JOURNAL OF PUBLIC ECONOMIC THEORY
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Submission Number:JPET-08-00163

Imitation and the Incentive to Contribute early in a Sequential Public Good game

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Abstract

Whether motivated by reciprocity or conformity, imitation is common in public good contexts. We consider the incentive for an agent to contribute to a public good if he expects imitation from others. Using a sequential public good game with exogenous ordering, we show that agents early enough in the sequence who believe imitation to be sufficiently likely would want to contribute. By contributing, they expect total contributions to increase significantly. We also show that preferences determine how early an agent need be, that the observed share of imitators in experiments is sufficiently high to warrant contribution and that an increase in group size reduces the incentive to contribute.

We would like to thank participants at the PET Workshop on Leadership, Altruism and Social Organization for their helpful questions and comments about the paper. The Authors are indebted to two anonymous referees and the Associate Editor, Ted Bergstrom, for many helpful comments, particularly in improving the Proof of Lemma 1. Edward Cartwright also acknowledges the support of ESRC Grant RES-000-22-1999, `Why some people choose to be leaders'.
Citation: Edward Cartwright and Amrish Patel, (2012) "Imitation and the Incentive to Contribute early in a Sequential Public Good game ", *Journal of Public Economic Theory*, Vol. 12 No. 4 pp. 691-708.
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Submitted: August 05, 2008. **Published:** March 08, 2012.

1 Introduction

That individuals are willing to contribute to public goods despite the incentive to free ride has long been known (Ledyard 1995). For example, in a sequential public good game (where individuals make observable contributions in order) any contribution by earlier individuals should crowd out contributions by later individuals (Varian 1994). The evidence, however, shows so called crowding in, whereby the contributions of later individuals are positively correlated with earlier contributions (Fischbacher, Gaechter and Fehr 2001, List and Lucking-Reiley 2002, Guth et al. 2007). This results in both higher contributions than theory would suggest and higher contributions in sequential public good games than are observed in simultaneous public good games (Erev and Rapoport, 1990, Moxnes and van der Heijden, 2003, Masclet and Willinger, 2005, Potters, Sefton and Vesterlund 2005, 2007).

Clearly it is of interest to understand why individuals do contribute to public goods. In a sequential setting we suggest that it is helpful to split the problem into two related but distinct questions. (i) Why do some individuals contribute early on in the sequence, and, (ii) Why are contributions of later individuals positively correlated with those of earlier individuals. Clearly both questions are important but we shall, in this paper, avoid the second question by simply assuming that some individuals reciprocate or conform to previous contributions. We believe that this is a justifiable starting point given the large literature, both theoretical and experimental, that suggests a significant proportion of individuals do indeed imitate their predecessors (Sugden 1984, Fehr and Gaechter 2000, Fischbacher, Gaechter and Fehr 2001, Bardsley and Strausgruber 2005, Burlando and Guala 2005). If we set out to avoid question (ii) then we clearly hope to have something interesting to say about question (i). Before explaining what this is we need to recognize the variety of individual behaviors observed in public good games.

The literature suggests that there is significant heterogeneity in individual behavior, and so while some individuals do reciprocate or conform, others do not (Fischbacher and Gaechter 2008). Various methods have been used to try and categorize individuals into different types (e.g. Offerman, Sonnemans and Schram 1996, Fischbacher, Gaechter and Fehr 2001, Brandts and Schram 2001, Burlando and Guala 2005 and Bardsley and Moffat 2007). While all of these categorizations are open to criticism they all identify the existence of stable behavioral types.[1] These include cooperators, who invest

[1] A secondary claim is that these behavioral types generalize to other settings. Fis-

at the Pareto efficient level, defectors, who contribute little, reciprocators or imitators, who invest some multiple of the average contribution observed so far, and strategists, who behave strategically to maximize own payoff. Reciprocators and imitators are typically found to be the most common, in experiments (for example, 63% of subjects in Kurzban and Houser 2005) but it can be difficult to distinguish a strategist from the other three types. Bardsley and Moffat (2007) estimate that strategists make up 39% of participants in their experiments.

With this heterogeneity in mind, we can now better explain the motivation for our paper. We wish to question *what an individual should do in order to maximize his expected payoff if he expects that some individuals will potentially reciprocate, or imitate, previous contributions.* Answering this question seems crucial in understanding the incentives to contribute to a public good. More specifically, in asking this question we hope to shed some light on (i) by showing that an individual may want to contribute a lot and early to the public good in order to encourage higher contributions from others. Why is this? The individual knows that if he were to contribute, this may motivate some of his successors, who are imitators, to contribute more. Furthermore, the extra contribution of these imitators, may motivate subsequent imitators to invest even more, and so on. The strategist needs to work out how much extra others will contribute in total if he contributes himself. That is he needs to ask how much subsequent imitation will 'multiply' his own contribution. He can then weigh the benefits and costs of contributing and may want to contribute.

We solve explicitly for the extra contribution that there will be (Lemma 1) and can thus say when a strategist should and should not contribute to the public good (Theorem 1). This is done in the setting of a standard linear public good game with an exogenous sequence in which agents invest. Each agent is assumed to randomly, and independently, be one of three possible types: strategist, imitator or independent. An imitator contributes to the public good an amount equal to the average of previous contributions; this is consistent with reciprocation or conformity. An independent agent contributes an amount that is independent of previous contributions; this is a 'catch all' category for any agent who is not influenced by others, such as defectors. A strategist acts to maximize his expected payoff and is therefore

chbacher and Gaechter (2008) provide evidence that elicited social preferences accurately predict contributions in public goods experiments. Laury and Taylor (2008) find that the behavior of experimental subjects is a good predictor of their behavior in real-life social dilemmas. A further claim is that manipulating the types of individuals in a group can influence contributions (Gaechter and Thoni 2005, Ones and Putterman 2007).

the agent of most interest to us.

What we find (Theorem 1 and Corollary 1) is that a strategist who is 'early enough' in the sequence and thinks that subsequent imitation is 'sufficiently likely' should contribute to the public good, while a strategist who is 'late' in the sequence should not contribute to the public good. What is 'early enough' and 'late' depends in a surprisingly simple way on the strategists preference for the public good (Corollary 2). More specifically, if the strategist gets m in payoff for every unit of the public good provided, then 'early enough' is to be one of the first m agents to contribute and late is to be otherwise. Saying what is 'sufficiently likely' is less simple, but we do show (Corollary 3) that for 'plausible parameter values' a strategist may well wish to contribute to the public good. Interestingly, however, we find that the more agents there are, the less incentive for a strategist to contribute (Corollary 4). More agents does mean more agents that could potentially imitate a high contribution, but it also means more agents to share the public good with. This latter effect dominates. Finally, we demonstrate that the decision of those early in the sequence to contribute or not, can have a very large effect on total contributions. For this reason, a strategist may well do best if he is first in the sequence, and so can contribute a lot and influence future imitators (Corollary 5).

The idea that a 'leadership effect' may exist is not new in the literature. Usually, however, leadership effects are seen to result from either imperfect or incomplete information. For example, Andreoni (1998) argues that when there is uncertainty over whether a public project will take-off (due to insufficient funds) a non-contribution equilibrium is frequently observed. 'Leadership givers', those who contribute first, can, however, reassure others that the provision threshold will be reached and thus ensure the public project is well funded. A related story can be told where the quality of a charity or public good is unknown. In this setting the contribution of one individual can signal good quality and thus encourage subsequent contributions (Vesterlund 2003, Andreoni 2006, Potters, Sefton and Vesterlund 2005, 2007). Alternatively, the uncertainty may be over individuals' private valuations of the public good. Announcing early contributions gives contributors an incentive to contribute repeatedly and agents with high valuations an incentive to contribute early to conceal their type (Bag and Roy 2008).

The arguments made in this paper do not rely on a lack of information. Instead, the leadership effect results from the heterogeneity in behavior. In particular, the expectation that some individuals will imitate provides the incentive for a strategist to lead. Masclet and Willinger (2005) make a related argument when analyzing experimental data in which early contributors appear to invest more so as to influence subsequent contributors. They do not, however, highlight the importance of heterogeneous behavior, but this does appear crucial: If everyone is a strategist then no-one would ever contribute to the public good and if everyone is an imitator then contributions should not fall over time. So, at least in the model considered in this paper, we do need both strategists and imitators to obtain a leadership effect.

The rest of the paper is structured as follows. We introduce a sequential public good game with exogenous sequencing in Section 2. Solving for a rational agent's optimal strategy in Section 3, we present our results. In Section 4 we conclude with possible generalizations and qualifications to our results.

2 Model

There exists a set of *agents* $N = \{1, 2, ..., n\}$. Each agent $i \in N$ is endowed with e units of a *private good* . There are n *time periods* indexed $t = 1, ..., n$. In period t, agent $i = t$ must decide how much of the private good to invest in a *public project*. He can invest any amount in the interval $[0, e]$. The amount invested by agent i will be denoted k_i. After period n total investment is calculated

$$K := \sum_{i \in N} k_i \tag{1}$$

and the payoff of player i is

$$u_i(k_i, K) = e - k_i + \frac{m_i K}{n} \tag{2}$$

where m_i is some real number between 1 and n that measures preference for the public project. It should be clear that this is a standard linear public good game with sequential investments. Agents, therefore, derive utility from the private good and their share of the returns from the public good. Note that we do permit agent heterogeneity in preference for the public project m_i. We could also allow heterogeneity in endowment of the private good but only at the cost of significantly complicating the analysis.

2.1 Agent types

We shall distinguish three different types of agent: strategist, imitator and independent. A *strategist* will choose the level of investment that maximizes his expected payoff. More on this below. An *imitator* will invest g if he is agent 1, where $g \in [0, e]$ is some real number that can be thought of as the *generosity* of an imitator, and will invest $\overline{k}_i$ if he is agent $i > 1$ where[2]

$$\overline{k}_i := \frac{1}{i-1} \sum_{j=1}^{i-1} k_j.$$

We see, therefore, that unless the imitator is the first to invest, he will invest an amount equivalent to the mean contribution so far. Imitation could arise for different reasons, such as a desire for reciprocity, or because it is a simple rule of thumb to use in a complex game.[3] The main thing to note for our purposes is that the investment of an imitator will depend on previous investments. This is not the case for an independent agent. An independent agent will invest an amount that does not depend on previous investments. For example, he may be a 'cooperator' who invests e, a 'defector' who invests 0 or a 'could not care' who invests a random amount. Basically, the agent type 'independent' is a catch all for any agent whose investment is not influenced by previous investments. Indeed we do not rule out the possibility that a strategist is also an independent agent. This will be true if the optimal strategy does not depend, as will actually prove to be the case, on others investments.

How types are actually distributed amongst players is not of great concern for us here. What is important is how likely a strategist believes it is that others are imitators. We shall assume throughout that *if agent i is a strategist then he believes that the probability player $j > i$ is an imitator is* p_i (and agent j's type is independent of the types of agents $1, ..., j-1$). Furthermore, p_i is assumed to not depend on the investment of agents $1, ..., i-1$. This assumption is consistent with the possibility that each agent is, independently, an imitator with some probability p and strategists know this. In this case we would have $p_i = p$ for any strategist i. The assumption is also consistent with each strategist having his own subjective prior on the probability of imitation. In this case a relatively high p_i indicates that i considers subsequent imitation likely.[4]

[2] Generosity g will prove largely irrelevant for us in saying what a strategist will do. We shall, however, see that it is very important in determining total investment.

[3] More subtle imitator behavior is observed in experiments. For example, individuals may invest some fraction/multiple of mean investment and/or a function of other statistics of previous investments such as median, mode and variance (Carpenter 2004, Burlando and Guala 2005, Fischbacher and Gaechter 2008). The simple rule that we assume has the benefit of tractability and transparency. Also, perhaps because of its simplicity, it does appear to be consistent with how many people do behave in public good settings (Fischbacher, Gaechter and Fehr 2001).

[4] As we discuss further in the conclusion, our results do also extend to situations with

3 A strategist's optimal strategy

While an imitator or independent agent follows a simple rule of thumb when investing we clearly have to do more work to find the investment of a strategist. In particular, the strategist has to contend with two countervailing incentives. On the one hand, investing involves a cost in terms of the private good. On the other hand, investing may increase the investment of others because of imitation (or, perhaps more importantly, not investing may decrease the investment of others because of imitation). Whether or not the strategist wishes to invest will depend on the relative magnitude of these incentives.

The cost of investing is clear but the benefit of investing is not so clear. What we need to know is the, ceteris paribus, extra investment that agent i expects others to make if he invests more himself. That is, we need to know how much the investment of agent i will be *multiplied* by subsequent imitation. In order to derive this we denote by $Y(i, r, k_i, \overline{k}_i)$ agent i's expectation of the total investment of agents $i, i+1, ..., i+r$ if he invests k_i and the average investment to date is $\overline{k}_i$.[5] The following lemma characterises the multiplier.

Lemma 1: If strategists are independent agents then, for any $r > 1, i, k_i$ and $\overline{k}_i$,

$$Y(i, r, k_i, \overline{k}_i) - Y(i, r, 0, \overline{k}_i) = k_i \mu(i, i+r)$$

where.

$$\mu(i, i+r) := \frac{(i-1)!}{(i+r-1)!} \prod_{j=0}^{r-1} (i + p_i + j) \tag{3}$$

Proof: By assumption, agent i's choice of investment will not influence the amount invested by subsequent agents if they are independents or strategists.

Bayesian updating. In this case the strategist can update his prior as he observes the investment of others, and p_i will reflect his posterior belief. The complication in doing this is that p_i will depend on previous investments and so a strategist may want to 'behave like' an imitator. Our results can only be applied if there is a 'separating equilibrium' where strategists would not want to behave like imitators. Our prior is that this should typically be the case.

[5] That is,

$$Y(i, r, k_i, \overline{k}_i) = k_i + E\left[\sum_{j=i+1}^{i+r} k_j \middle| \overline{k}_{i+1} = \frac{\overline{k}_i + k_i}{i}\right]$$

where $E[\cdot]$ is the expectations operator.

All that matters, for our purposes, therefore, is whether agents $i+1$ to $i+r$ are imitators or not. There are 2^r possible sequences of imitator and not imitator. Let ε_l denote a typical sequence. For any given sequence ε_l one can calculate the increased total contributions v_l (if agent i invests k_i rather than 0) and the probability of the sequence q_l.[6] Let, $\Delta Y(i,r,k_i,\overline{k}_i) := Y(i,r,k_i,\overline{k}_i) - Y(i,r,0,\overline{k}_i)$ we can write

$$\Delta Y(i,r,k_i,\overline{k}_i) = \sum_{l=1}^{2^r} q_l v_l.$$

In order to derive an iterative relation we next ask what agent $i+r+1$ can be expected to do. For any sequence ε_l, of agents $i+1$ to $i+r$, agent $i+r+1$ will invest an extra $\frac{v_l}{i+r}$ if he is an imitator and 0 if he is not. Given that agent $i+r+1$ is expected by agent i to be an imitator with probability p_i the expected increase in total investment, conditional on sequence ε_l, is, therefore,

$$v_l + p_i \frac{v_l}{i+r} = v_l \left(1 + \frac{p_i}{i+r}\right).$$

Thus, the expected increase in total contributions is,

$$\begin{aligned} \Delta Y(i,r+1,k_i,\overline{k}_i) &= \sum_{l=1}^{2^r} \left(q_l v_l \left(1 + \frac{p_i}{i+r}\right)\right) = \left(1 + \frac{p_i}{i+r}\right) \sum_{l=1}^{2^r} q_l v_l \quad (4) \\ &= \left(1 + \frac{p_i}{i+r}\right) \Delta Y(i,r,k_i,\overline{k}_i). \quad (5) \end{aligned}$$

It is simple to see that

$$\Delta Y(i,1,k_i,\overline{k}_i) = k_i \left(1 + \frac{p_i}{i}\right). \quad (6)$$

Recursive use of (4) together with (6) gives that

$$\Delta Y(i,r,k_i,\overline{k}_i) = k_i \left(1 + \frac{p_i}{i+r-1}\right) \left(1 + \frac{p_i}{i+r-2}\right) ... \left(1 + \frac{p_i}{i+1}\right) \left(1 + \frac{p_i}{i}\right) \quad (7)$$

We can simplify equation (7),

$$\begin{aligned} k_i \left(1 + \frac{p_i}{i+r-1}\right) ... \left(1 + \frac{p_i}{i}\right) &= k_i \left(\frac{i+r-1+p_i}{i+r-1}\right) ... \left(\frac{i+1+p_i}{i+1}\right) \left(\frac{i+p_i}{i}\right) \quad (8) \\ &= k_i \frac{(i-1)!}{(i+r-1)!} \prod_{j=0}^{r-1} (i+p_i+j) \quad (9) \end{aligned}$$

[6] The value v_l includes the extra investment k_i of agent i.

to give the desired result.■

Lemma 1 demonstrates that the extra investment agent i expects agents $i, i+1, ..., n$ to make if he invests one extra unit in the public project is

$$\mu(i,n) = \frac{(i-1)!}{(n-1)!} \prod_{j=0}^{n-i-1} (i + p_i + j) \tag{10}$$

Note that the (certain) extra investment agent i is included and so $\mu(i,n) \geq 1$. The higher is $\mu(i,n)$ then the more is i's investment multiplied by subsequent imitators. To put Lemma 1 into some context, Figure 1 plots $\mu(i,n)$ for combinations of i, r and p_i. A value of 3, for example, means that an extra investment of 1 by agent i leads to an expected increase in total investment (including the original 1) of 3. So, the extra investment is effectively tripled.

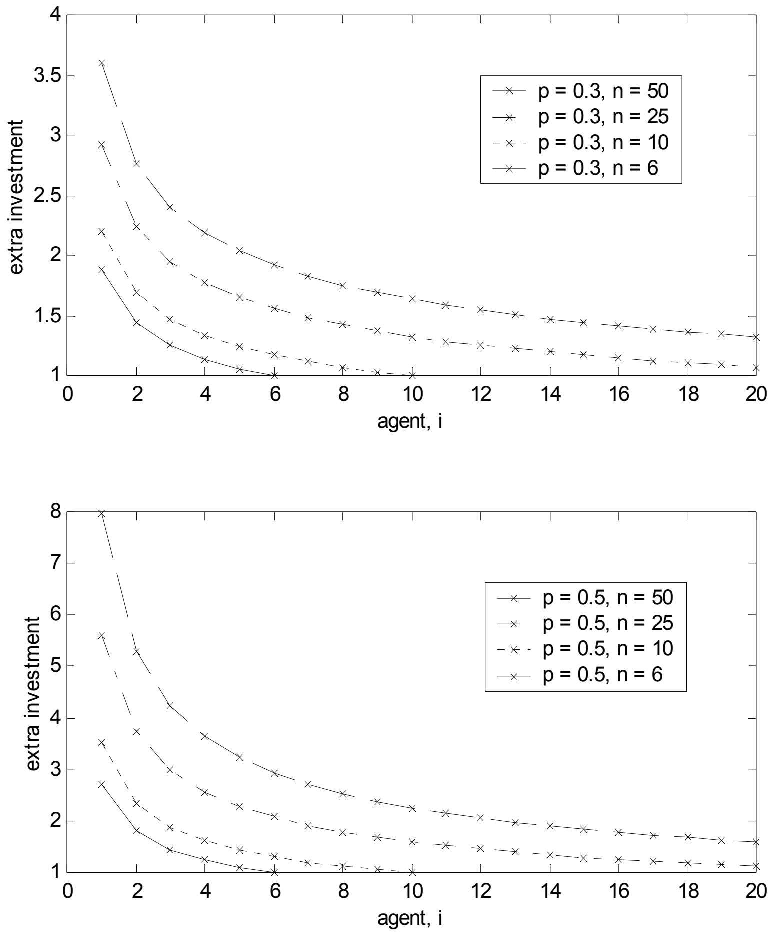

Figure 1: The extra investment if agent i invests 1 rather than 0 (i.e. $\mu(i, n)$) for different values of p_i and n.

The most striking thing we learn from Figure 1 is how quickly the expected increase in investment falls with i. This is because investment by agent 1 can be expected to result in significantly more subsequent investment than would investment by agent 2, and so on. More specifically, agent 1 will determine $\overline{k}_2$, and thus can significantly influence the investment of agent 2, and those that follow. Agent 2, by contrast, can merely change

$\overline{k}_3$, and so will have relatively less influence on the investment of agent 3, and those that follow. Agent 3 has even less opportunity to change $\overline{k}_4$, and so on.[7] A second notable feature of Figure 1 is that the extra investment can be relatively large, particularly if n and/or p_i is large. For example, a value of $\mu(i,n) = 2$, meaning that any investment by agent i is doubled, significantly increases the incentive for agent i to invest.

The trade-offs that a strategist i will face can now be more clearly explained. Investing k_i will cost k_i in private good and will yield benefit of $\frac{m_i}{n} k_i \mu(i,n)$ from the public project. This leads immediately to our main result.

Theorem 1: If agent i is a strategist then he should invest e in the public project if

$$\frac{m_i}{n}\mu(i,n) > 1$$

and 0 if this inequality is reversed.

Proof: If agent i invests k_i in the public project then his expected payoff is

$$\pi_i(k_i, \overline{k}_i) := e - k_i + \frac{m_i}{n}\left[(i-1)\overline{k}_i + Y(i, n-i, k_i, \overline{k}_i)\right].$$

Thus, for any $k_i' < k_i$,

$$\pi_i(k_i, \overline{k}_i) - \pi_i(k_i', \overline{k}_i) = k_i' - k_i + \frac{m_i}{n}[\Delta Y(i, n-i, k_i, \overline{k}_i) - \Delta Y(i, n-i, k_i', \overline{k}_i)]$$

If strategists are also independent agents then using Lemma 1 we get that

$$\begin{aligned}\pi_i(k_i, \overline{k}_i) - \pi_i(k_i', \overline{k}_i) &= k_i' - k_i + \frac{m_i}{n}\mu(i,n)\left(k_i - k_i'\right) \\ &= \left(k_i - k_i'\right)\left(\frac{m_i}{n}\mu(i,n) - 1\right).\end{aligned}$$

So, agent i should invest the maximum e if $m_i\mu(i,n) > n$, the minimum 0 if this inequality is reversed, and is indifferent otherwise. We are not quite done yet because we have assumed that strategists are independent. This can, however, be seen by using backward induction from agent n. Specifically: If agent n is a strategist then he will invest 0. This means that if agent $n-1$ is a strategist he knows that his investment will not influence a strategist agent n. Thus, he invests if $m_{n-1}\mu(n-1,n) > n$ and does not invest if

[7] This does not imply that a strategist would rather be first, i.e. agent 1, but more on this below.

$m_{n-1}\mu(n-1,n) < n$. This means that if agent $n-2$ is a strategist he knows that his investment will not influence a strategist agent $n-1$ and/or agent n. So, he invests if $m_{n-2}\mu(n-2,n) > n$ and does not invest if $m_{n-2}\mu(n-2,n) < n$. And so on.■

Theorem 1 confirms that an expectation of subsequent imitation may provide sufficient incentive for a strategist to invest in the public project. As one would intuitively expect $\mu(i,n)$ is increasing in both m_i and p_i and so a strategist has a greater incentive to invest the higher his preference for the public project and the higher the chance of subsequent imitation. For example, if the strategist believes that subsequent imitation is unlikely (as in the 'textbook public good story') then $\mu(i,n)$ is close to 1 and the strategist will not invest (because $\frac{m_i}{n} < 1$). If the strategist believes that subsequent imitation is more likely then he may wish to invest. Less obvious when looking at relation (10) but more easily apparent from looking at (9) is that a strategist also has a greater incentive to invest the earlier he appears in the sequence. This leads to the following simple corollary of Theorem 1.

Corollary 1: If $p_i = p$ and $m_i = m$ for some p and m and all strategists i then: If $\frac{m}{n}\mu(1,n) < 1$ all strategists invest 0 while if $\frac{m}{n}\mu(1,n) > 1$ there exists some $i^* \geq 1$ such that any strategist $i \leq i^*$ would invest e and any strategist agent $i > i^*$ would invest 0.

If, therefore, all strategists are homogenous, strategists early in the sequence invest their full endowment while strategists later in the sequence invest zero. The reason for this is the diminishing influence, clearly seen in Figure 1, that a strategist later in the sequence will have on imitators. Strategists early enough in the sequence invest e because of an expectation that their investment (for better or worse) will significantly impact on subsequent investment. Strategists late enough in the sequence, by contrast, will invest 0 because of an expectation that their investment will not impact enough to matter.

This raises the question of what is early and late in the sequence. The following result shows that preference for the public project m_i is a good measure. Specifically, if $i > m_i$ then agent i would never invest while if $i \leq m_i$ agent i would want to invest if he believes subsequent imitation is sufficiently likely. In interpretation it may be useful to think of m_i/n rather than m_i. For example, if $m_i = 0.5n$ then agent i may invest if in the first half of the sequence but would definitely not invest if in the last half of the sequence.

Corollary 2: A strategist agent i would invest 0 if $m_i < i$. A strategist agent i would invest e if $m_i \geq i$ and p_i is sufficiently high.

Proof: Putting $p_i = 1$ into (10) gives

$$\mu(i,n) = \frac{(i-1)!}{(n-1)!} \prod_{j=1}^{n-i} (i+j) = \frac{(i-1)!}{(n-1)!} \frac{n!}{i!} = \frac{n}{i}$$

and so, by Theorem 1, we see that agent i invests e if $m_i \geq i$ and will invest 0 if $m_i < i$. Given that agent i's incentive to invest is increasing in p_i we get the desired result.■

Whether or not a strategist who is early in the sequence would want to invest still depends on how likely he believes will be subsequent imitation. For example, in an experimental setting Moxnes and van der Heijden (2003) found that the high investment of a leader did lead to increased subsequent investment but not enough to pay back the cost of leading.[8] So, how much imitation is needed for a strategist to want to invest? In order to get some feel for this question we can look at the choice facing agent 1. Substituting $i = 1$ into (10) and applying Theorem 1 we get

Corollary 3: A strategist agent 1 would invest e if

$$\frac{m_1}{n} \geq \prod_{j=1}^{n-1} \frac{j}{p_i + j}. \tag{11}$$

and 0 otherwise.

From Corollary 3 we can calculate how high need be p_1 for agent 1 to want to invest in the public project. Table 1 gives some representative values, and recall that typical estimates for the proportion of imitators are 0.4 to 0.65 (Fischbacher, Gaechter and Fehr 2001, Kurzban and Houser 2005) Table 1 illustrates that for 'plausible parameter values' a strategist agent 1 may or may not want to invest in the public project. In particular, the strategists preference for the public good does need to be 'high enough', where a preference of $m_1 = 2$ is not enough and that of 5 to 8 does seem enough. Whichever way we look at it, however, the figures in Table 1 do suggest that the possibility of imitation can make it worthwhile for a strategist to invest in the public project.

[8] Interestingly it seems that many leaders did not invest a high amount seemingly anticipating that the rewards would not warrant it.

Table 1: The minimum value of p_1 such that agent 1 would want to invest.

$n \backslash m_1$	2	3	5	8	15
6	0.56	0.33	0.08	0.00	0.00
10	0.66	0.48	0.26	0.08	0.00
25	0.76	0.63	0.46	0.32	0.14
50	0.81	0.70	0.56	0.44	0.28
100	0.84	0.74	0.63	0.52	0.39

One notable thing we observe in Table 1 is how a, ceteris paribus, increase in n makes it less likely that agent 1 would want to invest in the public project. This may seem surprising but is a general property.

Corollary 4: An increase in n, keeping m_i/n constant, makes it more likely that a strategist will invest in the public project. An increase in n, keeping m_i constant, makes it less likely that a strategist agent would invest in the public project.

Proof: The first part of the Corollary is a trivial consequence of Theorem 1. For the second part we can use (10) and compare

$$\begin{aligned}\frac{\mu(i,n+1)}{n+1} - \frac{\mu(i,n)}{n} \quad : \quad &= \frac{(i-1)!}{(n+1)!}\prod_{j=0}^{n-i}(i+p_i+j) - \frac{(i-1)!}{n!}\prod_{j=0}^{n-i-1}(i+p_i+j) \\ &= \frac{(i-1)!}{n!}\prod_{j=0}^{n-i-1}(i+p_i+j)\left[\frac{n+p_i}{n+1}-1\right] < 0\end{aligned}$$

to give the desired result.■

At first sight, an increase in n would seem to increase the incentive for a strategist to invest in the public project because 'there is one more agent to influence'. This is captured by the first part of Corollary 4. An increase in n does, however, mean there is 'one more agent to share any returns from the public project with'. A priori it does not seem too obvious which effect will be larger, particularly if p_i is large, but Corollary 4 shows that the latter effect dominates. The basic reason for why this proves to be the case is that the strategist has relatively little influence on the '$n+1$'th agent' but for sure has to share the public project with him. Overall this means less reason to invest in the public project.

Of principal interest is clearly how much will be invested in the public project. If the first few agents in the sequence are strategists (who do invest in the public project), or 'initially generous imitators', then we can expect high total investment in the public project. If, by contrast, the first few investors invest little (because, for example, they are strategists who do not invest, or are less generous imitators) then we should expect low total investment in the public project. Whether or not total investment is high will thus depend on the generosity of imitators (and independent agents) and chance, which determines the type of early agents.

It is simple to come up with examples where the type of the first agent can result in a difference in total investment as large as ne. For instance, compare a scenario where agent 1 is a strategist and agents $2, ..., n$ are imitators, versus a scenario where agent 1 invests 0 (for whatever reason) and agents $2, ..., n$ are imitators. In the first scenario total investment is ne and in the later 0. Lemma 1 and Figure 1 illustrate the 'more realistic' differences in investment that can result if an agent is a strategist (who invests) versus an agent who does not invest. To put this into context we provide an additional Figure 2. The top panel of Figure 2 gives the expected total investment, for varying values of p, n and generosity g.[9] Combining this with the data from Figure 1, the bottom panel of Figure 2 gives the potential influence that agent 1 can have relative to expected total investment. For example, when $p = 0.5, n = 10$ and $g = 0.5$, expected total investment is around 2 (from the top panel of Figure 2), agent 1 can expect to increase total investment by 3.5 if he invests in the public project (see Figure 1), and so the relative influence of agent 1 is around 1.75.

[9] The values are calculated assuming that $e = 1$, $m_i = n/2$ for all i and that with probability 0.3 each agent is an independent agent who invests 0.

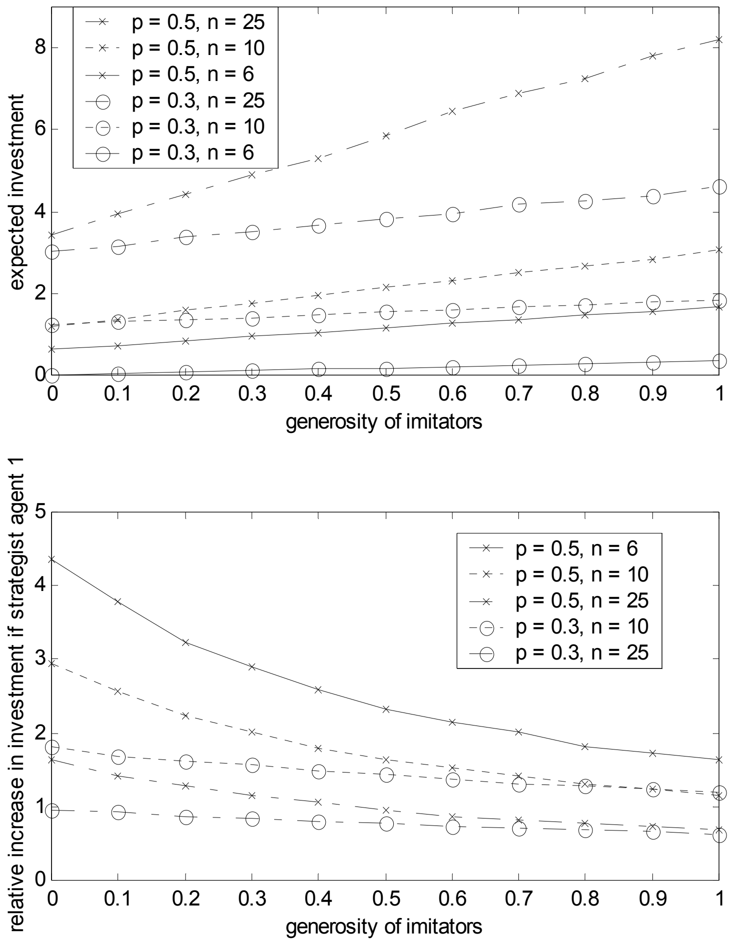

Figure 2: Total expected investment and the relative influence of agent 1 for varying values of p, n and g assuming that $e = 1$, $m_i = n/2$ for all i and an agent is independent and invests 0 with probability 0.3.

Figure 2 suggests that agent 1 does have a relatively large influence on the success of the public project. As we would expect, the larger is n the less this influence, but it continues to remain relatively high. Not only, therefore, can the investment of a strategist agent 1 'pay itself back to agent

1' it can also have a big say on the 'relative success' of the public project. This is consistent with the history dependence observed in laboratory and field experiments where leaders' contributions are critical (List and Lucking-Reiley 2002, Moxnes and van der Heijden 2003, Guth et al. 2007).

That agent 1 is critical in determining total investment leads us to one final issue: If a strategist could choose where he is in the sequence then where would he choose? A first point to note is that the investment of a strategist will depend only on his position in the sequence and not on previous investments. This means, that given a preference for the public project m and number of agents n, we can find an i^* such that a strategist agent i would invest e if $i \leq i^*$ and would invest 0 if $i > i^*$. The expected payoff of the strategist (taken before any agent has invested) must be decreasing in i for all $i \leq i^*$. This is because the strategist is going to invest the maximum amount e and so the earlier he is in the sequence the more imitators he can potentially influence by his high investment. The expected payoff of the strategist must, however, be increasing in i for all $i > i^*$. This is because the strategist is going to invest the minimum 0 and so the later he is in the sequence the less imitators he will potentially influence with his low investment. This leads to the following result.

Corollary 5: The expected payoff of a strategist is highest if either agent 1 or agent n.

The strategist, therefore, would either like to be first or last in the sequence. He would want to be first if he expects low initial investments to lead to low total investment because his investment can influence things positively. By contrast, he would want to be last if he expects high initial investments that will lead to a high total investment because he can free-ride and not influence anyone. Related, is the issue of first mover advantage. In a sequential public good game, with non-linear returns from the public good, the Nash equilibrium typically suggests a first mover advantage. This is because the first agent in the sequence should contribute little meaning that those later in the sequence do need to contribute (Varian 1994). The evidence, however, is that first movers may not gain as much as predicted by moving first, and may even be at a disadvantage (Gächter et. al. 2009). Imitation and reciprocity are one clear reason why, and Corollary 3 captures this, by showing that the first agent should contribute if he believes subsequent imitation is likely. It is worth noting, therefore, that Corollary 5 does not imply an advantage to being first. It does reflect how a strategist would rather be first than, say, second in the sequence, because he will then have more influence. A strategist who is first must, however, get a weakly lower payoff than any other strategist, because he will invest all his endowment in the public project. There is, therefore, a first mover disadvantage.

4 Conclusion

Our objective in this paper was to see how a 'rational agent' should behave in a sequential public good game if he expects imitation by others. We have considered a simple linear public good game and characterized the optimal strategy. What we find is that an agent should invest in the public good if he is 'early enough' in the sequence and the probability of imitation is 'high enough', but should not invest otherwise. 'Early enough' can be measured by the ratio of the agent's return to investing in the public good and the number of players. We have also showed that total investment can depend significantly on early investments meaning that the early investment of a rational agent can have a significant effect on the 'success' of the public good.

Given that we have considered a simple linear public good game with an exogenous sequence there are plenty of avenues for generalization and/or further study. For instance, we may want to allow diminishing returns to investment in the public good. This would presumably nuance the optimal strategy in several ways. We should observe, for example, optimal investments that fall between the extremes of everything or nothing. We should also expect the optimal strategy to be conditional on the investment to date, with lower investments encouraging a rational agent to invest more.

Two further distinct issues worthy of note are that of Bayesian updating on the probability of imitation and an endogenous sequence to investments. Taking up the first of these issues, we assumed that a rational agent had an *exogenous* belief that others would be imitators. In reality a rational agent may update his belief while observing the investments of others. This is not a problem for our model, unless there is an incentive for a rational agent to invest 'as if' an imitator in order that future rational agents will consider imitation more likely and therefore invest more. That is, our results do not allow for 'pooling equilibria' where rational agents try to look like imitators. It is fairly simple to come up with examples that show such pooling is possible.[10] In most instances, however, it seems highly unlikely that it

[10] Consider a strategist agent i deciding whether to invest. He 'can see' that if agent $i+1$ is a strategist then he will 'very marginally' decide to invest 0. If agent i invests $\overline{k}_i$ then agent $i+1$ will slightly increase his belief p_{i+1} that others are imitators. [Note that

would happen, and so while we do consider this an interesting avenue to explore we do not think it diminishes our results to ignore such possibilities.

The possibility of endogenizing the sequence of investments is an intriguing one. We have shown (Corollary 5) that a rational agent would prefer to be first or last if the sequence is exogenous. This, however, is not easily extended to obtain a result with an endogenous sequence. With an exogenous sequence a rational agent must invest at his specified point in the sequence and so faces the choice of *how much to invest* knowing that this will influence others. For example, in an exogenous sequence a rational agent who invests zero will likely influence others and so this creates an incentive to invest more than zero. By contrast, in an endogenous sequence, a rational agent can choose *when to invest*. He can, for instance, wait until last to invest zero, and, therefore, influence nobody. This is not to say, of course, that rational agents will delay and invest zero because we have seen that by investing more they can significantly increase total investment. We leave further exploration of this issue to future work.

References

[1] Andreoni, J., 1998, "Toward a theory of charitable fund raising," *Journal of Political Economy*, 106(6): 1186-1213.

[2] Andreoni, J., 2006, "Leadership giving in charitable fund-raising," *Journal of Public Economic Theory*, 8(1): 1-22.

[3] Bag, P. K. and Roy, S., 2008, "Repeated charitable contributions under incomplete information," *Economic Journal*, 118: 60-91.

[4] Bardsley, N. and Moffat, P.G., 2007, "The experimetrics of public goods: inferring motivations from contribution," *Theory and Decision*, 62: 161-193.

[5] Bardsley, N. and Strausgruber, R., 2005, "Conformity and reciprocity in public good provision," *Journal of Economic Psychology*, 26: 664-681.

[6] Brandts, J. and Schram, A., 2001, "Cooperation and noise in public goods experiments: applying the contribution function approach," *Journal of Public Economics*, 79: 399-427.

this will happen even if $i+1$ knows that a strategist would invest $\overline{k}_i$.] This slight increase in p_{i+1} may tip the balance so that a strategist agent $i+1$ would now invest in the public project. This may mean that i has a higher expected payoff from investing $\overline{k}_i$.

[7] Burlando, R.M. and Guala, F., 2005, "Heterogeneous agents in public goods games," *Experimental Economics*, 8: 35-54.

[8] Carpenter, J. P., 2004, "When in Rome: conformity and the provision of public goods," *Journal of Socio-Economics*, 33: 395-408.

[9] Erev, I. and Rapoport, A., 1990, "Provision of step-level public goods: the sequential contribution mechanism," *Journal of Conflict Resolution*, 34(3): 401-425.

[10] Fehr, E. and S. Gaechter, 2000, "Fairness and Retaliation: The Economics of Reciprocity," *Journal of Economic Perspectives,* 14(3): 159-181.

[11] Fischbacher, U., S. Gaechter and E. Fehr, 2001, "Are people conditionally cooperative? Evidence from a public goods experiment," *Economics Letters*, 71: 397-404

[12] Fischbacher, U. and S. Gaechter, 2008, "Heterogeneous social preferences and the dynamics of free-riding in public good experiments," CeDEx Discussion Paper No. 2008-07.

[13] Gaechter, S. and Thoni, C., 2005, "Social learning and voluntary cooperation among like-minded people," *Journal of the European Economic Association*, 3: 303-314.

[14] Gaechter, S., D. Nosenzo, E. Renner and M. Sefton, 2009, "Sequential versus simultaneous contributions to public goods: Experimental evidence" CeDEX Discussion Paper No. 2009-07.

[15] Guth, W., Levati, M. V., Sutter, M. and van der Heijden, E., 2007, "Leading by example with and without exclusion power in voluntary contribution experiments," *Journal of Public Economics*, 91: 1023-1042.

[16] Kurzban, R. and Houser, D., 2005, "Experiments investigating cooperative types in humans: a complement to evolutionary theory and simulations," *Proceedings of the National Academy of Sciences*, 102(5): 1803-1807.

[17] Laury, S. K. and Taylor, L. O., 2008, "Altruism spillovers: are behaviors in context-free experiments predictive of altruism toward a naturally occurring public good," *Journal of Economic Behavior and Organization*, 65(1): 9-29.

[18] Ledyard, J. O., 1995, “Public goods: A survey of experimental research,” in *The Handbook of Experimental Economics* J. Kagel and A. Roth (Eds.) Princeton University Press.

[19] List, J. A. and Lucking-Reiley, 2002, "The effects of seed money and refunds on charitable giving: experimental evidence from a university capital campaign," *Journal of Political Economy*, 110(1): 215-233.

[20] Masclet, D. and Willinger, M., 2005, "Does contributing sequentially increase the level of cooperation in public goods games, An experimental investigation," working paper.

[21] Moxnes, E. and van der Heijden, E., 2003, "The effect of leadership in a public bad experiment," *Journal of Conflict Resolution*, 47(6): 773-795.

[22] Offerman, T., Sonnemans, J. and Schram, A., 1996, "Value orientations, expectations and voluntary contributions in public goods," *Economic Journal*, 106: 817-845.

[23] Ones, U. and Putterman, L., 2007, "The ecology of collective action: a public goods and sanctions experiment with controlled group formation," *Journal of Economic Behavior and Organization*, 62: 495-521.

[24] Potters, J., Sefton, M. and Vesterlund, L., 2005, "After you - endogenous sequencing in voluntary contribution games," *Journal of Public Economics*, 89: 1399-1419.

[25] Potters, J., Sefton, M. and Vesterlund, L., 2007, "Leading-by-example and signalling in voluntary contribution games: an experimental study," *Economic Theory* 33: 169-182.

[26] Sugden, R., 1984, "Reciprocity: the supply of public goods through voluntary contributions," *Economic Journal*, 94: 772-787.

[27] Varian, H. R., 1994, "Sequential contributions to public goods," *Journal of Public Economics*, 53: 165-186.

[28] Vesterlund, L., 2003, "The informational value of sequential fundraising," *Journal of Public Economics*, 87: 627-657.