\partial b\partial l_i} = (1-t)\,s_i\widetilde{u}_{11}-\widetilde{u}_{12}, \quad (9.2)$$

$$\frac{\partial^2 u}{\partial l_i^2} = (1-t)^2\,s_i^2\widetilde{u}_{11}-2\,(1-t)\,s_i\widetilde{u}_{12}+\widetilde{u}_{22}. \quad (9.3)$$

From (4.2a), (4.2b) and (9.2) we get

$$\frac{\partial^2 u}{\partial b\partial l_i}\leq 0, \quad (9.4)$$

and from (4.2a), (4.2b), (4.2c) and (9.3) we get

$$l_i>0\Rightarrow\frac{\partial^2 u}{\partial l_i^2}<0. \quad (9.5)$$

First, consider the case $t = 1$. From (2.11), $u(l_i; 1, b, s_i) = \widetilde{u}(b, 1 - l_i)$. From (4.1b), $\widetilde{u}(b, 1 - l_i)$ is a strictly decreasing function of l_i on $(0, 1]$. By continuity, $\widetilde{u}(b, 1 - l_i)$ must be a strictly decreasing function of l_i on $[0, 1]$. Hence, the optimum must be

$$l_i = 0 \text{ at } t = 1. \tag{9.6}$$

Now suppose $t \in [0, 1)$. From (2.1), (4.1a), (4.1c) and (9.1) we get:

$$\frac{\partial u}{\partial l_i}(0; t, b, s_i) = (1 - t) s_i \widetilde{u}_1(b, 1) - \widetilde{u}_2(b, 1) = (1 - t) s_i \widetilde{u}_1(b, 1) > 0,$$

and, using (4.1d),

$$\frac{\partial u}{\partial l_i}(1; t, b, s_i) = (1 - t) s_i \widetilde{u}_1((1 - t) s_i + b, 0) - \widetilde{u}_2((1 - t) s_i + b, 0) < \widetilde{u}_1(0, 0) - \widetilde{u}_2(0, 0) \leq 0.$$

Hence, a maximum is an interior point, i.e.,

$$0 < l_i < 1. \tag{9.7}$$

From (9.7) and (9.5) it follows that $\frac{\partial^2 u}{\partial l_i^2} < 0$. Hence, the maximum is unique and is given by

$$\frac{\partial u}{\partial l_i}(l_i; t, b, s_i) = 0. \tag{9.8}$$

Since, from (4.1c), $\widetilde{u}_2(b, 0) = 0$, it follows, from (9.1) and (9.6), that (9.8) also holds for $t = 1$. Hence, for any voter i, the labor supply,

$$l_i = l(t, b, s_i), t \in [0, 1], \tag{9.9}$$

can be found by solving (9.8).

Since u is twice continuously differentiable it follows, from (2.11) and (9.8) that $l(t, b, s_i)$ is continuously differentiable. If $t = 1$ then, from (9.6), $\frac{\partial l_i}{\partial b} = 0$. Now suppose $t < 1$. From (9.5), $\frac{\partial^2 u}{\partial l_i^2} < 0$. Hence, from (9.4) and the implicit function theorem (or differentiating the identity (9.8)), we get $\frac{\partial l_i}{\partial b} = -\left[\frac{\partial^2 u}{\partial b \partial l_i} / \frac{\partial^2 u}{\partial l_i^2}\right]_{l_i = l(t, b, s_i)} \leq 0$. Hence, for all $t \in [0, 1]$,

$$\frac{\partial l_i}{\partial b} \leq 0. \tag{9.10}$$

Let $f(b) = \frac{1}{n} t \Sigma_{i=1}^n s_i l(t, b, s_i)$. Since $f(b) \geq 0$, $f(b)$ is continuously differentiable (hence continuous) and $f'(b) \leq 0$ (from (9.10)), it follows that $f(b) = b$ has a unique solution, $b(t, \mathbf{s}) \geq 0$; and $b(t, \mathbf{s})$ is continuously differentiable. ■

Proof of Lemma 2: From (2.11), (4.3) and the envelope theorem (or Lemma 1 (d)), we get

$$\frac{\partial v(t, b, s)}{\partial b} = \left[\frac{\partial u(l; t, b, s)}{\partial b}\right]_{l = l(t, b, s)} = [\widetilde{u}_1((1 - t) sl + b, 1 - l)]_{l = l(t, b, s)}, \tag{9.11}$$

$$\frac{\partial v(t,b,s)}{\partial s} = \left[\frac{\partial u(l;t,b,s)}{\partial s}\right]_{l=l(t,b,s)} = \left[(1-t)\, l\widetilde{u}_1\left((1-t)\, sl+b, 1-l\right)\right]_{l=l(t,b,s)}, \tag{9.12}$$

$$\frac{\partial v(t,b,s)}{\partial t} = \left[\frac{\partial u(l;t,b,s)}{\partial t}\right]_{l=l(t,b,s)} = -\left[sl\widetilde{u}_1\left((1-t)\, sl+b, 1-l\right)\right]_{l=l(t,b,s)}. \tag{9.13}$$

Part (a) follows from (4.1a) and (9.11). Part (b) follows from (4.1a), Lemma 1(b,c) and (9.12). Part (c) follows from (4.1a), Lemma 1(b,c) and (9.13).

From (9.11) (or (9.12)), we get

$$\frac{\partial^2 v}{\partial b \partial s} = \frac{l(1-t)\left[\widetilde{u}_{11}\widetilde{u}_{22} - (\widetilde{u}_{12})^2\right] + (1-t)\,\widetilde{u}_1\widetilde{u}_{12} - (1-t)^2 s\widetilde{u}_1\widetilde{u}_{11}}{(1-t)^2 s^2\widetilde{u}_{11} - 2(1-t)\, s\widetilde{u}_{12} + \widetilde{u}_{22}}. \tag{9.14}$$

From (4.1), (4.2) and (9.14), we get $\frac{\partial^2 v}{\partial b \partial s} \leq 0$; and $\frac{\partial^2 v}{\partial b \partial s} = 0$ for $t = 1$. This establishes part (d). ■

Proof of Proposition 3: From (4.8),

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial t} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial t} = \left(1 - \frac{j\beta}{n-1} + \frac{(n-j)\,\alpha}{n-1}\right)\left(\frac{\partial v(t,b,s_{j+1})}{\partial t} - \frac{\partial v(t,b,s_j)}{\partial t}\right). \tag{9.15}$$

From (2.10), Assumption A1 and (9.15), it follows that, for $t \in [0,1)$,

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial t} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial t} < 0. \tag{9.16}$$

From (4.7)

$$\begin{aligned}\frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial b} = {} & \frac{\partial v(t,b,s_j)}{\partial b} - \frac{\alpha}{n-1}\sum_{k>j}\left[\frac{\partial v(t,b,s_k)}{\partial b} - \frac{\partial v(t,b,s_j)}{\partial b}\right] \\ & -\frac{\beta}{n-1}\sum_{i<j}\left[\frac{\partial v(t,b,s_j)}{\partial b} - \frac{\partial v(t,b,s_i)}{\partial b}\right]\end{aligned} \tag{9.17}$$

From Lemma 2(a,d) and (9.17)

$$\frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial b} > 0. \tag{9.18}$$

From (4.8)

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial b} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial b} = \left(1 - \frac{j\beta}{n-1} + \frac{(n-j)\,\alpha}{n-1}\right)\left(\frac{\partial v(t,b,s_{j+1})}{\partial b} - \frac{\partial v(t,b,s_j)}{\partial b}\right). \tag{9.19}$$

From (2.10), Lemma 2(d) and (9.19), it follows that

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial b} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial b} \leq 0. \tag{9.20}$$

Now,

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right)-\left(\frac{-\frac{\partial V_{j}}{\partial t}}{\frac{\partial V_{j}}{\partial b}}\right)=\frac{\left(\frac{\partial V_{j+1}}{\partial b}-\frac{\partial V_{j}}{\partial b}\right)\frac{\partial V_{j}}{\partial t}+\frac{\partial V_{j}}{\partial b}\left(\frac{\partial V_{j}}{\partial t}-\frac{\partial V_{j+1}}{\partial t}\right)}{\frac{\partial V_{j}}{\partial b}\frac{\partial V_{j+1}}{\partial b}}. \tag{9.21}$$

From (9.16), (9.18), (9.20), (9.21) and Assumption A2, we get that, for $t \in [0,1)$,

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right)-\left(\frac{-\frac{\partial V_{j}}{\partial t}}{\frac{\partial V_{j}}{\partial b}}\right)>0, \tag{9.22}$$

hence,

$$-\frac{\partial V_j}{\partial t}/\frac{\partial V_j}{\partial b} \text{ is strictly increasing in } j. \tag{9.23}$$

From Lemma 3 and (9.23) we get that 'single-crossing' holds. Hence, from Lemma 4, the median-voter is decisive, i.e., a majority chooses the tax rate that is optimal for the median-voter. This establishes Proposition 3. ■

Proof of Corollary 1: If voters are selfish, so that $\alpha = \beta = 0$, then Assumption A2 reduces to $\frac{\partial v}{\partial t} \leq 0$, which we know holds from Lemma 2(c). This establishes Corollary 1. ■

Proof of Lemma 5: The proof follows from (6.5) by direct calculation. ■

Proof of Lemma 6: Differentiating the first order condition, $\frac{\partial u}{\partial l}(l;t,b,s) = 0$, (or appealing to the implicit function theorem) gives

$$\frac{\partial l}{\partial b}(t,b,s) = -\left[\frac{\frac{\partial^2 u(l;t,b,s)}{\partial l \partial b}}{\frac{\partial^2 u(l;t,b,s)}{\partial l^2}}\right]_{l=l(t,b,s)}, \quad \frac{\partial l}{\partial s}(t,b,s) = -\left[\frac{\frac{\partial^2 u(l;t,b,s)}{\partial l \partial t}}{\frac{\partial^2 u(l;t,b,s)}{\partial l^2}}\right]_{l=l(t,s)}. \tag{9.24}$$

The result then follows from (9.24) and Lemma 5. ■

Proof of Lemma 7: From (4.3), Lemma 5 and the envelope theorem (or direct calculation), we get $\frac{\partial v}{\partial b} = \frac{\partial u}{\partial b} = 1$. This establishes part (a). From (4.3), Lemma 5 and the envelope theorem, we get $\frac{\partial v}{\partial t} = \frac{\partial u}{\partial t} = -sl(t,s)$. Hence, $\frac{\partial^2 v}{\partial s \partial t} = -\frac{\partial}{\partial s}[sl(t,s)] = -\left[l(t,s) + s\frac{\partial l}{\partial s}\right] = -\left[l(t,s) + \frac{s(1-t)}{g''(l(t,s))}\right]$ (using Lemma 6(b)). Part (b) then follows from (6.3) and Lemma 1(b,c). This completes the proof. ■

Proof of Proposition 4: It is straight forward to check that assumptions (4.1) and (4.2) are satisfied in the quasi-linear case (6.1)-(6.3). From Lemma 7(b), Assumption A1 holds. From Lemma 7(a), $\frac{\partial v(t,b,s)}{\partial b} = 1$. From this, and (9.17), we get that $\frac{\partial V_j}{\partial b} = 1$. Hence, (9.21) reduces to

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right)-\left(\frac{-\frac{\partial V_{j}}{\partial t}}{\frac{\partial V_{j}}{\partial b}}\right)=\frac{\partial V_j}{\partial t}-\frac{\partial V_{j+1}}{\partial t}>0, \tag{9.25}$$

where the inequality in (9.25) comes from (9.16). Hence, (9.23) again holds, but we have not used Assumption A2. ■

Proof of Lemma 8: Recall that labour supply results from maximizing own utility subject to the budget constraint, $c = (1-t)\, sl + b$. Rewrite this budget constraint as:

$$c + pl = b, \tag{9.26}$$

where

$$p = -(1-t)\, s. \tag{9.27}$$

From Roy's identity (or direct calculation) we get

$$l = -\frac{\partial v / \partial p}{\partial v / \partial b}. \tag{9.28}$$

From (9.27) we get

$$\frac{\partial v}{\partial p} = \frac{1}{s}\frac{\partial v}{\partial t}. \tag{9.29}$$

From (9.28) and (9.29) we get

$$\frac{\partial v}{\partial t} = -sl\frac{\partial v}{\partial b}, \tag{9.30}$$

and, hence,

$$\frac{\partial^2 v}{\partial s \partial t} = -\left[l\frac{\partial v}{\partial b} + s\frac{\partial l}{\partial s}\frac{\partial v}{\partial b} + sl\frac{\partial^2 v}{\partial b \partial s}\right]. \tag{9.31}$$

Rearranging (9.31) gives (7.5). ■

Proof of Lemma 9: From (4.7) we get

$$\frac{\partial V_j}{\partial t} = \frac{\partial v}{\partial t}(t,b,s_j) - \frac{\alpha}{n-1}\sum_{k>j}\left[\frac{\partial v}{\partial t}(t,b,s_k) - \frac{\partial v}{\partial t}(t,b,s_j)\right] - \frac{\beta}{n-1}\sum_{i<j}\left[\frac{\partial v}{\partial t}(t,b,s_j) - \frac{\partial v}{\partial t}(t,b,s_i)\right]. \tag{9.32}$$

Rearranging (9.32) gives

$$\frac{\partial V_j}{\partial t} = \frac{1}{n-1}\left\{[n-j+(j-1)(1-\beta)]\frac{\partial v}{\partial t}(t,b,s_j) + \beta\sum_{i<j}\frac{\partial v}{\partial t}(t,b,s_i) - \alpha\sum_{k>j}\left[\frac{\partial v}{\partial t}(t,b,s_k) - \frac{\partial v}{\partial t}(t,b,s_j)\right]\right\} \tag{9.33}$$

Hence

$$\frac{\partial V_j}{\partial t} \leq 0 \Leftrightarrow [n-j+(j-1)(1-\beta)]\frac{\partial v}{\partial t}(t,b,s_j) + \beta\sum_{i<j}\frac{\partial v}{\partial t}(t,b,s_i) \leq \alpha\sum_{k>j}\left[\frac{\partial v}{\partial t}(t,b,s_k) - \frac{\partial v}{\partial t}(t,b,s_j)\right]. \tag{9.34}$$

If A1 holds, then

$$\frac{\partial v}{\partial t}(t,b,s_k) - \frac{\partial v}{\partial t}(t,b,s_j) < 0, \tag{9.35}$$

and (7.6) follows from (9.34) and (9.35). ■

Proof of Lemma 10: Starting from $s/(\partial v/\partial b)\frac{\partial}{\partial s}(\partial v/\partial b) = -1$, separate variables to get $\partial(\partial v/\partial b)/(\partial v/\partial b) = -\partial s/s$. Integrate both sides to get $\ln(\partial v/\partial b) =$

$-\ln s + \ln f(t,b)$, where $f(t,b)$ is an arbitrary function of t and b. Take anti-logs to get $\partial v/\partial b = s^{-1} f(t,b)$. Integrate with respect to b, to get $v(t,b,s) = G(t,s) + s^{-1}\int f(t,b)\,db$, where $G(t,s)$ is an arbitrary function of s and t. However, not all such functions will be economically relevant. From the budget constraint, $c = (1-t)\,sl + b$, we see that s and t enter through the combination $s(1-t)$. It follows that G must be a function of $s(1-t)$: $G(t,s) = G(s(1-t))$. It also follows that $f(t,b) = (1-t)^{-1} f(b)$ (to make $s^{-1}\int f(t,b)\,db$ a function of $s(1-t)$). Letting $F(b) = \int f(b)\,db$, we get $v(t,b,s) = s^{-1}(1-t)^{-1} F(b) + G(s(1-t))$. ■

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