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Ex post welfare under alternative health care systems

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Abstract

The implications of a societal aversion to inequality for the optimal structure of the health care system are studied. Agents are assumed to be ex ante identical, but to differ ex post in the state of their health. Inequality aversion is introduced by postulating a strictly concave ex post social welfare function. It is shown that the optimal public health care system allocates health care differently than would private health insurance; specifically, people who are relatively unhealthy with and without treatment receive more health care, and people who are relatively healthy with and without treatment receive less health care. The aggregate quantity of health care under the optimal public health care system can be either greater or less than under private health care insurance. If the public health care system is optimally designed, allowing agents to purchase supplementary private health care insurance cannot raise social welfare and is likely to decrease it.

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1. Introduction

Although public health care developed largely in response to concerns over equity, these concerns are missing from almost all of the existing studies of the allocative effects of health care programs.[1] The intent of the current paper is to address this omission, with an emphasis on two questions. First, how does inequality aversion affect the structure of a public health care program? Second, does inequality aversion cause societies to prefer public health care to private health care?

Two modelling issues must be resolved if these questions are to be answered. The first issue is the manner in which inequality aversion is introduced. One option is Tobin's (1970) specific egalitarianism, under which society's preference for relatively equal distributions extends only to particular goods, health care being one example. Another option is Pauly's (1971) health care externality, under which each agent cares about the other agents' health as well as his own. The externality approach is interesting because it implements specific egalitarianism without forsaking the tools of welfare economics, but it can be analytically cumbersome. A third option—the one that is adopted here—assumes that each agent cares only about his own health, and that the policy-maker maximizes a social welfare function that is increasing and strictly concave in the utilities. The strict concavity of the social welfare function implies inequality aversion.

The second issue is the characterization of the agents' uncertainty about the state of their own future health. The assumption made here is that there is individual risk but no aggregate risk. (The number of agents who will break their legs is known, but their identities are not known.) This assumption is not uncommon in the health literature, but its implications are unusually significant in the present instance. There are competing concepts of social welfare whenever uncertainty is present. The two most common ones are the *ex ante social welfare function*, which is a function of the agents' expected utilities, and the *ex post social welfare function*, which is the expected value of a function of the agents' realized utilities.[2] These functions are mathematically equivalent when the social welfare function is a weighted sum of the

[1] An exception is Fleurbaey's (2005) discussion of the socially optimal health care system, which assumes that the reduction of income inequality is socially desirable. For the evolution of public health care in Europe and elsewhere, see Cutler (2002)

[2] The early study of these function is contained in Starr (1973), Mirrlees (1974), Harris (1978) and Hammond (1981).

utilities, but they are not equivalent under the strict concavity assumption. Arguments from Broome (1984) and Fleurbaey (2005) show that the ex post social welfare function is the appropriate one when there is no aggregate risk. These arguments are discussed in detail in the next section.

It is sometimes argued that public health care is preferred to private health care because it corrects a market imperfection, typically an informational asymmetry. Nichols and Zeckhauser (1982), Blackorby and Donaldson (1988), and Besley and Coate (1991) show that the public provision of some private good (such as health care) relaxes the selection constraints when the government does not have perfect information about the attributes of agents. Rochet (1991) and Cremer and Pestieau (1996) show that social insurance (health care is again one example) can increase the extent of redistribution if there is a negative correlation between income and the risks to health. Their arguments have been extended by Petretto (1999) and by Boadway, Leite-Monteiro, Marchand and Pestieau (2003, 2006).

The model presented here sets out an entirely different basis for comparing these two systems. It assumes that the information available to the government is the same as the information available to the agents. Under a system of private health care insurance the agents choose the insurance contract that maximizes their expected utilities. Under a public health care system the government designs its health care program to maximize a strictly concave social welfare function. If this function were the ex ante function (i.e., a function of the expected utilities), the choices that maximize expected utility would also maximize social welfare and public health care would not be preferred to private health care. However, the appropriate social welfare function in the absence of aggregate risk is the ex post function (i.e., the expected value of a function of realized utilities). The choices that maximize expected utility do not maximize this function, causing public health care to be socially preferred to private health care.

The difference in social welfare under private and public health care is, of course, a reflection of differences in the way health care is allocated under the two systems. The model assumes that the agents are ex ante identical but uncertain about their future health. Future health is represented by a triplet whose elements are the state of health without treatment, the state of health with treatment, and the cost of treatment. A production possibility frontier describes the combinations of aggregate health care and aggregate consumption that the economy can produce. Each health care system is characterized by the kinds of agents that are treated under that system. This decision directly determines aggregate health care and aggregate consumption. The treatment groups under the two systems are different: a public health care system shifts health care away from those who are relatively healthy with and without treatment, and towards those who are relatively unhealthy with and without treatment.[3] Aggregate health care could be either larger or smaller under public health care than under private health care.

There is a laissez-faire argument that a system of private health care insurance, operated in conjunction with the public health care system, must raise welfare: those who purchase private health insurance are better off, while those who do not purchase it are unaffected. This argument does not hold here. A parallel system of private health insurance moves the economy toward the allocation that would occur under a purely private health insurance system, and that system does not maximize ex post social welfare. An attempt to supplement public health care with private health insurance can have only two outcomes: either no one purchases it or ex post welfare falls.

The government's first-best policy—defined as its policy in the absence of active private insurers—takes a very simple form: the social value of another unit of health care is equated to the social value of the consumption forgone when this unit is produced. This policy is also its second-best policy—defined as its policy in the presence of active private insurers—if the marginal cost of health care is constant. However, if the marginal cost of health care is increasing, the second-best policy pushes the social value of another unit of health care *below* the social value of the forgone consumption. This policy reduces the amount of health care provided by the private insurers, increasing the government's control over health care.

2. Ex Ante and Ex Post Social Welfare

One of the main requirements of a social welfare function is that it be non-paternalistic. In the absence of uncertainty non-paternalism means that

[3] The idea that public health care can be preferred to private health care on equity grounds is implicit in Weitzman's (1977) demonstration that rationing can allocate a commodity better than the price system. The current paper extends this result in several directions. It places the issue on more solid welfare foundations, makes aggregate health care an endogenous variable, and generalizes Weitzman's concept of "need" into a multidimensional variable. Most importantly, it compares what are today the three most important health care systems—public health care, private health care insurance and a mixture of the two—whereas Weitzman compared only spot markets and proportional rationing.

the planner respects each agent's preferences over commodity bundles. Its meaning is less clear if there is uncertainty. One definition simply extends the idea of preferences. Each agent is assumed to have preferences over lotteries of commodity bundles, rather than over the commodity bundles themselves, and non-paternalism means that the planner respects these preferences. This definition immediately implies that the arguments of the social welfare function are the agents' expected utilities, so that only the ex ante social welfare function can be non-paternalistic. Although this definition is widely accepted, it is sometimes at odds with our intuitive understanding of non-paternalism.

Consider, for example, Broome's (1984) rescue island example. He imagines that a volcano has erupted on a small island and that the island will soon be covered with molten lava. The inhabitants gather at the island's northern and southern tips, with a larger group gathering at the northern tip. Only one boat is sufficiently near the island to carry out a rescue. This boat is not large enough to carry away all of the people at either end of the island, so wherever the boat lands, a lottery will determine who is saved and who is left to perish. Should the boat be sent to the northern tip or the southern tip? Maximizing an additive and strictly concave ex post social welfare function leads to the conclusion that it does not matter where the boat goes. Maximizing an additive and strictly concave ex ante social welfare function sends the boat to the northern tip: it is better to save a certain number of people from a larger group than the same number of people from a smaller group.[4] By extension this procedure will sometimes find that it is better to save a smaller number of people from a larger group than a larger number of people from a smaller group.

Since each person's sole interest is his survival, one would expect a non-paternalistic measure of social welfare to prefer the policy that saves the greater number of people. A strictly concave ex post social welfare function has this property, while a strictly concave ex ante social welfare function does

[4] Ex post welfare depends only on the number saved, and that number is the same at both tips. Ex ante welfare rises with the number of people in the group reached by the boat, and with the probability with which each person in this group will be rescued. These two effects are opposing because each person's probability of rescue is inversely proportional to the number of people in the group. The concavity of the social welfare function causes the former effect to dominate. (This result assumes that the social "payoff" associated with an agent's death is not infinitely negative, so that social welfare is always a finite number.)

not.

These functions differ only in the way in which they incorporate information. The ex ante approach requires the planner to handle information in the same way as the agents do. Since some of the structure of the problem is irrelevant to a single agent (specifically, that someone else will die if he is saved and someone else will be saved if he dies), he discards it, forcing the planner to do the same. The ex post analysis yields the "non-paternalist" result by releasing the planner from this constraint.[5]

Fleurbaey's (2005) work provides further insight into the nature of this problem. His "omniscient evaluator" approach is based upon the premise that agents operating under imperfect information do not necessarily make the best choice. A planner should not mimic the agents' flawed choices; instead, the planner should implement (wherever possible) the choices that the agents would have made if they had been fully informed. The planner is *not* better informed than the agents, but he does fully exploit the available information. His advantage over the agents varies with the characteristics of the allocation problem. Fleurbaey's approach reduces to ex ante evaluation if the planner has no advantage over the agents. It reduces to ex post evaluation if the planner has a consistent advantage over the agents, and it does not reduce to either in the remaining cases.

The planner has a consistent advantage in Broome's rescue example. This example falls within a class of allocation problems that Fleurbaey defines as containing only "micro risk": the array of realized utilities is the same in every state, but the assignment of the utilities to the agents varies from state to state.[6] If some state s° were to occur, the planner could use a conventional social welfare function to evaluate that state's allocation. But every other state differs from s° only in the assignment of utilities to agents, and by the anonymity principle, changes in the assignment should not affect social welfare. It follows that even though the planner does not know which state will occur—because he is no better informed than the agents—he can

[5] Another way to obtain this result is to compare two problems under the assumption that the social welfare function is strictly concave. The first problem assumes that S randomly selected agents in a group of N will be saved, and the second problem assumes that each agent in a group of N is saved with a fixed and independent probability S/N. These problems have the same ex ante social welfare function, but the ex post functions differ.

[6] In Broome's example, S of N people live and the rest die in every state. The states differ only in the identities of the survivors.

behave *as if* he were perfectly informed. Moreover, behaving in this fashion is equivalent to evaluating ex post welfare.

Now consider the implications of these findings for the analysis of health care systems. Assume that society is somewhat averse to inequality so that the appropriate social welfare function is strictly concave in the utilities. Imagine that there is a continuum of agents who are ex ante identical, and adopt the standard assumption that their health outcomes are uncorrelated. If an ex ante social welfare function is chosen in the belief that it and only it is non-paternalistic, no health care system can dominate private health care insurance.[7] But these assumptions imply that there is only "micro risk". The appropriate social welfare function is therefore the ex post function, and its use leads to very different policy prescriptions.[8]

3. The Model

An agent is identified by his type a, where

$$a \equiv (h_0, h_1, m).$$

Here, h_0 is the agent's health without medical treatment, h_1 is health with treatment, and m is the cost of treatment (measured in units of health care).

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[7] Private insurance maximizes expected utilities, which are the arguments of the social welfare function.

[8] Fleurbaey shows that the ex ante social welfare function is appropriate *only if* the uncertainty is a "macro risk". In a population of ex ante identical agents who are uncertain about their future health, macro risk means that everyone's health realization is the same. There would be one state in which everyone is afflicted with Parkinson's disease, another in which everyone breaks a leg, and so on. As this assumption is clearly untenable, it is not reasonable to use a social welfare function whose arguments are the individual expected utilities.

The particular assumption adopted here is that uncertainty over future health is a "micro risk": we can anticipate how many people will break their legs but not which people will do so. This assumption is more reasonable, and it is convenient because it implies that the familiar ex post social welfare function is the appropriate one.

In the middle ground in which uncertainty over health is neither a macro risk nor a micro risk, the social welfare function would take yet another form. Although the details of the analysis would change, the principal finding of this paper—that the allocation under optimal public health care differs from the allocation under competitive insurance—would remain. This result follows directly from the use of a social welfare function whose arguments are *not* the expected utilities and, as noted above, the social welfare function will have this property whenever the uncertainty is not a macro risk.

Each element of the sample space of agent types, A, satisfies the conditions

$$0 \leq h_0 < h_1, 0 < m.$$

The sample space is assumed to be compact and connected. The utility of each agent is

$$U = h + u(c)$$

where h is health (either h_0 or h_1) and c is consumption.[9] The function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be increasing and strictly concave, with

$$u'(0) = \infty, \lim_{c \to \infty} u'(c) = 0.$$

Let $F(X)$ be the measure of the agents whose types are contained in X (where X is any measurable subset of A). The set of all types that are not treated is A_0 and the set of all types that are treated is A_1. By definition,

$$F(A_0 \cap A_1) = 0,$$

$$F(A_0 \cup A_1) = 1. \tag{1}$$

Health care and consumption goods are produced under constant returns to scale using some vector of inputs. There is a fixed supply of each input. The efficient allocation of these inputs between the industries gives rise to a production possibility frontier that is concave but not necessarily strictly concave. The frontier is

$$C + G(M) = 0 \qquad G' > 0, G'' \geq 0$$

where C and M are aggregate consumption and aggregate health care respectively:

$$C \equiv \int_A c dF,$$

$$M \equiv \int_{A_1} m dF. \tag{2}$$

[9] An addivitively separable utility function has been imposed here to make the analysis tractable. The main results are unlikely to to be significantly different under a more general utility function, because these results follow directly from the use of a social welfare function whose arguments are not the expected utilities.

Let M_{all} be the quantity of health care provided when agents of all types are treated, and let $M_{\max}$ be the maximum quantity of health care that the society is capable of providing:

$$M_{\text{all}} \equiv \int_A m dF,$$

$$M_{\max} \equiv G^{-1}(0).$$

Treating all of the agents is feasible if and only if M_{all} is less than or equal to $M_{\max}$.

The inputs are owned by the agents, who inelastically offer them for sale. Both goods are produced by competitive and privately owned firms. Since the firms are price-takers in both the input and output markets, the total income of the agents is equal to the market value of the produced goods. Total income, measured in consumption goods, is

$$Y \equiv C + G'(M)M.$$

Each agent has an equal claim on total income.

4. Private Health Care Insurance

Assume that the agents, prior to learning their own types, are able to contract with competitive health care insurers. Each insurance policy specifies a set of types for which treatment will be provided. The insurer collects premiums from all insurees, and provides treatment by purchasing health care from the health care producers. The insurers compete, both in terms of the nature of the contracts they offer and the prices at which they are offered.

Proposition 1 *Competition among the insurers gives rise to policies with the following properties:*

1. *No policy provides health care to a positive measure of types for which*

$$h_1 - h_0 < u'(c)G'(M)m.$$

The cost of treating these types would exceed the additional premiums that could be collected by including their treatment in a policy.

2. *The set of types for which*

$$h_1 - h_0 > u'(c)G'(M)m \tag{3}$$

and for which no policy provides coverage is measure zero. If a positive measure of such types existed, some insurer would be able to earn profits by offering a policy that covered them.

3. *Competition among insurers reduces each insurer's profits to zero. That is, the total premiums collected by an insurer are equal to the cost of providing treatment to its insurees.*

The last result implies that, in equilibrium, every agent benefits from the purchase of insurance if the set of types satisfying (3) has positive measure. The total premiums collected by the insurers are equal to $G'(M)M$. After paying these premiums, the agents have just enough income to purchase all of the consumption goods. Since each agent has equal income,

$$c = C = -G(M) \tag{4}$$

for all agents.

Define the composite function

$$v(M) \equiv u(-G(M)).$$

It is decreasing and strictly concave, and its first derivative is the utility of the consumption forgone when aggregate health care rises by an arbitrarily small amount:

$$v'(M) = -u'(-G(M))G'(M) < 0.$$

The sets of untreated and treated types under private health care insurance are[10]

$$A_0(M) \equiv \{a \in A : h_1 - h_0 < -v'(M)m\} \qquad 0 \leq M \leq M_{\max},$$

$$A_1(M) \equiv \{a \in A : h_1 - h_0 \geq -v'(M)m\} \qquad 0 \leq M \leq M_{\max}.$$

Then:

[10] The set of agents for whom $h_1 - h_0$ is just equal to $-v'(M)m$ has measure zero, so the placement of these agents has no impact on any aggregate variable. Their placement in A_1 is an arbitrary choice of no consequence.

Proposition 2 *Under a private insurance system the equilibrium values of C and M are unique. M is equal to zero and no insurance is sold if*

$$F(A_1(0)) = 0$$

and M is positive and every agent purchases insurance if

$$F(A_1(0)) > 0.$$

Also, M is equal to M_{all} (i.e, every type is treated) if and only if treating all of the agents is feasible and

$$F(A_1(M_{all})) = 1.$$

5. The Public Health Care System

Now assume that all health care is provided by the government. It divides the types into two groups, those that will be treated and those that will not.[11] It provides treatment by purchasing health care from the health care producers. Since the producers are competitive (and hence equate price to marginal cost), the total cost of health care is $G'(M)M$. The government finances its purchase of health care by imposing a lump-sum tax on each agent. The collective after-tax income of the agents is equal to C, which is just enough to enable them to purchase all of the available consumption goods. Each agent has an equal share of income, so (4) again holds.

The government designs the health care system to maximize an ex post social welfare function W that incorporates inequality aversion:[12]

$$W = \frac{1}{\alpha}\int_A U^\alpha dF = \frac{1}{\alpha}\int_{A_0}(h_0 + v(M))^\alpha dF + \frac{1}{\alpha}\int_{A_1}(h_1 + v(M))^\alpha dF. \quad (5)$$

The degree of inequality aversion is measured by α, which is non-zero and less than one. A larger value of α implies less inequality aversion. Setting α

[11] The absence of aggregate risk implies that it makes no difference whether the government chooses these groups before or after the individual health outcomes are known. Also, remember that ex post welfare is an expectation—not a realization—based on the same information as the expected utilities. The differences between private insurance and public health care do not have anything to do with the timing of actions or the availability of information. They arise strictly from differences in objectives.

[12] See Atkinson (1970) for a discussion of geometric social welfare functions.

equal to 1 reduces W to the Benthamite social welfare function. The optimal health care system maximizes (5) subject to (2).

A necessary condition for the maximization of W is that the government treat only the agents for whom the benefit—the increase in the value of W—per unit of health care is greatest. Consequently, the sets of untreated and treated agents take the form[13]

$$A_0(k,M) \equiv \{a \in A : (h_1 + v(M))^\alpha - (h_0 + v(M))^\alpha < \alpha k m\},$$
$$A_1(k,M) \equiv \{a \in A : (h_1 + v(M))^\alpha - (h_0 + v(M))^\alpha \geq \alpha k m\}. \tag{6}$$

for all $k \geq 0$ and $0 \leq M \leq M_{\max}$. Here, k is the government's policy instrument: the government treats only agents for whom the benefit per unit of health care is at least k.

The government's choice of k determines the remaining variables. Specifically, c and M are determined by conditions (2), (4) and (6). Since (4) shows that c is determined entirely by M, it is useful to focus on the relationship between k and M. Define the function

$$\Psi(M;k) \equiv \int_{A_1(k,M)} m dF.$$

Aggregate health care M under any policy k is a fixed point of the equation

$$M = \Psi(M;k). \tag{7}$$

There is some uncertainty about the uniqueness of a fixed point of (7) because an increase in M reduces consumption, expanding the set of types that qualify for treatment under any policy k and causing Ψ to rise. A sufficient condition for uniqueness is that, for each k,

$$\frac{\partial \Psi}{\partial M} < 1 \qquad \text{for all } M \in [0, M_{\max}]. \tag{8}$$

This condition can be interpreted as restricting the density of the agents on the sample space (so that a decrease in c does not shift many agents into the treatment group), or as placing a sufficiently tight upper bound on $G'(M)$ (so that an increase in M does not greatly reduce c). It will be assumed henceforth that this condition is satisfied.

[13] The placement of marginal agents again has no impact on the aggregate variables C, M and W. Their assignment to the treatment group is arbitrary and without consequence.

The useful variation in k is limited. Let $\overline{k}$ be the smallest value of k such that

$$A_1(k, 0) = \varnothing.$$

Increasing k from $\overline{k}$ would tighten the criteria for treatment, but this tightening would be futile because the treatment group at $\overline{k}$ is already empty.[14] If $M_{\max}$ is greater than M_{all}, define $\underline{k}$ to be the largest value of k such that

$$A_1(k, M_{\text{all}}) = A.$$

Reducing k from $\underline{k}$ would loosen the criteria for treatment, but this loosening would also be futile because the treatment group at $\underline{k}$ includes everyone. If $M_{\max}$ is not greater than M_{all}, define $\underline{k}$ to be the unique solution to the equation

$$M_{\max} = \Psi(M_{\max}, k).$$

Now the treatment group at $\underline{k}$ is just large enough that all of the society's resources are allocated to health care. There is no solution to (7) at any lower value of k, because loosening the criteria for treatment creates a treatment group too large to be treated with society's limited resources. It is therefore sufficient to consider only values of k lying in the interval $[\underline{k}, \overline{k}]$.

Proposition 3 *Assume that (8) is satisfied for all $k \in [\underline{k}, \overline{k}]$. There exists a unique solution to (7) for each k in the interval $[\underline{k}, \overline{k}]$. Let this solution be*

$$M = \widetilde{M}(k).$$

The function $\widetilde{M}$ is continuous and decreasing on its domain $[\underline{k}, \overline{k}]$, with $\widetilde{M}(\underline{k}) = \min[M_{all}, M_{max}]$ and $\widetilde{M}(\overline{k}) = 0$.

Consumption and the groups of treated and untreated agents can now be expressed in terms of k alone:

$$c = \widetilde{c}(k) \equiv -G\left(\widetilde{M}(k)\right) \qquad k \in [\underline{k}, \overline{k}],$$

$$\widetilde{A}_i(k) \equiv A_i(k, \widetilde{M}(k)) \qquad k \in [\underline{k}, \overline{k}]; i = 0, 1.$$

The function $\widetilde{c}$ is continuous and increasing. The connectedness of A implies that $\widetilde{A}_1(k)$ contracts and $\widetilde{A}_0(k)$ expands as k rises. Now consider the government's optimal policy.

[14] The number $\overline{k}$ is bounded above by the maximum value of $(h_1 - h_0)/m$, which is finite by the compactness of A.

Proposition 4 *Let k^* be the policy that maximizes social welfare, and let $\mu(k)$ be the social value of a marginal increase in every agent's utility under the policy k:*

$$\mu(k) \equiv \int_{\widetilde{A}_0(k)} \left(h_0 + v(\widetilde{M}(k))\right)^{\alpha-1} dF + \int_{\widetilde{A}_1(k)} \left(h_1 + v(\widetilde{M}(k))\right)^{\alpha-1} dF.$$

If $\underline{k} < k^* < \overline{k}$, k^* satisfies the condition

$$k = -\mu(k)v'(\widetilde{M}(k)).$$

The social benefit of an additional unit of health care is k (by definition). The social cost of a unit of health care is $-\mu v'(M)$, the social value of the consumption lost when one more unit of health care is provided. If the social benefit is greater than the social cost, social welfare can be increased by reducing k (i.e, relaxing the requirements for treatment), which increases aggregate health care. Likewise, social welfare rises with k if the social cost is greater than the social benefit. There is no small change in k that raises welfare when the social cost and social benefit of additional health care are equal.

5.1 Health Care without Inequality Aversion

If the social welfare function does not display inequality aversion ($\alpha = 1$), the treatment group under the optimal policy is simply

$$A_1 = \{a : h_1 - h_0 \geq -v'(M)m\}$$

which is also the treatment group under private health care insurance. The allocation of resources under the optimal public insurance program is exactly the same as the allocation under private insurance. The only rationale for government involvement in health care in this model is inequality aversion. If there is no inequality aversion, it does not matter whether health care is a public or private institution.

Suppose that there is no aversion to inequality, and that there is a demand for private insurance when a public health care system is in place. Would acceding to that demand be welfare improving? It would, but only because it would signal that the public health care program is not optimally designed. Optimally restructuring that program would have the same impact as introducing private insurance.

5.2 Health Care with Inequality Aversion

Resources are allocated differently under public and private health care if there is inequality aversion. Figure 1 shows a cross section of the sample space (m is fixed at an arbitrary value) under the assumption that M is the same under both private and public care. Under private insurance, an agent is treated if his type places him above the EU-max locus. Under public insurance, an agent is treated if his type places him above the W-max locus. Since W-max cuts EU-max from below, there are types that are treated under private insurance but not under public insurance.[15] These types are contained in the darkly shaded region. Likewise, there are types that are treated under public insurance but not under private insurance.

Replacing private insurance with public insurance shifts health care away from those who have good health, whether they are treated or not, and towards those who have poor health, whether treated or not. If there is inequality aversion, public insurance exists precisely to engineer this kind of reallocation of health care resources.

It is difficult to compare the resource allocations implied by these two systems, but the range of outcomes can be illustrated by two examples. The first example assumes that

$$h_1 = m = 1$$

and that, within the population, h_0 is uniformly distributed on the interval $[0, 1]$. Imagine that some but not all types are treated under private health care insurance. The agents purchase insurance that provides treatment if and only if h_0 is less than $1 + v'(M^\circ)$, where M° satisfies

$$M = \int_0^{1+v'(M)} dh_0$$

or

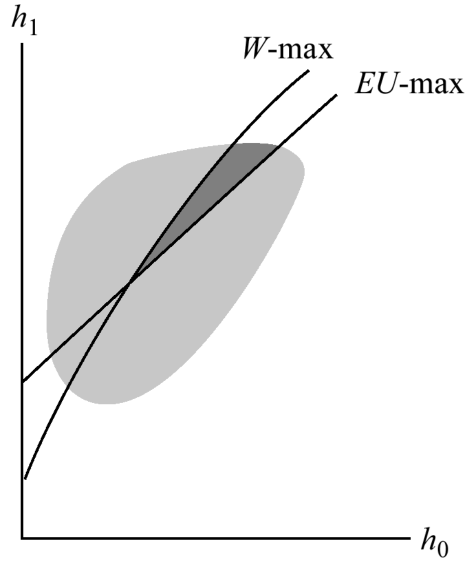
$$M = 1 + v'(M). \tag{9}$$

[15] The EU-max locus has a slope of 1 while the slope of the W-max locus is

$$\frac{dh_1}{dh_0} = \left(\frac{h_1 + v}{h_0 + v}\right)^{1-\alpha} > 1$$

The loci do not necessary intersect in every cross section; but since total health care is assumed to be the same under each regime, they must intersect in some cross sections.

Figure 1: A cross section of the sample space. Types above EU-max would be treated under private insurance; types above W-max would be treated under public insurance.



The solution to this equation is unique. Now consider public health care. An agent is treated if and only if h_0 lies on some interval $[0, t]$. Social welfare is

$$\omega(t) = \frac{1}{\alpha}\left\{t[1 + v(M(t))]^{\alpha} + \int_t^1 [h_0 + v(M(t))]^{\alpha} dh_0\right\}$$

where

$$M(t) = t.$$

Then

$$\omega'(t) = \frac{1}{\alpha}[1 + v(M(t))]^{\alpha} - [t + v(M(t))]^{\alpha} + \\ v'(M(t))\left\{t[1 + v(M(t))]^{\alpha-1} + \int_t^1 [h_0 + v(M(t))]^{\alpha-1} dh_0\right\}.$$

Let t° be the value of t under private health care insurance:

$$t^{\circ} \equiv 1 + v'(M^{\circ}). \tag{10}$$

Evaluating $\omega'(t)$ at t° gives, after some manipulation,

$$\omega'(t^\circ) = (1-t^\circ)t^\circ \left\{ \left[\frac{1}{1-t^\circ}\right] \int_{t^\circ}^{1} [h_0 + v(M^\circ)]^{\alpha-1} dh_0 - [1 + v(M^\circ)]^{\alpha-1} \right\} > 0.$$

It follows that the optimal public health care policy sets t above t°, so that aggregate health care is greater under the public health care system than under private health insurance.

The difference between these two systems is, of course, the way in which the treatment costs are allocated. Equation (10) states that an agent of type $(t^\circ, 1, 1)$ feels that the benefit of treatment is just offset by the cost of treatment. A public health system spreads this cost across all of society. Since an agent of this type has a lower utility than any other agent,[16] spreading the cost of his treatment across the whole of society raises social welfare. Thus, social welfare rises when this agent is treated under public health care, but does not change when he is treated under private health insurance.

The second example assumes that

$$h_0 = 0, m = 1$$

and that h_1 is uniformly distributed on $[0, 1]$. Under private health care insurance, every agent purchases insurance that provides treatment if and only if h_1 is greater than $-v'(M^\circ)$, where M° is again the solution to (9).[17] Under public health care, an agent is treated if and only if h_1 lies within some interval $[t, 1]$. Social welfare is now

$$\omega(t) = \frac{1}{\alpha} \left\{ \int_t^1 [h_1 + v(M(t))]^\alpha dh_1 + tv(M(t))^\alpha \right\}$$

where

$$M(t) = 1 - t.$$

Then

$$\omega'(t) = -v'(M(t)) \left\{ \int_t^1 [h_1 + v(M(t))]^{\alpha-1} dh_1 + tv(M(t))^{\alpha-1} \right\} + \frac{1}{\alpha} \left\{ v(M(t))^\alpha - [t + v(M(t))]^\alpha \right\}.$$

[16] An agent with a smaller h_0 is treated and therefore healthier; an agent with a larger h_0 is healthier even though he is also untreated.

[17] As before, the fraction of the types that are treated is $1 + v'(M^\circ)$.

Table 1: The coverage of private health care insurance and the optimal public health care system in the second example

t°	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.86
t^*	0	0.09	0.18	0.27	0.37	0.49	0.62	0.76	0.91	1.0

As before, let t° be the value of t under private health care insurance:

$$t^\circ \equiv -v'(M^\circ).$$

Evaluating the derivative at t° gives

$$\omega'(t^\circ) = t^\circ \left\{ \int_{t^\circ}^{1} [h_1 + v(M(t))]^{\alpha-1} dh_1 + t^\circ v(M^\circ) - \left(\frac{1}{t^\circ}\right) \int_0^{t^\circ} [h_1 + v(M(t))]^{\alpha-1} dh_1 \right\}.$$

Inspection of this expression shows that the sign of the derivative is not certain. It is negative if t° is near zero and it is positive if t° is near one, so that the optimal public health system plan might treat either more types or fewer types than the private health care system. Suppose, for example, that the utility function is

$$u(c) = c$$

and that the production possibility frontier is

$$c + \beta M = 1.$$

Then t° is equal to β, while the socially optimal value of t (call it t^*) depends upon both α and β. Table 1 shows the relationship between t^* and t° when α is equal to 0.6. The public health care plan is more extensive than the private health care plan (as it was in the first example) if β is smaller than some value near 0.55; but if β is larger than this value, the private health care plan is less extensive.

As in the first example, the relationship between t° and t^* depends upon the welfare effect of spreading one agent's treatment cost across all of society. If t° is small, an agent of type $(1, t^\circ, 1)$ would be less healthy—and have lower utility—than most members of society, even after he has been treated.[18] Shifting his treatment cost to the whole of society raises social welfare. Treating agents of this type under the public system raises social welfare, even though treating them under private insurance has no impact on welfare. The conclusion is reversed if t° is large. An agent of type $(1, t^\circ, 1)$ would then be healthier than most members of society after he has been treated. Shifting his treatment cost to the whole of society reduces social welfare. Treating an agent of this type under the public system reduces social welfare, while treating him under private insurance has no impact on social welfare.

It seems that anything goes. The optimal public health care system might treat everyone who would be treated under private health care, or it might not. Aggregate health care might be greater under the optimal health care system than under private health care, or it might not.

5.3 Infinite Inequality Aversion

Aversion to inequality becomes infinitely great as α approaches $-\infty$. In an economy with a finite number of agents, the optimal policy when there is infinite inequality aversion is the policy that maximizes the lowest utility. A very similar result holds when there is a continuum of agents. The economy's distribution of utilities can be represented by a non-decreasing function $U : [0,1] \rightarrow \mathcal{R}_+$. The argument of this function, $i \in [0,1]$, does not denote a particular individual, but rather a position in the utility distribution. $U(.2)$, for example, is the utility of the agent at the top of the bottom quintile. Then:

Proposition 5 *Let the functions U^1 and U^2 represent the distribution of utility under policies 1 and 2, and assume that there is some $j \in (0,1)$ such that*

$$U^1(i) < U^2(i) \qquad \text{for all } i \in [0,j].$$

In the limiting case of infinite risk aversion, policy 2 generates a higher level of social welfare than does policy 1.

This result allows the characterization of the optimal public health policy under infinite inequality aversion.

[18] He would be healthier than every untreated agent but less healthy than every treated agent. Since t° is small, the second group of agents outnumbers the first group.

Proposition 6 *Define the parameter*

$$\overline{h} \equiv \min_{a \in A} h_1.$$

In the limiting case of infinite risk aversion, social welfare is maximized by a policy of the form

$$\mathcal{A}_0(h^*) = \{a \in A : h_0 > h^*\}, \tag{11}$$

$$\mathcal{A}_1(h^*) = \{a \in A : h_0 \leq h^*\}, \tag{12}$$

implying an aggregate level of health care

$$M(h^*) \equiv \int_{\mathcal{A}_1(h^*)} mdF$$

The optimal value of h^* *satisfies the conditions*

$$1 + v'\left(M(h^*)\right) M'(h^*) \geq 0, h^* \leq \overline{h},$$

$$\left[1 + v'\left(M(h^*)\right) M'(h^*)\right] \left[\overline{h} - h^*\right] = 0.$$

The optimal policy is to treat everyone whose health (without treatment) falls below the threshold h^*, regardless of the cost or effectiveness of treatment. Social welfare rises when the health of the least healthy individuals rises. Any policy that treats only some of these people (the ones for whom treatment is cheapest or most effective) does not increase the minimum level of health and hence does not raise social welfare.

The choice of the threshold h^* is likewise chosen to maximize the well-being of the least healthy agent. Increasing the threshold raises the minimum level of health (by treating more people), but the additional health expenditures shift resources away from consumption. If the unhealthiest person is untreated, the optimal threshold is the one at which the increase in minimum health is exactly offset by a decline in the utility from consumption. If the least healthy agent is already being treated, increasing the threshold does not raise the minimum level of health, but it does reduces the least healthy agent's consumption. That agent's consumption (and utility) is maximized by choosing not to treat anyone whose health without treatment is better than that of the least healthy agent.

6. A System with Both Private and Public Insurance

Assume that there is aversion to inequality ($\alpha < 1$), and imagine that a public health system is in place and that private health care insurance is allowed. The private insurers offer policies that cover treatment for types that are not covered under the public system. The government must recognize the response of the private insurers when it designs its own system.

The government finances its program by imposing a lump-sum tax on each agent. If the private insurers are viable (they need not be), each agent will also choose to purchase private insurance, and each agent will pay the same premium. Consequently, each agent's consumption is again determined by (4).

The set A is now split into three subsets: the set of untreated agents A_0, the set of agents treated under public health care A_1, and the set of agents treated under private insurance A_2. As before, the government maximizes social welfare by treating the agents for whom the increase in social welfare per unit of health care is greatest. The set of types treated under its program is again given by (6). The private insurers will cover all of the remaining types for which the benefit of treatment exceeds the cost. All other types are untreated.

The set of types treated by the private insurers is

$$A_2(k, M) \equiv \{a \in A \backslash A_1(k, M) : h_1 - h_0 \geq -v'(M)m\} \tag{13}$$

for all $k \geq 0$ and $0 \leq M \leq M_{\max}$. The quantity of health care that they provide is

$$\Gamma(M; k) \equiv \int_{A_2(k,M)} m dF.$$

Aggregate health care is the sum of the health care provided by the government and the health care provided by private insurers:

$$M = \Psi(M; k) + \Gamma(M; k). \tag{14}$$

The equilibrium level of health care associated with any given k is a fixed point of this equation.

As before, M varies with k only when k lies within some finite interval. The lower bound of this interval is $\underline{k}$, at which either everyone is treated or only health care is produced. The upper bound of the interval, denoted k^I, is the smallest value of k at which the public health care system is so restrictive that it treats no-one, leaving health care provision in the hands of the private insurers.

Proposition 7 *Assume that (8) is satisfied at every* $k \in [\underline{k}, k^I]$. *Then:*

1. *There exists a unique solution to (14) for every* $k \in [\underline{k}, k^I]$.

2. *Let the solution to (14) be* $\widehat{M}(k)$, *and let* M^I *be the quantity of health care under a private insurance system. Then* $\widehat{M}(k)$ *is continuous and non-increasing on its domain* $[\underline{k}, k^I]$, *with* $\widehat{M}(\underline{k}) = \min[M_{all}, M_{max}]$ *and* $\widehat{M}(k^I) = M^I$.

3. *The set*
$$\widehat{A}_1(k) \equiv A_1(k, \widehat{M}(k))$$
either contracts or remains unchanged as k *rises, and the set*
$$\widehat{A}_2(k) \equiv A_2(k, \widehat{M}(k))$$
either expands or remains unchanged as k *rises.*

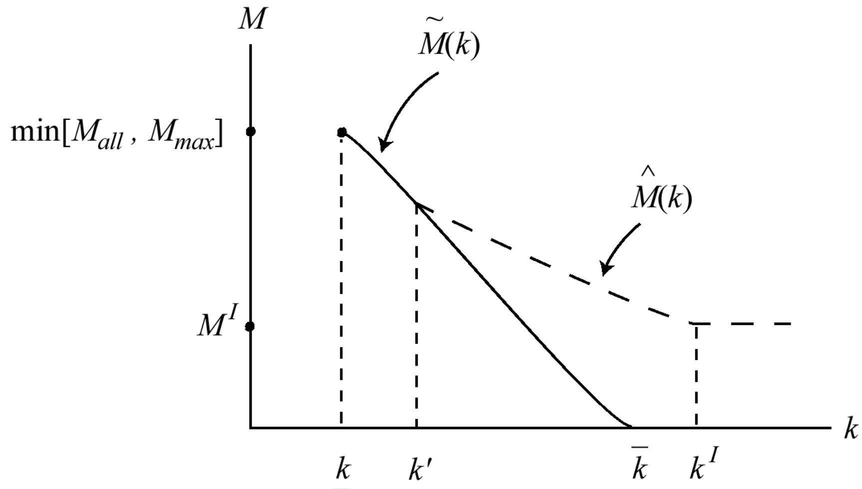
4. *There exists some* $k' \geq \underline{k}$ *such that the private insurers are inactive when* $k \in [\underline{k}, k']$ *and active when* $k \in (k', k^I]$. *Consequently,* $\widehat{M}(k) = \widetilde{M}(k)$ *for all* $k \in [\underline{k}, k']$ *and* $\widehat{M}(k) > \widetilde{M}(k)$ *for all* $k \in (k', k^I]$.

Figure 2 illustrates these results. The public health care system provides the greatest possible coverage at $\underline{k}$. The coverage of this system contracts as k rises above $\underline{k}$ until, when k reaches k', private insurers begin to provide supplementary insurance. Further increases in k result in greater private coverage and reduced public coverage. The public program disappears at k^I, and there is only private coverage at all greater values of k. At every k greater than k', aggregate health care is greater under parallel systems of health care than under public health care alone.

Little can be said about the set of all treated types, $\widehat{A}_1(k) \cup \widehat{A}_2(k)$. An increase in k transfers some types from A_1 to A_2; but some of the types added to A_2 were not previously included in A_1 and some of the types dropped from A_1 are not added to A_2. Even though aggregate health care $\widehat{M}$ is non-increasing in k, it is not certain whether the measure of all treated types rises or falls as k rises.

An optimally designed public health care system is characterized by a value of k that lies somewhere between $\underline{k}$ and $\overline{k}$. Can this value be so low that there would be no latent demand for private health care? Can it be so high that aggregate health care is less than it would be under private health care? Yes, it can, as the examples of section 4.2 have shown.

Figure 2: Aggregate health care under a public health care system, and under a combined public and private health care system.



6.1 Welfare Implications

A parallel system of private health insurance reduces welfare:

> Assume that both private and public health care are permitted, and that the private insurers offer some or all of the coverage ($k \geq k'$). Then there is a public health care system under which social welfare W is higher.

Suppose that both private and public health care are permitted, and that the government sets k at some k^* that is greater than k'. Aggregate health care is then equal to $\widehat{M}(k^*)$. If private health care is not permitted, there is a smaller value of k (call it k^{**}) such that $\widetilde{M}(k^{**})$ is equal to $\widehat{M}(k^*)$. The only difference between the "mixed" equilibrium at k^* and the purely public equilibrium at k^{**} is the manner in which health care is allocated. All of the health care is allocated to maximize W in the purely public equilibrium, but only a part of the health care is allocated to this end in the mixed equilibrium. It follows that maximal social welfare is higher when private health care insurance is not permitted.

This result suggests that the role of private insurance in an inequality-averse society is extremely limited.

- If the government chooses the optimal value of k, and if that k is no greater than k', there is no demand for private health care insurance.
- If the government chooses the optimal value of k, and if that k is above k', there is a demand for private health care insurance. Acceding to that demand would necessarily reduce welfare. Aggregate health care would be greater if private insurance were permitted (and k were not changed), but it would be both too large and badly allocated.
- If the government sets k at a value that is not optimal, there might (or might not) be a demand for private health care insurance. Acceding to this demand would always be a worse strategy than simply optimizing the government program.

If the social welfare function displays an aversion to inequality, the observation that there is a demand for private insurance does not imply that its introduction would be welfare improving. If the public program is optimally designed, introducing private insurance would lower welfare; if the public program is not optimally designed, improving the public program would yield a larger increase in welfare.

6.2 Second-Best Policy

How would the public health care program adjust to the introduction of private health care insurance? This question has usually been answered by imagining that there is an innate difference in efficiency between the public and private sectors, or by investigating the manner in which the incentives of doctors and other health care providers are changed. The current model implies a different basis for comparison. The private sector allocates health care in a manner that does not maximize social welfare. The public sector might be able to influence the extent of the misallocation by strategically altering its own program. This possibility is described by the following lemma.

Proposition 8 *Assume that both public and private health care is permitted. Let $\mu(k)$ be the social value of a marginal increase in every agent's consumption:*

$$\mu(k) \equiv \int_{\widehat{A}_0(k)} (h_0 + v(\widehat{M}(k)))^{\alpha-1} dF + \int_{\widehat{A}_1(k) \cup \widehat{A}_2(k)} (h_1 + v(\widehat{M}(k)))^{\alpha-1} dF.$$

Let k^* be the welfare-maximizing value of the instrument k, and assume that both the private and public insurers are active at k^* (that is, $k' < k^* < k^I$). Then

$$k^* = -\mu(k^*)v'(\widehat{M}(k^*))$$

if G'' is equal to zero , and

$$k^* < -\mu(k^*)v'(\widehat{M}(k^*))$$

if G'' is positive.

If the marginal cost of health care is constant at the optimum, the rule that characterizes the optimal design of the public program is the same whether there are private insurers or not. Specifically, the social value of another unit of health care, k, is equal to the social value of the consumption that must be given up to obtain that unit of health care, $-\mu v'(M)$.[19] However, this rule does not characterize the optimal design of the public program if the marginal cost of health care is increasing and the private insurers are active.

The reasoning behind this result can be understood by referring once again to the cross section in Figure 1. If G'' is zero in the neighbourhood of k^*, a small change in k leaves the position of the EU-max locus unchanged. Every type that lies above the locus is treated under one of the two programs. The only role of the instrument k is to divide the types that lie below this locus into two groups, those who are treated under the public program and those who are untreated. A decrease in k shifts W-max downward, shifting some types from the untreated group to the public treatment group. The social value of additional health care is equal to its social opportunity cost under the optimal policy. Now suppose that G'' is positive in the neighbourhood of k^*. A decrease in k shifts W-max downward; but it also shifts EU-max upward, so that some types are moved from the private treatment group to the untreated group. From the perspective of the public insurer (which conscientiously maximizes W), the latter shift is welfare improving: the private insurers allocate resources badly, so social welfare rises when their program is curtailed. The optimally designed public program exploits this effect by pushing k below $-\mu v'(M)$.

[19] The form and interpretation of the rule are the same whether or not private insurers are allowed. However, the optimal values of k and M differ in these two cases because the functions that relate μ and M to k differ.

7. Conclusions

An economy in which the public health system is designed to mitigate inequality has been examined. Replacing private health care insurance with this kind of public system shifts health care away from those who are relatively healthy with or without treatment and towards those who are relatively unhealthy with or without treatment. Aggregate health care can either rise or fall. If such a system is in place, introducing a parallel system of private insurance cannot raise welfare. If the insurers are active, their presence must reduce welfare.

An important limitation of these results is that the agents were assumed to be ex ante identical, and in particular, to have equal claims on output. It is not clear what impact the introduction of ex ante income disparities would have on the results. These disparities would create greater inequality across the population, so that the welfare gain to using health care as a redistributive mechanism rises. At the same time, income disparities lead to different demands for health care, so that a "one size fits all" public health care system entails welfare losses. The net effect on welfare is difficult to predict. However, if there are income disparities, it might be misleading to draw inferences from a model in which health care is the only redistributive instrument. One could imagine using the tax system to reduce the inequalities that arise from ex ante income disparities, and the public health care system to reduce the disparities that arise from ex post differences in health status. The practice of combining ex ante and ex post redistributive instruments is already familiar to us; for example, a large amount of redistribution occurs through progressive income taxation, but there is further redistribution towards those who find themselves unemployed or disabled.

Appendix

Proof of Proposition 1: The third item follows immediately from the assumption that the insurers engage in price competition. It implies that, in equilibrium, the price that agents pay for each unit of insurance coverage is equal to its marginal cost of production:

$$p = G'(M).$$

The agents take this price as given when choosing their coverage. To show the first item, suppose that the insurers offer for sale, at cost, an insurance contract that covers a positive measure of types for which

$$h_1 - h_0 < u'(c)pm. \tag{15}$$

The expected utility of an agent under this contract is

$$EU = \int_{A_0} h_0 dF + \int_{A_1} h_1 dF + u(Y - pM)$$

where

$$M = \int_{A_1} m dF.$$

This contract is an equilibrium contract only if no insurer can earn positive profits by offering an alternative contract. This requirement is not satisfied. Let z be a compact set of types for which (15) is satisfied. Identify a type a° within this set, and define a mapping $\psi : \mathcal{R}_+ \to z$ such that (i) $\psi(0) = a^\circ$, (ii) $\psi(x') \subset \psi(x)$ for all $x' < x$, (iii) $C \equiv F \circ \psi$ is continuous and differentiable at all $x \in \mathcal{R}_+$. Also, define the functions

$$H(x) \equiv \int_{\psi(x)} (h_1 - h_0) dF,$$

$$L(x) \equiv \int_{\psi(x)} m dF.$$

Suppose that an agent does not purchase the original contract. In its place he purchases, again at cost, an insurance contract from which coverage for the types in $\psi(x)$ has been deleted. The increase in his expected utility is

$$\omega(x) = u(Y - p\,[M - L(x)]) - u(Y - pM) - H(x).$$

Differentiating this expression and evaluating it at 0 gives

$$\omega'(0) = u'(Y - pM)pL'(0) - H'(0) = u'(c)G'(M)L'(0) - H'(0).$$

Since the types deleted from the contract when x is equal to 0 satisfy (15), $\omega'(0)$ is positive. It follows that there are small but positive values of x such that, if both contracts are offered at cost, the agents prefer the contract that treats the types in $A_1 \backslash \psi(x)$ to the contract that treats the types in A_1. An insurer that offered the less extensive contract at a price somewhat greater than cost would find willing buyers, and would earn positive profits. Since insurers can profitably deviate from a contract that provides coverage for a positive measure of types satisfying (15), a contract of that sort is not an equilibrium contract. The proof of the second item proceeds on similar lines. Suppose that the insurers offer for sale, at cost, a contract that provides treatment for a positive measure of types that satisfy the condition

$$h_1 - h_0 > u'(c)pm. \tag{16}$$

This contract is an equilibrium contract only if there is no other contract that an insurer can profitably offer in its place. This requirement is not satisfied. Let z be a compact set of these types, and define $\psi(x)$ and C as above. If an agent purchases at cost an insurance contract which provides treatment for the types in $\psi(x)$ as well as the types in A_1, the increase in his welfare is

$$\omega(x) = u(Y - p\,[M + L(x)]) - u(Y - pM) + H(x).$$

Differentiating this expression and evaluating it at 0 gives

$$\omega'(0) = -u'(Y - pM)pL'(0) + H'(0) = -u'(c)G'(M)L'(0) + H'(0).$$

Since the types added to the treatment set when x is equal to 0 satisfy (16), $\omega'(0)$ is positive. It follows that there are small but positive values of x such that, if both contracts are offered at cost, the agents prefer the contract that treats the types in $A_1 \cup \psi(x)$ to the contract that treats the types in A_1. An insurer that offered the more extensive contract at a price somewhat greater than cost would find willing buyers, and would earn positive profits. Since insurers can profitably deviate from a contract that provides coverage for a positive measure of types satisfying (16), a contract of that sort is not an equilibrium contract. ■

Proof of Proposition 2: Define the function

$$\Phi(M) \equiv \int_{A_1(M)} m dF.$$

An increase in M either contracts $A_1(M)$ or leaves it unchanged, so Φ is non-increasing in M. By (2) the equilibrium value of M is the fixed point of

$$M = \Phi(M).$$

Since Φ is non-increasing, the fixed point is unique. If $F(A_1(0)) = 0$, $\Phi(0) = 0$ so that the equilibrium values of M and C are 0 and $-G(0)$. If $F(A_1(0)) > 0$,

$\Phi(0) > 0$, so the equilibrium value of M must be positive. It must be smaller than M_{all} if treating all of the agents is not feasible. There are two cases if treating all of the agents is feasible. First, if $F(A_1(M_{\text{all}})) < 1$, $\Phi(M_{\text{all}}) < M_{\text{all}}$, implying that the fixed point must satisfy the condition $0 < M < M_{\text{all}}$. Second, if $F(A_1(M_{\text{all}})) = 1$, $F(A_1(M)) = 1$ for all $M \in [0, M_{\text{all}}]$ and hence $\Phi(M) = M_{\text{all}}$ for all $M \in [0, M_{\text{all}}]$. The fixed point is then M_{all}. In both of these cases, the equilibrium value of C is $-G(M) > 0$. ■

Proof of Proposition 3: An increase in k shrinks $A_1(k, M)$, so Ψ is decreasing in k. An increase in M expands $A_1(k, M)$, so Ψ is increasing in M. The function $\Psi(M; k)$ is continuous and maps $[0, M_{\max}]$ into $[0, M_{\max}]$, so a solution exists, and by assumption, that solution is unique. An increase in k shifts downward the graph of Ψ (against M), so that the value of M satisfying (7) declines. ■

Proof of Proposition 4: Social welfare under any k° can be expressed as the difference between social welfare when the treatment group is fixed at $\widetilde{A}_1(k)$, and the social welfare lost because the actual treatment group $\widetilde{A}_1(k^\circ)$ differs from $\widetilde{A}_1(k)$.

$$W(k^\circ; k) = \frac{1}{\alpha} \sum_{i=0}^{1} \left(\int_{\widetilde{A}_i(k)} \left[h_i + v(\widetilde{M}(k^\circ)) \right]^\alpha dF \right) - J(k^\circ; k).$$

Here, $J(k^\circ; k)$ is the change in social welfare caused by the removal of agents from the treatment group as k° rises above the value k. $J(k^\circ)$ is positive when k° is greater than k, zero when k° is equal to k, and negative when k° is smaller than k. Setting the policy instrument equal to k maximizes social welfare if any deviation of k° from k reduces welfare, or equivalently, if $W(k^\circ; k)$ has a stationary point at k:

$$\left. \frac{\partial W(k^\circ; k)}{\partial k^\circ} \right|_{(k,k)} = 0.$$

Differentiating W with respect to k° and evaluating the resulting expression at k gives

$$\left. \frac{\partial W(k^\circ; k)}{\partial k^\circ} \right|_{(k,k)} = \mu(k) v'(\widetilde{M}(k)) \widetilde{M}'(k) - J'(k).$$

An arbitrarily small increase in k removes from the treatment group the agents who are marginal candidates for treatment under the current policy.

The characteristic of these agents is that

$$\frac{1}{\alpha}\left\{[h_1 + v(\widetilde{M}(k))]^\alpha - [h_0 + v(\widetilde{M}(k))]^\alpha\right\} = km.$$

Since the left-hand side of this equation is the social benefit of moving an agent into the treatment group, integrating over all of the agents moved *out* of the treatment group by an arbitrarily small increase in k gives

$$J'(k) = -k\widetilde{M}'(k).$$

Then

$$\left.\frac{\partial W(k^\circ;k)}{\partial k^\circ}\right|_{(k,k)} = \left[k + \mu(k)v'(\widetilde{M}(k))\right]\widetilde{M}'(k).$$

Since $\widetilde{M}'$ does not switch signs, a stationary point only occurs when the bracketed expression is equal to zero. ■

Proof of Proposition 5: The increase in welfare when policy 1 is replaced by policy 2 is

$$\begin{aligned}
\Delta W &= \frac{1}{\alpha}\int_0^1 \left[\left(U^2(i)\right)^\alpha - \left(U^1(i)\right)^\alpha\right] di \\
&= \int_0^1 \left[\int_{U^1(i)}^{U^2(i)} z^{\alpha-1} dz\right] di \\
&= \left[U^1(j)\right]^{\alpha-1}\left\{\int_0^j \left[\int_{U^1(i)}^{U^1(j)} \left(\frac{U^1(j)}{z}\right)^{1-\alpha} di + \int_{U^1(j)}^{U^2(i)} \left(\frac{U^1(j)}{z}\right)^{1-\alpha} di\right]\right. \\
&\quad \left. + \int_j^1 \int_{U^1(i)}^{U^2(i)} \left(\frac{U^1(j)}{z}\right)^{1-\alpha} di\right\}.
\end{aligned}$$

The sign of ΔW is the sign of the bracketed expression. Since the value of the first integral approaches ∞ as α approaches $-\infty$, while the values of the second and third integrals are contained in the interval $[-1,1]$, the bracketed expression is positive for all sufficiently small values of α. ■

Proof of Proposition 6: Consider first the form of the optimal policy. For concreteness, assume that A_1 is closed. Assume that A_0 and A_1 are connected sets, and that a positive measure of agents would have to be moved between the sets to convert them into the kind of sets described by (11) and (12). The assumption of connectedness implies that the function $U : [0,1] \to$

$\mathcal{R}_+$ is continuous over some arbitrarily small interval $[0, \varepsilon]$. The agent who has the lowest utility could be either treated or untreated. If he is treated, let $\widehat{h}$ be his health; if he is untreated, let $\widehat{h}$ be the greatest lower bound of the health of untreated agents. Then

$$U(0) = \widehat{h} + v(M(\widehat{h})).$$

Since A_0 does not contain any types for which h_0 is less than $\widehat{h}$, moving all of the types for which h_0 is greater than $\widehat{h}$ into the set A_0 converts the original sets into sets that satisfy (11) and (12). Now consider the impact of this change on the utilities $U(i)$ for all i in some small interval $[0, j]$. Aggregate health care falls, so each agent's consumption rises, pushing utilities upward. The expansion of A_0 also potentially reduces the health of the agents on the interval $[0, j]$, pushing it to or toward $\widehat{h}$. However, since the health of the agent at $i = 0$ is fixed at $\widehat{h}$, and since health varies continuously over some interval $[0, \varepsilon]$, the reduction in the health of the agent at $i = j$ falls to 0 as j approaches 0 ($j \leq \varepsilon$). It follows that the increase in the utility from consumption is greater than the reduction in health (if any) for any sufficiently small j. Since this change raises the utilities on some interval $[0, j]$, Proposition 5 immediately implies that the change increases social welfare. Since switching to a policy that satisfies (11) and (12) raises social welfare, social welfare is maximized by a policy that satisfies these conditions. Now consider the optimal value of h^*. If h^* were greater than $\overline{h}$, reducing h^* to $\overline{h}$ would raise $U(i)$ for all i contained in a sufficiently small interval $[0, j]$. The health of these agents cannot fall below $\overline{h}$, and the decline in aggregate health care raises their consumption. As before, the consumption effect dominates the health effect for any sufficiently small j. Proposition 5 therefore implies that reducing h^* to $\overline{h}$ raises social welfare. The optimal value of h^* therefore lies on the interval $[0, \overline{h}]$; to find it, note that the continuity of $U(i)$ near $i = 0$ implies that an increase in $U(0)$ necessarily increases the utilities $U(i)$ over a sufficiently small interval $[0, j]$, so that social welfare rises. The value of h^* that maximizes social welfare is therefore the value that maximizes $U(0)$. That is, it is the solution to the problem

$$\max_{h^* \in [0, \overline{h}]} U(0) = h^* + v(M(h^*)).$$

This solution is described by the Kuhn-Tucker conditions stated in the Lemma.
∎

Proof of Proposition 7: The function $\Psi(M) + \Gamma(M)$ is continuous and maps $[0, \min[M_{\text{all}}, M_{\max}]]$ into $[0, \min[M_{\text{all}}, M_{\max}]]$, so there is at least one fixed point of (14) for each k. This fixed point is unique. An increase in M expands $A_1(k, M)$ or leaves it unchanged, and increases $-v'(M)$. An expansion of $A_1(k, M)$ or an increase in $-v'(M)$ shrinks $A_2(k, M)$, or at least does not expand it, so $A_2(k, M)$ either contracts or remains unchanged as M rises. It follows that $\Gamma(M)$ is non-increasing in M. Since $\Psi(M)$ rises less quickly than M under (8), $\Psi(M) + \Gamma(M)$ also rises less quickly than M, implying uniqueness of the fixed point.

Now consider the endpoints of $\widehat{M}(k)$. Suppose that $M_{\text{all}} \leq M_{\max}$, so that $A_1(\underline{k}, M_{\text{all}}) = A$ and $A_2(\underline{k}, M_{\text{all}}) = \varnothing$. Then $\Psi(M_{\text{all}}; \underline{k}) = M_{\text{all}}$ and $\Gamma(M_{\text{all}}; \underline{k}) = 0$, so that M_{all} is the fixed point. Now suppose that $M_{\max} < M_{\text{all}}$, so that $\Psi(M_{\max}; \underline{k}) = M_{\max}$. Since $v'(M_{\max})$ is infinite while $(h_1 - h_0)/m$ has a finite upper bound, $A_2(\underline{k}, M_{\max})$ is empty and $\Gamma(M_{\max}; \underline{k}) = 0$. These observations imply that $M_{\max}$ is the fixed point. Thus, $\widehat{M}(\underline{k}) = \min[M_{\max}, M_{\text{all}}]$. The parameter k^I is the smallest k satisfying the condition $A_1(k, M^I) = 0$. Then $\Psi(M^I; k^I) = 0$ while $\Gamma(M^I; k^I) = M^I$, implying $\widehat{M}(k^I) = M^I$. Between $\underline{k}$ and k^I, an increase in k tends to contract $A_1(k, M)$, although it could remain unchanged. The types removed from $A_1(k, M)$ are not always added to $A_2(k, M)$, so $A_1(k, M) \cup A_2(k, M)$ tends to contract as k rises, causing the graph of $\Psi(M) + \Gamma(M)$ to shift upwards. Consequently, $\widehat{M}(k)$ either falls or remains unchanged as k rises. The connectedness of A implies that $\widehat{M}(k)$ varies continuously with k.

Now consider $A_1(k, \widehat{M}(k))$. An increase in k tightens the criteria for treatment under the public program, so that $A_1(k, M)$ tends to contract. This effect is reinforced by a fall in $\widehat{M}(k)$. Any contraction of $A_1(k, \widehat{M}(k))$ leaves more scope for private insurers, so that $A_2(k, \widehat{M}(k))$ tends to rise. There is an additional expansion of $A_2(k, \widehat{M}(k))$ if $-v'(\widehat{M}(k))$ falls as k rises.

The final result in this proposition follows immediately if there is some k' such that $\widehat{A}_2(k)$ is empty when $k \leq k'$ and non-empty when $k > k'$. But such a k' must exist, because $\widehat{A}_2(k)$ does not contract as k rises, and is empty at $\underline{k}$ but non-empty at k^I. ■

Proof of Proposition 8: Let $\widehat{M_1}(k)$ and $\widehat{M_2}(k)$ be the aggregate quantities of health care provided by the public and private insurers respectively.

Social welfare under any policy k° can be written as

$$W(k^\circ;k) = \frac{1}{\alpha}\int_{\widehat{A}_0(k)} \left[h_0 + v(\widetilde{M}(k^\circ))\right]^\alpha dF + \\ \frac{1}{\alpha}\int_{\widehat{A}_1(k)\cup\widehat{A}_2(k)} \left[h_1 + v(\widetilde{M}(k^\circ))\right]^\alpha dF - J(k^\circ;k) + L(k^\circ;k).$$

The first two terms describe social welfare when the public and private treatment groups are artificially fixed at $\widehat{A}_1(k)$ and $\widehat{A}_2(k)$ respectively. The next term describes the change in welfare induced by changing the public treatment group from $\widehat{A}_1(k)$ to $\widehat{A}_1(k^\circ)$, and the last term describes the change in welfare induced by changing the private treatment group from $\widehat{A}_2(k)$ to $\widehat{A}_2(k^\circ)$. Since $\widehat{A}_1(k^\circ)$ shrinks as k° rises, $J(k^\circ;k)$ is positive (negative) when k° is greater than (less than) k. Since $\widehat{A}_2(k^\circ)$ expands as k° rises, $L(k^\circ;k)$ is also positive (negative) when k° is greater than (less than) k. (Note that these two terms enter $W(k^\circ;k)$ with opposite signs.) Setting the policy instrument equal to k maximizes social welfare if any deviation of k° from k reduces welfare, which requires $W(k^\circ;k)$ to have a stationary point at k:

$$\left.\frac{\partial W(k^\circ;k)}{\partial k^\circ}\right|_{(k,k)} = 0.$$

Differentiating W with respect to k° and evaluating the resulting expression at k gives

$$\left.\frac{\partial W(k^\circ;k)}{\partial k^\circ}\right|_{(k,k)} = \mu(k)v'(\widehat{M}(k))\widehat{M}'(k) - J'(k) + L'(k).$$

An arbitrarily small increase in k removes from $\widehat{A}_1(k)$ the agents who are marginal candidates for treatment under the policy k. The characteristic of these agents is that

$$\frac{1}{\alpha}\left\{[h_1 + v(\widehat{M}(k))]^\alpha - [h_0 + v(\widehat{M}(k))]^\alpha\right\} = km.$$

Since the left-hand side of this equation is the social benefit of moving an agent into the treatment group, integrating over all of the agents moved *out* of the treatment group by an arbitrarily small increase in k gives

$$J'(k) = -k\widehat{M}_1'(k).$$

Then

$$\left.\frac{\partial W(k^\circ;k)}{\partial k^\circ}\right|_{(k,k)} = \left[k + \mu(k)v'(\widehat{M}(k))\right]\widehat{M}_1'(k) + \left[L'(k) + \mu(k)v'(\widehat{M}(k))\widehat{M}_2'(k)\right]$$

or

$$\left.\frac{\partial W(k^\circ;k)}{\partial k^\circ}\right|_{(k,k)} = \left[k + \mu(k)v'(\widehat{M}(k))\right]\widehat{M}'(k) + \left[L'(k) - k\widehat{M}_2'(k)\right]. \tag{17}$$

Consider the two cases in turn.

Suppose that $G''(k) = 0$. Inspection of (13) shows that an increase in k causes $\widehat{A}_2(k)$ to expand only because some of the types removed from $\widehat{A}_1(k)$ are added to $\widehat{A}_2(k)$. It follows that all of the agents added to $\widehat{A}_2(k)$ have the same characteristic as the agents deleted from $\widehat{A}_1(k)$:

$$\frac{1}{\alpha}\left\{[h_1 + v(\widehat{M}(k))]^\alpha - [h_0 + v(\widehat{M}(k))]^\alpha\right\} = km. \tag{18}$$

Integrating over all of the agents moved into $\widehat{A}_2(k)$ by an arbitrarily small increase in k gives

$$L'(k) = k\widehat{M}_2'(k).$$

Substituting this expression into (17) shows that the stationary point occurs where

$$k = -\mu(k)v'(\widehat{M}(k)).$$

Now suppose that $G'' > 0$. When k rises by some arbitrarily small amount, some types are moved from $\widehat{A}_1(k)$ to $\widehat{A}_2(k)$, as before, but others are transferred from $\widehat{A}_0(k)$ to $\widehat{A}_2(k)$. The latter transfer occurs because an increase in k reduces aggregate health care $\widehat{M}(k)$, which in turn reduces the marginal cost of health care $G'(\widehat{M}(k))$. The types moved from public to private health care satisfy (18). The types shifted from no health care to private health care must satisfy the condition

$$\frac{1}{\alpha}\left\{[h_1 + v(\widehat{M}(k))]^\alpha - [h_0 + v(\widehat{M}(k))]^\alpha\right\} < km$$

because they are not contained in $\widehat{A}_1(k)$. Since $L'(k)$ is the increase in social welfare generated by treating all of the agents added to $\widehat{A}_2(k)$ when k rises marginally,

$$L'(k) < k\widehat{M}_2(k).$$

Inspection of (17) shows that the condition

$$k < -\mu(k)v'(\widehat{M}(k))$$

characterizes the stationary point. ■.

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