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Leadership in Public Good Provision: a Timing Game Perspective

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Abstract

We address in this paper the issue of leadership when two governments provide public goods to their constituencies with cross border externalities as both public goods are valued by consumers in both countries. We study a timing game between two different countries: before providing public goods, the two policymakers non-cooperatively decide their preferred sequence of moves. We establish conditions under which a first- or second-mover advantage emerges for each country, highlighting the role of spillovers and the strategic complementarity or substitutability of public goods. As a result we are able to prove that there is no leader when, for both countries, public goods are substitutable. When public goods are complements for both countries, both countries may emerge as the leader in the game. Hence a coordination issue arises. We use the notion of risk-dominance to select the leading government. Lastly, in the mixed case, the government for whom public goods are substitutable becomes the leader.

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1 Introduction

The issue of leadership is much studied in industrial economics, in particular in relation with duopoly theory, but much less in public economics. Yet this issue cannot be underestimated in this domain of economics. There are many examples of interdependent decisions made by independent public authorities. To name a few examples, let us mention the case of cross-border externalities, federations, military alliances, environmental issues, transnational public goods. These public authorities clearly rule differentiated jurisdictions. It is common to oppose core vs periphery jurisdictions, large vs small governments, or differences in capacities (in military alliances for example). These differences often translate in strategic asymmetries, as one jurisdiction assumes a leading role with respect to others, setting the agenda, deciding and constraining other jurisdictions, considered and acting as followers.

The present paper aims at formally addressing the issue of leadership in public economics, focusing on the problem of providing public goods in the presence of cross-jurisdiction spillovers.[1] We aim at understanding who is leader (follower) in providing public goods and why. We also want to characterize the consequences of leadership relative to its absence for each jurisdiction involved in this setting. In order to answer this question, we adopt a game-theoretic approach and define leadership as the action of moving first.[2]

As our approach is parallel to the one adopted in industrial economics to study leadership in duopolies, a brief summary of this research is relevant here. First, taking as given the order of moves in a duopoly, in other words the presence of a leader, industrial economists were interested in assessing the respective advantages of each firm: that is, in comparing the first-mover and second-mover advantages, defined as the payoffs for this

[1] Of course, there are many more issues in public economics which can be related to the problem of leadership. We return to this point in the conclusion.

[2] See Kempf and Taugourdeau (2005) for a first exploration of Stackelberg games over fiscal decisions in a two-country model.

firm of playing either first or second. Once it was recognized that the two roles of leader and follower lead to different advantages, the next step was to endogenize the moves, that is, to consider the sequence of moves as the equilibrium result of non-cooperating strategies played by the two firms given their own relative characteristics and their prospective advantage as a leader or as a follower. D'Aspremont and Gerard-Varet (1980), Gal-Or (1985) and Dowrick (1986) proposed an initial analysis, which has been extended by Hamilton and Slutsky (1990), Amir and Grillo (1999), van Damme and Hurkens (1999) and more recently by van Damme and Hurkens (2004) or Amir and Stepanova (2006). The determination of simultaneity versus sequentiality of moves, as well as the assignment of roles of the players in the latter case, has then been made fully endogenous.

In this paper, we shall follow the same logic. First, using a simple yet fairly general two-jurisdiction model of public good provision and externalities (linked to the public goods) across jurisdictions, we shall characterize the first-mover and second-mover advantages. Then we shall set-up a timing game, or equivalently the two-period action commitment game proposed by Hamilton and Slutsky (1990). In the first stage, the policymaker of each jurisdiction states which role (leading or following) it prefers. Once the solution of this stage is obtained, that is when the two roles are attributed to the two jurisdictions, given their statements, the resulting game is played in the second stage of the timing game.[3] It may happen that two equilibria emerge as the outcome of the first stage: that is, two sequences of moves are solutions of the two-stage game. In this case, in order to break-up this multiplicity, we resort to the concepts of Pareto-dominance and risk-dominance offered by Harsanyi and Selten (1988). Applying this concept to two different specifications of our general model, we show how it allows a simple and straightforward explanation of which jurisdiction ends up as the leader.

Compared to the IO literature,[4] we consider both negative and positive externalities

[3] Another presentation of this game has been proposed by van Damme and Hurkens (1999): each player has to move in one of two periods; choices are simultaneous, but if one player choose to move early while the other moves late, the latter behaves as a Stackelberg follower, the former as a leader.

[4] Amir and Stepanova (2006) consider a Bertrand competition which exhibits positive externali-

between players. This reflects the wide diversity of public goods. As a consequence, the issue of leadership in public good provision is more complex. We prove that both the magnitude of the moves' advantages and the existence and identity of a leader depend on the characteristics of public goods in the utility function. When for both countries the public goods are substitutes, both jurisdictions experience a first-mover advantage and the solution of the timing game is the Nash simultaneous game. When for both countries the public goods are complements, at least one jurisdiction benefits from a second-mover advantage when public goods are complements and the two sequential "basic games" are solutions of the timing game. Finally, in the mixed case, the country for which the public goods are substitutes benefits from a first-mover advantage and is the leader. Then we prove that the use of the risk-dominance criterion always allows us to identify the leader and provides an explanation of this leadership in each of the two specifications of the utility function we consider. Overall, the issue of leadership is solved for all cases.

Our paper highlights the role of inter-jurisdictional spillovers. Several recent contributions can be questioned through our results. The literature on centralization and international unions only considers simultaneous games where countries choose to cooperate or not (see for instance, Lockwood, 2002, Besley and Coate, 2003 or Alesina *et al.* 2005). It can be deduced from our analysis that the usual benchmark used to appreciate political centralization, i.e. the simultaneous Nash equilibrium, may not be relevant when public goods are complements. Studies on global public goods (Kaul *et al.* 1999 or Barrett, 2007) are also related to our analysis since we establish a taxonomy of international interactions and their consequences in term of their global provision and the "natural" (since endogenous) emergence of a leader. We emphasize that the free rider issue is not as strong as predicted in the literature, when public goods are complements. Indeed, in this situation, a sequential situation emerges as a Subgame Perfect Equilibrium (SPE), which involves

ties among firms, while van Damme and Hurkens (1999) focus on Cournot competition with negative spillovers.

a higher provision of public goods to this at the simultaneous Nash equilibrium.[5]

Finally, our analysis might be fruitful in political science where the concepts of hegemony and leadership, often confounded, remain a hot topic (see Keohane, 1984 or Pahre, 1999). By considering the leadership as endogenous, we are able to suggest a clear distinction between hegemony and leadership: hegemony would be a structural variable (an *ad hoc* assumption in the utility functions), while leadership would characterize the solution of a timing game.

The next section sets up the two-jurisdiction model we use and studies the three non-cooperating games, with either synchronous or sequential moves, that can be played, and derives the first- and second-mover advantages for a very general specification of the utility function. The third section tackles the selection of a leader by means of a timing game. The fourth section resorts to the concept of risk-dominance in order to solve the multiplicity issue, arising when public goods are complements. The last section concludes.

2 Public good provision non-cooperating games

We consider an economy consisting of two jurisdictions or countries (A and B). There is no mobility accross countries. Their populations are normalized to 1. Each country i ($\in \{A, B\}$) provides a local public good, in quantity g_i, which generates some externalities for the other country, namely country j ($\neq i$). There is perfect information. Inhabitants in country i are assumed to have preferences that can be described by the following utility function:

$$U_i\left(g_i, g_j\right) = y - g_i + \Psi^i\left(g_i, g_j\right), \tag{1}$$

[5] Our analysis may also be linked to the work of Varian (1994), who considers a sequential game of private contribution. This author highlights that the ability to commit to a contribution reinforces the free-rider problem and he concludes that the amount of public good sequentially provided is never larger than the amount simultaneously provided. We extend his analysis in two ways: first we consider the leader as endogenous; second, since Varian (1994) focuses exclusively on the contribution game proposed by Bergstrom *et al.* (1986), he considers only public goods as substitutes, we study here all the possible configurations. Whereas the conclusions of Varian (1994) remain valid for substitutes and an endogenous leadership, they do not hold anymore as soon as complement public goods are considered.

where y is the gross income (identical for each country) and g_i the level of public good provided by country i. The function $\Psi^i(g_i, g_j)$ represents the public goods provision function, entering linearly into the utility function of the representative agent in country i. We make the following assumptions:

$$\forall i = A, B, \qquad \begin{array}{ll} \Psi_1^i(g_i, g_j) \equiv \frac{\partial \Psi^i(g_i,g_j)}{\partial g_i} > 0, & \Psi_2^i(g_i, g_j) \equiv \frac{\partial \Psi^i(g_i,g_j)}{\partial g_j} > \text{ or } < 0, \\ \Psi_{11}^i(g_i, g_j) \equiv \frac{\partial^2 \Psi^i(g_i,g_j)}{\partial g_i^2} < 0, & \Psi_{12}^i(g_i, g_j) \equiv \frac{\partial^2 \Psi^i(g_i,g_j)}{\partial g_i \partial g_j} > \text{ or } < 0. \end{array} \tag{2}$$

The derivative $\Psi_{12}^i(g_i, g_j)$ plays a crucial role in this paper. When for any (g_i, g_j), $\Psi_{12}^i(g_i, g_j)$ is positive, we shall say that the two public goods are complements: an increase in the provision of g_j increases the marginal utility of good i. When it is negative, they are substitutes. When it is nil, the three studied games yield the same equilibrium payoffs. This last assumption is often implicit in the literature on inter-jurisdictional spillovers and it appears as an important limitation of these analysis (see for instance Lockwood, 2002 or Besley and Coate, 2003).[6][7]

Despite the use of a quasi-linear utility function which is the workhorse of this literature (see Batina and Ihori, 2005), our formalization remains quite general. It allows us to encompass several issues, as transnational or global public good provisions, the functioning of international organizations or fiscal federalism where a jurisdiction corresponds to subnational governments. The quasi-linear form involves a strict equivalence between the function $\Psi_1^i(g_i, g_j)$ and the marginal rate of substitution (MRS) between private and public consumptions in country i.

We consider three possible situations for the determination of non-cooperative national policies, defining three "basic games": the simultaneous Nash game (G^N) and the two Stackelberg games (G_i^S where country i is leader and country j follower).

[6] Lockwood (2002) specifies $\Psi^i(g_i, g_j) = g_i + g_j$; Besley and Coate (2003), $\Psi^i(g_i, g_j) = (1-\alpha)\log g_i + \alpha \log g_j$ with $\alpha \in [0, 1]$.

[7] Since we consider continuously differentiable functions, we do not have to resort to lattice theory.

2.1 Characterizing the simultaneous game (G^N)

At the simultaneous non-cooperative equilibrium, each country chooses its own policy taking as given the provision of the other public good, that is, without taking into account the externalities its decision generates on the other country. We denote by $\left(g_A^N, g_B^N\right)$ the Nash equilibrium of this game. This pair must satisfy the following set of definitions:

$$\begin{cases} g_A^N \equiv \arg\max_{g_A \geqslant 0} U_A\left(g_A, g_B\right), & g_B \text{ given,} \\ g_B^N \equiv \arg\max_{g_B \geqslant 0} U_B\left(g_B, g_A\right), & g_A \text{ given.} \end{cases} \tag{3}$$

The non-cooperative equilibrium levels of public good provisions are implicitly given by:

$$\forall i \in \{A, B\}, \quad -1 + \Psi_1^i\left(g_i^N, g_j^N\right) = 0 \tag{4}$$

Following **?**, a sufficient condition for best-replies to be contractions is: $\frac{\partial^2 U_i(g_i,g_j)}{\partial g_i^2} + \left|\frac{\partial^2 U_i(g_i,g_j)}{\partial g_i \partial g_j}\right| < 0$, which yields in our model

$$\Psi_{11}^i\left(g_i, g_j\right) + \left|\Psi_{12}^i\left(g_i, g_j\right)\right| < 0, \tag{5}$$

This insures the existence and the uniqueness of the Nash Equilibrium. In the following, we always assume that condition (5) is satisfied.

2.2 Characterizing the Stackelberg game (G_i^S)

Under this scenario, we assume that one of the two jurisdictions denoted by i is the first player to set its provision of g_i, and then jurisdiction j (the follower F) chooses its own level g_j. In other words, jurisdiction i behaves as a Stackelberg leader (L).

Applying backward induction, we first consider the maximization program of the follower which is given by:

$$g_j^F(g_i) \equiv \arg\max_{g_j} U_j(g_j, g_i), \quad g_i \text{ given.} \tag{6}$$

The FOC, which is equivalent to (4) for country j, yields:

$$-1 + \Psi_1^j\left(g_j^F(g_i), g_i\right) = 0 \tag{7}$$

Applying the envelop theorem, we remark that:

$$\frac{dg_j}{dg_i} = -\frac{\Psi_{12}^j(g_j, g_i)}{\Psi_{11}^j(g_j, g_i)} \lesseqgtr 0 \Leftrightarrow \Psi_{12}^j(g_j, g_i) \lesseqgtr 0 \tag{8}$$

The leader solves the following program:

$$g_i^L \equiv \arg\max_{g_i} U_i\left(g_i, g_j^F(g_i)\right) \tag{9}$$

which implies the following FOC:

$$-1 + \Psi_1^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) + \frac{dg_j^F(g_i)}{dg_i}\Psi_2^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) = 0 \tag{10}$$

or equivalently,

$$-1 + \Psi_1^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) - \frac{\Psi_{12}^j\left(g_j^F\left(g_i^L\right), g_i^L\right)}{\Psi_{11}^j\left(g_j^F\left(g_i^L\right), g_i^L\right)}\Psi_2^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) = 0 \tag{11}$$

The SOC is assumed to be satisfied.

2.3 Comparison of the levels of public good provisions

Our analysis offers a taxonomy of inter-jurisdictional interactions. Indeed, we consider six cases depending on the sign of the spillovers and the presence of complementarity or substitutability among national public goods.[8] The literature focuses essentially on one kind of interactions, when either complementarity or substitutability is considered. Indeed, many authors implicitly assume a standard technology of aggregation of jurisdiction's contribution, the weighted summation, which may be related to the canonical model of Bergstrom *et al.* (1986). This hypothesis involves substitutability among national public goods. Thus, the standard formalization of interjurisdictional interactions corresponds to the case where $\Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) < 0$ (for any $i \in \{A, B\}$). Under these assumptions, Bloch and Zenginobuz (2007) extends the analysis of the issue of local public good provision with spillovers and highlights their effects. Ellingsen (1998), Redoano and Sharf (2004) and Alesina *et al.* (2005) consider the issues of international agreements or centralization under similar assumptions.[9] The case of substitutable goods has been much studied in the literature. In particular, Hirshleifer (1983) has developed two interesting cases, relying on a "weakest-link" specification or a "best-shot" one.[10] These specifications have been used in the context of global or transnational public goods (see Kaul *et al.*, 1999 or Barrett, 2007).

The case of complementary goods with positive spillovers ($\Psi_2^{i,j}(.) > 0$ and $\Psi_{12}^{i,j}(.) > 0$) may be illustrated with the analysis of defence expenditures between two allied countries (see Ihori, 2000) and some works on the global public goods, highlighting a better-shot aggregation technology (see Cornes, 1993 and Barrett, 2007). The mixed case with negative spillovers may be apprehended as a generalization of the Contest Success Function used in the rent-seeking games or R&D races (see Dixit, 1987 and Baik and Shogren, 1992), while, to our knowledge, the other mixed case has not been considered in the literature. Different public goods can logically be linked to different cases, depending on their complementarity or substitutability and the sign of their induced spillovers. The

[8] We do not assume that the signs of the spillovers differ for the two countries.

[9] These authors use a more restrictive function than our, since they assume: $\Psi^i(g_i, g_j) = v(g_1 + \beta g_2)$, with $0 < \beta < 1$ and $v''(.) < 0$.

[10] See Hirshleifer (1985) for a correction. These two cases generate several difficulties since the underlying functions are not differentiable.

following table offers examples of various public goods and (loosely) relates them to these characteristics.

	$\Psi_2^{A,B}(.) > 0$	$\Psi_2^{A,B}(.) < 0$
$\Psi_{12}^{A,B}(.) > 0$	*Defence expenditures among allied countries. International civil aviation network.*	*Defence expenditures among rival countries. Competition for an international standard.*
$\Psi_{12}^{A,B}(.) < 0$	*Local public goods. Pollution abatement programs. Biodiversity conservation.*	*Industry supporting infrastructure. Public support for fisheries.*
$\Psi_{12}^{B}(.) < 0 < \Psi_{12}^{A}(.)$	*International drug enforcement cooperation.*[11]	*Contest Success Function.*

The comparison of the levels of public goods requires to consider six cases depending on the sign of $\Psi_2(.)$ and $\Psi_{12}(.)$ (when it is assumed to be non-null).[12] In the appendix, we establish the following PROPOSITION:

Proposition 1 *The public good provision levels solutions of the Nash and Stackelberg games are such that:*
(i) If $\Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) > 0$ (positive spillovers and complements):

$$\begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^L > g_j^F > g_j^N \end{cases} \quad or \quad \begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^F > g_j^L > g_j^N \end{cases}$$

[11] Consider the cooperation on drug enforcement between the US and Colombia. Externalities are positive for both countries. For the US, an increase in Colombian intelligence effort increases the efficiency of its public effort, whereas an increase in the US effort (more helicopters) diminishes the marginal efficiency of the Colombian effort on the ground.

[12] The equilibria of the different games are identical when $\Psi_{12}^i(.) = 0$, that is in the absence of any interaction between the two countries.

(ii) If $\Psi_2^i(.) < 0$ *and* $\Psi_{12}^i(.) > 0$ *(negative spillovers and complements):*

$$\begin{cases} g_i^N > g_i^F > g_i^L \\ g_j^N > g_j^F > g_j^L \end{cases} \quad or \quad \begin{cases} g_i^N > g_i^L > g_i^F \\ g_j^N > g_j^F > g_j^L \end{cases}$$

(iii) If $\Psi_2^i(.) > 0$ *and* $\Psi_{12}^i(.) < 0$ *(positive spillovers and substitutes):*

$$\begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^F > g_j^N > g_j^L \end{cases}$$

(iv) If $\Psi_2^i(.) < 0$ *and* $\Psi_{12}^i(.) < 0$ *(negative spillovers and substitutes):*

$$\begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^L > g_j^N > g_j^F \end{cases}.$$

(v) If $\Psi_2^A(.) > 0$, $\Psi_2^B(.) > 0$ *and* $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$ *(positive spillovers for both countries, complements for country* i *and substitutes for country* j*):*

$$\begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^L > g_j^F > g_j^N \end{cases} \quad or \quad \begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^F > g_j^L > g_j^N \end{cases}$$

(vi) If $\Psi_2^A(.) < 0$, $\Psi_2^B(.) < 0$ *and* $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$ *(negative spillovers for both countries, complements for country* i *and substitutes for country* j*):*

$$\begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^L > g_j^F \end{cases} \quad or \quad \begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^F > g_j^L \end{cases}$$

Proof. See APPENDIX A.1. ■

Consider the first case ($\Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) > 0$). When the leader, say A, increases its level of provision relative to the Nash equilibrium value, it induces its follower, B, to increase its own provision level because of the complementarity property between the two public goods. In turn, this increases the leader's payoff because of the positive externality assumption. Hence we get $g_A^L > g_A^N$ and $g_B^F > g_B^N$. However it may happen that $g_A^F > g_A^L$. This comes from the differences in the $\Psi^i(g_i, g_j)$ functions. It may happen that the externalities and interaction effects are much stronger from B to A, than from A to B. Then g_A^L is very close to g_A^N and g_B^L is very far from g_B^N as well as g_A^F from g_A^N. This explains the two possible rankings obtained for (i). The other cases may be explained by means of similar reasoning, that is by the interplay between the externality effect and the interaction effect.[13]

2.4 First-mover and second-mover advantages

Given these rankings, we can compute the first- and second-mover advantages. These advantages have been extensively used and discussed in the duopoly theory since Gal-Or (1985). Here they will allow us to understand the stakes linked to the possible existence of leadership in public good provision. Following Amir and Stepanova (2006) we define the notions of "first-" and "second-mover advantage" as follows:

Definition 1 *Country i has a first-mover advantage (a second-mover advantage) if its equilibrium payoff in the Stackelberg game in which it leads is higher (lower) than in the Stackelberg game in which it follows.*

Formally, country i benefits from a first-mover advantage when $U_i\left(g_i^L, g_j^F\right) > U_i\left(g_i^F, g_j^L\right)$ and from a second-mover advantage when $U_i\left(g_i^F, g_j^L\right) > U_i\left(g_i^L, g_j^F\right)$. Given this definition we offer the following PROPOSITION:

Proposition 2 *(i) If public goods are complements in both countries* $\left(\Psi_{12}^i(.) > 0\right)$*, at least one country has a second-mover advantage;*
(ii) If public goods are substitutes in both countries $\left(\Psi_{12}^i(.) < 0\right)$*, each country has a first-mover advantage;*
(iii) If $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$*, country j, the country for which the public goods are substitutes, has a first-mover advantage. Country i has a first-mover advantage (a second-mover advantage) when externalities from the foreign public good are positive (negative).*

Proof. see APPENDIX A.2. ■

Let us first focus on the case when the public goods are complements for both countries (though affecting them differently) and spillovers are positive. At least one player prefers to be a second-mover as then, it optimally benefits from the higher provision decided by the first-mover, which generate positive externalities. To be second-mover allows this

[13] When the two countries are symmetrical, that is when $\Psi^i(g_i, g_j) = \Psi(g_i, g_j)$, $\forall i$, the mixed cases disappear. The coexistence of two rankings in cases (i) and (ii) vanishes too. For instance, PROPOSITION 1 yields immediately the following ranking $g^L > g^F > g^N$ when public goods are complements and involve positive spillovers.

country to reduce its own provision of public good, in other terms it free-rides the leader. However this free-riding is less than in the simultaneous Nash game where spillovers are positive. The other cases can be explained by similar reasonings.

3 Selecting a leader through a timing game

Clearly, given these first- and second-mover advantages, the existence and identity of a leader matter. This puts to the fore the issue of determining whether a leader emerges, and if yes, who she is. In order to address this issue, we want to endogenously define the order of moves, and not take it as given as in the previous section by resorting to a timing game, following the seminal study of Hamilton and Slutsky (1990).

A timing game is a sequential game in the first stage of which players non-cooperatively choose their preferred order of moves. Once the order of moves has been defined, players act accordingly in the second stage, that is, non-cooperatively choose their level of public good provision, applying the order of moves selected in the first stage. In other words, a timing game is an extended game $\widetilde{G}$ which encompasses the preceding games. Following Hamilton and Slutsky (1990) and Amir and Stepanova (2006) we restrict our attention to the SPE of $\widetilde{G}$.

More precisely, $\widetilde{G}$ is defined as follows: at the first stage, the *preplay* stage to the "basic game", players simultaneously and independently of each other decide whether they prefer to move early or late in the "basic game". In the same way as Hamilton and Slutsky (1990), we assume that if country i chooses leadership (strategy *Leads*), it commits itself to setting its national policy as leader and if it chooses to be a follower (*Follows*), it commits itself to following the other country's decision. Once the timing choice of each player is announced, the order of moves is defined according to the following rules: if countries choose compatible roles, their preferred moves are enforced and one of the two possible Stackelberg games will emerge. If both choose lead or follow, as their decisions are inconsistent, it is decided that the simultaneous non-cooperative game will be enforced.

That is, the second stage corresponds to the realization of the selected "basic game": it is one of the three games studied in the previous section.

Notice that, in choosing their role (leader or follower), the two players also choose which kind of behavior they prefer. Therefore this game has the following normal form:[14]

		Country B	
		Leads	*Follows*
Country A	*Leads*	U_A^N, U_B^N	U_A^L, U_B^F
	Follows	U_A^F, U_B^L	U_A^N, U_B^N

where $U_i^N = U_i\left(g_i^N, g_i^N\right)$, $U_i^F = U_i\left(g_i^F, g_i^L\right)$ and $U_i^L = U_i\left(g_i^L, g_i^F\right)$.

The solution to this reduced form game is equivalent to characterizing the solution to the leadership problem. There is no leader when both governments choose the same action; a leader emerges when they choose compatible roles of leader and follower. The result of the timing game can be related to the nature of the interactions between the two countries. We obtain the following Proposition:

Proposition 3 *Whatever is the sign of spillovers, we have*
(i) If public goods are complements, the SPEs are the two Stackelberg situations;
(ii) If public goods are substitutes, the SPE is the simultaneous moves situation;
(iii) If public goods are complements for country i and substitutes for country j, the SPE is the Stackelberg situation where country j leads and country i follows.

Proof. See Appendix A.3. ■

[14] We remark that the literature on endogenous timing remains divided to qualify the situation where both players choose to lead. Indeed, Dowrick (1986) and more recently van Damme and Hurkens (1999) consider a Stackelberg warfare, where both countries apply their action as a leader. In contrast, Hamilton and Slutsky (1990) or Amir and Stepanova (2006) apprehend this situation as the static Nash game. Hamilton and Slutsky (1990, p. 42) emphasize that Stackelberg warfare can occur only through error, since the underlying strategy of one player is not consistent with the other player's strategy.

The following table summarizes the results stated in Propositions (2) and (3):[15]

$\Psi_2^{A,B}(.) \gtrless 0$	$\Psi_{12}^{B}(.) > 0$	$\Psi_{12}^{B}(.) < 0$
$\Psi_{12}^{A}(.) > 0$	Second-mover advantage for A or B SPE: (L, F) or (F, L).	First-mover advantage for B SPE: (F, L).
$\Psi_{12}^{A}(.) < 0$	First-mover advantage for A SPE: (L, F).	First-mover advantage for A and B SPE: (L, L) or (F, F).

We remark that the nature of the spillovers (positive or negative) does not affect the presence of a first- or second-mover advantage, nor the kind of the SPE. Given this proposition, it is straightforward to see that there is no leader in the case of substitutable public goods as the solution to the timing game is the Nash game, and that the leader in the mixed case is the government for whom the public goods are substitutable. In the former case, this outcome comes from the fact that both countries would like to exploit a first-mover advantage and suffer as a second-mover compared to the Nash solution. Consequently *Leads* is a dominant strategy for each of them and no leader emerges from the timing game.

In the latter case (the mixed case), the presence of a leader comes from the fact that the government which benefits from complementary public goods prefers being a follower to the Nash case, whereas the government which benefits from substitutable public goods always prefers to lead. There is then a unique SPE.

[15] We denote by (L, F) the SPE where country A leads and B follows, and by (F, L) the reverse.

On the other hand, when the public goods are complements, there are two possible Stackelberg equilibria solutions to the timing game. This comes from the fact that in any possible case with complement public goods, both the first- and second-movers are better off than under a Nash equilibrium. This raises a coordination issue: how to solve the tie? In the next section, we provide a solution for the selection of a leader by means of the risk-dominance criterion.

4 Solving the multiplicity issue in the complementarity case

To solve the coordination issue when the public goods are complements, more formally, when $\Psi^i_{12}(.) > 0$, two criteria can be used for the selection of the leader in this case: the Pareto-dominance and the risk-dominance. Given PROPOSITION 2, it is clear that no equilibrium Pareto-dominates the other one when both countries have a second-mover advantage. Hence, following Harsanyi and Selten (1988), we have to turn to the risk-dominance criterion. It amounts to a minimization of the risk of a coordination failure due to strategic uncertainty.[16]

Harsanyi and Selten (1988) define the concept of risk-dominance as follows:

Definition 2 *An equilibrium risk-dominates another equilibrium when the former is less risky than the latter, that is the risk-dominant equilibrium is the one for which the product of the deviation losses is the largest.*

Risk-dominance allows a simple characterization for 2×2 games with two Nash equilibria: the equilibrium *(A Leads, B Follows)* risk-dominates the equilibrium *(A Follows, B Leads)* if the former is associated with the larger product of deviation losses, denoted

[16] This uncertainty comes from the fact that a player is always unsure of the other player's move because of the multiplicity of solutions. Consider then a mixed-strategy equilibrium, where p and q are the probabilities corresponding to the choice of *Leads* by countries A and B, respectively. A pure-strategy equilibrium risk-dominates the other one if it has a larger basin of attraction in the (p, q) space.

by Π. More formally, we have: *(A Leads, B Follows)* risk-dominates *(A Follows, B Leads)* if and only if:

$$\Pi \equiv \left(U_A^L - U_A^N\right)\left(U_B^F - U_B^N\right) - \left(U_B^L - U_B^N\right)\left(U_A^F - U_A^N\right) > 0. \tag{12}$$

From the comparisons of U_i^L, U_i^F and U_i^N, the preceding inequality is also equivalent to

$$\frac{U_A^L - U_A^N}{U_A^F - U_A^N} > \frac{U_B^L - U_B^N}{U_B^F - U_B^N}. \tag{13}$$

This inequality is easy to understand: the leader happens to be the player who has relatively (compared to being a follower) more to gain from being in a Stackelberg game.

In our framework, Pareto-dominance always involves risk-dominance, but the reverse is not true. There is here no trade-off between risk and payoff-dominance. When $g_i^L > g_i^F > g_i^N$ for both countries, Pareto-dominance is not relevant. We then have to consider only the notion of risk-dominance to solve the coordination issue.[17] But, as stressed by Amir and Stepanova (2006), a resolution of the problem does not appear possible without resorting to an explicit specification of the payoff functions.

We will consider in the following sub-sections two cases in the presence of asymmetries between countries, relying on two specifications of the $\Psi^i(.)$ functions. These cases are characterized by complementary public goods, but they exhibit different assumptions on externalities.

[17] Obviously this criterion is not relevant in the symmetric case. In the symmetric case, both possible equilibria lead to the same value of the product of the deviation losses. Therefore, the risk-dominance criterion does not apply. A solution could then be to select an equilibrium randomly. But notice that the two equilibria are not equivalent from the point of view of a particular government. Depending on which equilibrium is chosen, one country loses and the other gains with respect to the discarded equilibrium.

4.1 A Cobb-Douglas specification.

We consider a Cobb-Douglas specification of the function $\Psi^i(.)$, that is, we assume:[18]

$$\Psi^i(g_i, g_j) = g_i^{\alpha_i} g_j^{\gamma_i}, \quad \text{with} \quad \begin{cases} (\alpha_i, \alpha_j, \gamma_i, \gamma_j) \in]0,1[^4 \\ \alpha_i + \gamma_i < 1 \\ \alpha_j + \gamma_j < 1 \end{cases} \qquad (14)$$
$$i, j \in \{A, B\}, \quad j \neq i.$$

The externality effect is always positive for both countries. We obtain the following values for the levels of public good provision:

	Nash Equilibrium G^N	Country A leads G_A^S	Country B leads G_B^S
Country 1	$g_A^N = \alpha_A^{\frac{1-\alpha_B}{\delta}} \alpha_B^{\frac{\gamma_A}{\delta}}$	$g_A^L = \alpha_B^{\frac{\gamma_A}{\delta}} \Omega_A^{\frac{1-\alpha_B}{\delta}}$	$g_A^F = \alpha_A^{\frac{1-\alpha_B}{\delta}} \Omega_B^{\frac{\gamma_A}{\delta}}$
Country 2	$g_B^N = \alpha_A^{\frac{\gamma_B}{\delta}} \alpha_B^{\frac{1-\alpha_A}{\delta}}$	$g_B^F = \alpha_B^{\frac{1-\alpha_A}{\delta}} \Omega_A^{\frac{\gamma_B}{\delta}}$	$g_B^L = \alpha_A^{\frac{\gamma_B}{\delta}} \Omega_B^{\frac{1-\alpha_A}{\delta}}$

where $\delta = (1-\alpha_A)(1-\alpha_B) - \gamma_A\gamma_B > 0$ since $\alpha_i + \gamma_i < 1$, $\Omega_A = \alpha_A + \frac{\gamma_A\gamma_B}{1-\alpha_B} < 1$ and $\Omega_B = \alpha_B + \frac{\gamma_A\gamma_B}{1-\alpha_A} < 1$. Since $\Omega_i > \alpha_i$, we have $g_i^N < g_i^L$ and $g_i^N < g_i^F$ without ambiguity. However, the comparison between g_i^F and g_i^L remains ambiguous. We obtain the three possible rankings established in Proposition 1.

The product of deviation losses is given by:

$$\begin{aligned} \Pi(\alpha_A, \alpha_B, \gamma_A, \gamma_B) = (1-\alpha_A)(1-\alpha_B)\, \alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}} \alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}} \\ \left\{ \left(\left(\frac{\Omega_A}{\alpha_A}\right)^{\frac{\gamma_B}{\delta}} - 1 \right) \left[\frac{1-\Omega_A}{1-\alpha_A} \left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}} - 1 \right] \right. \\ \left. - \left(\left(\frac{\Omega_B}{\alpha_B}\right)^{\frac{\gamma_A}{\delta}} - 1 \right) \left[\frac{1-\Omega_A}{1-\alpha_A} \left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}} - 1 \right] \right\} \end{aligned} \qquad (15)$$

[18] The condition (5) of existence and uniqueness of a simultaneous Nash equilibrium involves:

$$\forall g_A, g_B, \quad \frac{\gamma_B}{1-\alpha_B} < \frac{g_A}{g_B} < \frac{1-\alpha_A}{\gamma_A}$$

Restricting to the case where $\alpha_A = \alpha_B$, we obtain the following PROPOSITION:

Proposition 4 *Assuming $\alpha_A = \alpha_B$, the equilibrium (*A Leads, B Follows*) risk-dominates the other equilibrium (*A Follows, B Leads*) when $\gamma_A < \gamma_B$, and the equilibrium (*A Follows, B Leads*) risk-dominates the other equilibrium (*A Leads, B Follows*) when $\gamma_A > \gamma_B$.*

Proof. See APPENDIX A.4. ■

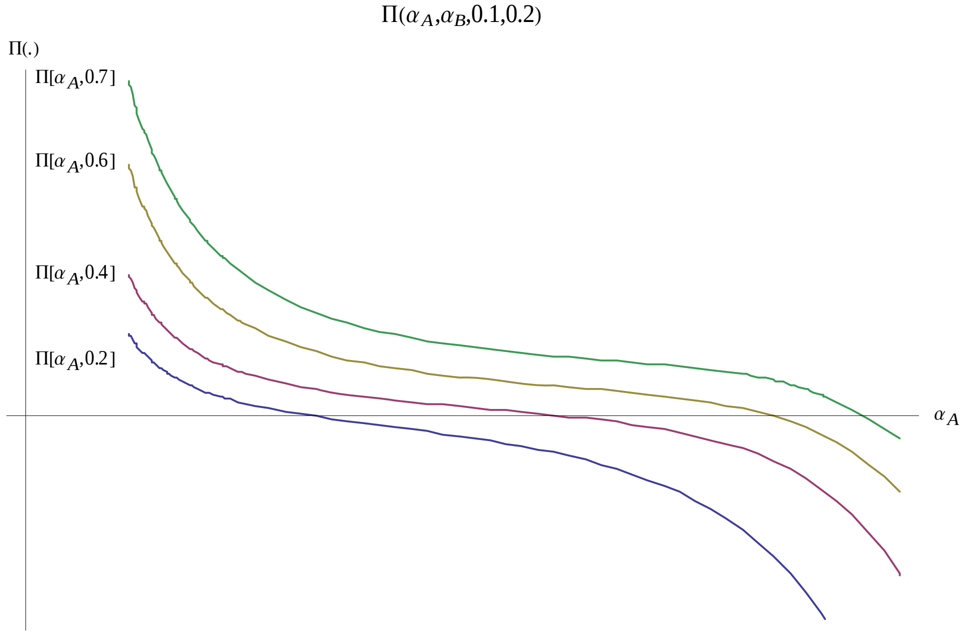
Notice that $\alpha + \gamma_i < 1$ involves $g_i^L > g_i^F$ $(i = A, B)$. When $\gamma_A < \gamma_B$, the spillover effect is smaller from B to A than from A to B: B values more the foreign public good than it is the case for A. Hence, B has more interest in *Follows* than country A. Both countries lose in case of simultaneous moves. Hence, since both players know that B is more interested in being the follower than A and that a consistent choice of moves delivers for both players positive advantages compared with the outcome of disagreements over moves, the equilibrium is that A chooses *Leads* and B *Follows*. B is able to push A to assume leadership by selecting *Follows* and thus can extract the second-mover advantage. The reverse explanation holds when $\gamma_A > \gamma_B$.

If the two relative coefficients weighing the home public goods (α_A and α_B) are sufficiently distant ($\alpha_A << \alpha_B$), then the country with the lowest coefficient, A, emerges as the leader from the timing game (see Kempf and Rota-Graziosi, 2008 for a detailed analysis of this point). As a leader, by increasing its own level compared to the Nash level,[19] it will trigger a larger increase in the other country's provision because of the complementarity assumption, which is beneficial to the leader because of the positive externality effect; on the other hand, if B is a leader, given that A as a follower will not tend to act much, i.e. increases its provision level (for opposite reasons), the gain of A as a follower with respect to the Nash solution, is not that large. The opposite reasoning explains why the country with the largest coefficient, B, has the more to lose from not being a follower. Since this is understood by both players, B chooses *Follows* and A *Leads*.

Resorting to simulations, we can also identify the roles assumed by the two countries in the equilibrium of the timing game. The following graph offers some illustrations of

[19] See PROPOSITION 1.

the function $\Pi(\alpha_A, \alpha_B, \gamma_A, \gamma_B)$. When the curb is under the horizontal axis, the risk-dominant equilibrium is the Stackelberg one where country A follows.



Graph 1: Risk-dominance with $\gamma_A = 0,1 < \gamma_B = 0,2$.

4.2 A log specification

Let us now assume a log specification of the $\Psi^i(.)$, that is:[20]

$$\Psi^i(g_i, g_j) = \theta_i \log(g_i + \beta g_j), \quad \text{with } -1 < \beta < 1. \tag{16}$$

The parameter β is the degree of spillovers generated by public goods. Depending on the sign of β, the externalities might be positive or negative. We consider here only negative spillovers ($\beta < 0$) since for $\beta > 0$, the public goods are substitutes and no coordination issue appears. Wlog, we assume that $\theta_A > \theta_B = 1$. Country A' s agents

[20]The condition (5) always holds for $\beta > -1$ since

$$\Psi^i_{11}(g_i, g_j) + \left|\Psi^i_{12}(g_i, g_j)\right| = -\frac{\theta_i(1+\beta)}{(g_i + \beta g_j)^2} < 0$$

value more the public good basket than agents in country 2, and therefore are more adversely affected by the other country's provision (ceteris paribus). An exemple of such public goods is defense spending among rival countries: inhabitants of country A are more sensitive to their national security. The following table gives the levels of public good in the different games.

	Nash Equilibrium G^N	Country A leads G_A^S	Country B leads G_B^S
Country 1	$g_A^N = \frac{\theta_A - \beta}{1-\beta^2}$	$g_A^L = \theta_A - \frac{\beta}{1-\beta^2}$	$g_A^F = \theta_A + \frac{\beta(\beta(\theta_A+\beta)-1)}{1-\beta^2}$
Country 2	$g_B^N = \frac{1-\beta\theta_A}{1-\beta^2}$	$g_B^F = 1 + \frac{\beta\left(\beta-\theta_A(1-\beta)^2\right)}{1-\beta^2}$	$g_B^L = 1 - \frac{\beta}{1-\beta^2}\theta_A$

We deduce the following rankings of the level of the public goods, directly from PROPOSITION 1:

$$\begin{aligned}
\forall\beta &\in \left]-1,0\right[, \quad g_A^N > g_A^F > g_A^L \\
\forall\beta &\in \left]-1,-\frac{1}{\theta_A}\right[, \quad g_B^N > g_B^L > g_B^F \\
\forall\beta &\in \left]-\frac{1}{\theta_A},0\right[, \quad g_B^N > g_B^F > g_B^L.
\end{aligned}$$

Let us denote by $\Pi(\theta_A,\beta)$ the difference of the products given in (12):

$$\Pi(\theta_A,\beta) = \frac{\beta^3}{\left(1-\beta^2\right)^2}\left(1-\theta_A^2\right)\left(\beta^2 + \left(1-\beta^2\right)\log\left(1-\beta^2\right)\right). \tag{17}$$

We then offer the following

Proposition 5 *Under the specification given in (16), if $\theta_A > \theta_B$, the equilibrium (*A Follows, B Leads*) is risk dominant and Pareto dominant.*

Proof. See APPENDIX A.5. ∎

Playing first is less risky for the country that values the less the basket of public goods. This safer equilibrium in which country A moves second is the neutral focal point and, adopting the risk-dominance concept, the players will coordinate on it. With the logarithm specification, we are able to establish that this equilibrium is also Pareto-dominant. In other terms, country B always has a first-mover advantage, while country A has a second-mover advantage.

Remark that the complementarity effect is higher for country A, as $\Psi_{12}^A(g,g) = \theta_A \Psi_{12}^B(g,g)$, as well as the negative externality effect (in absolute values) as $\Psi_2^A(g,g) = \theta_A \Psi_2^B(g,g)$. As a follower, by decreasing its own level compared to the Nash level, it will trigger a larger decrease in B's provision because of the lower complementarity effect for B: this is beneficial to A because of the negative externality effect ($\left(U_A^F - U_A^N\right)$ large). On the other hand, if A is a leader, given that B as a follower and given the negative externality, the gain of A as a leader with respect to the Nash solution, is not that large ($\left(U_A^L - U_A^N\right)$ small). This explains why the risk-dominance effect favors A as a follower.

While van Damme and Hurkens (1999, 2004) or Amir and Stepanova (2006) consider the two kinds of firm competition (Cournot for the formers and Bertrand for the latters), they both highlight that the low cost (or more efficient) firm will lead at the safer SPE. These authors assume a linear demand function. Here we have studied two particular forms of utility functions with public goods (a Cobb-Douglas and a log specification). We observe that our conclusions depend on the sign of spillovers as in the IO literature. Indeed, the identity of the leader at the safer SPE changes when spillovers are positive or negative: the country which values the most public goods, will follow (lead) when spillovers are positive (negative).[21]

5 Conclusion

The tools applied to study leadership in duopoly theory can be used to address the relevance of leadership in public economics. Doing so in the matter of public good provisions

[21] Notice that we have apprehended risk dominance through the product of deviation losses as Amir and Stepanova (2006), while van Damme and Hurkens (2004) consider the full tracing procedure.

by two interdependent yet non-cooperating jurisdictions generates neat and quite general results, which can easily be explained and may be applied to various situations. Formally, leadership is equated to moving first in a non-cooperative game, that is being a first-mover.

As a first step, a positive comparison between the equilibrium levels obtained in the simultaneous and sequential (non-simultaneous) games allows us to stress the role of externalities and the nature of the interactions between the public goods in the utility functions of the various agents (which are not assumed to be identical across countries), that is, whether they are complements or substitutes.

We can make explicit who benefits from a first- or second-mover advantage in the sequential game. We are then able to tackle the issue of the existence and if any, the identity, of a leader by means of a timing game. This solely depends on the complementarity or substitutability property of the public goods. It happens that being a leader (or a follower) is never beneficial when public goods are substitutes but is so when they are complements.

When public goods are complements, the timing game generates two equilibria with each country as a leader, respectively. We resort to the risk-dominance criterion to break the tie in two cases with particular specifications of the utility function, exhibiting complementarity. On the whole, the results we obtain are strikingly simple yet general, and easy to understand: the leader happens to be the public authority who, in an asymmetrical game, has relatively more to gain from being the leader than the other player, that is, the country that values less (more) public goods when spillovers are positive (negative).

Our results may be used to reconsider the literature on political integration or centralization, where the used benchmark is the simultaneous Nash equilibrium. When public goods are complements, it could be argued that the relevant benchmark for assessing the gains from centralization is not the simultaneous Nash equilibrium but a Stackelberg equilibrium as the two countries may be better off with such a non-cooperative equilibrium and may even resort to a timing game to select it.

This line of research, based on timing games, may be used to study other issues in public economics. An immediate candidate which comes to mind is the issue of tax competition either on production factors or on commodities. One also can think of applying them in the context of international trade as well as political economy problems. It is also possible to relax the assumption of perfect information for both players and adapt the notions we have just used to handle cases with imperfect information. These issues are left for future research.

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A Appendix

A.1 Proof of Proposition 1 (Comparison of the level of public goods).

By definitions of the Stackelberg and the Nash equilibria, we have

$$U_i\left(g_i^L, g_j^F\left(g_i^L\right)\right) \geqslant U_i\left(g_i^N, g_j^N\right) \tag{18}$$

The leader of the Stackelberg game always has a utility level superior or equal to the utility level obtained at the Nash equilibrium.

Moreover, we may establish from the FOCs at the Nash and Stackelberg equilibria that:

$$\Psi_1^i\left(g_i^F, g_j^L\right) = \Psi_1^i\left(g_i^N, g_j^N\right) = 1 \tag{19}$$

We distinguish six cases depending on the signs of $\Psi_2^i(.)$ and $\Psi_{12}^i(.)$ for each country. If $\Psi_{12}^i(.) > 0$, expression (19) then yields:

$$\begin{array}{c} g_i^F > g_i^N \Leftrightarrow g_j^L > g_j^N \\ g_i^F < g_i^N \Leftrightarrow g_j^L < g_j^N \end{array} \tag{20}$$

If $\Psi_{12}^i(.) < 0$, we have

$$\begin{array}{c} g_i^F > g_i^N \Leftrightarrow g_j^L < g_j^N \\ g_i^F < g_i^N \Leftrightarrow g_j^L > g_j^N \end{array} \tag{21}$$

If $\Psi_2^i(.) > 0$, we have from the definition of the Nash equilibrium:

$$U_i\left(g_i^N, g_j^N\right) \geqslant U_i\left(g_i^L, g_j^N\right) \tag{22}$$

The inequality $g_j^N > g_j^F$ then involves

$$U_i\left(g_i^N, g_j^N\right) \geqslant U_i\left(g_i^L, g_j^N\right) > U_i\left(g_i^L, g_j^F\right), \tag{23}$$

which contradicts the relation (18). We deduce that $\Psi_2^i(.) > 0 \Leftrightarrow g_j^F > g_j^N$.
In a similar way, if $\Psi_2^i(.) < 0$, we deduce from the definition of the Nash equilibrium that the inequality $g_j^N < g_j^F$ contradicts the relation (18), and we establish that $\Psi_2^i(.) < 0 \Leftrightarrow g_j^F < g_j^N$.

Combining our preceding results allows us to present the following table when both countries have the same signs of $\Psi_{12}^{i}(.)$ and $\Psi_{2}^{i}(.)$.

$\forall i \in \{A, B\}$	$\Psi_{12}^{i}(.) > 0$	$\Psi_{12}^{i}(.) < 0$
$\Psi_{2}^{i}(.) > 0$	$\begin{cases} g_i^N < g_i^F \\ g_i^N < g_i^L \end{cases}$	$\begin{cases} g_i^N < g_i^F \\ g_i^N > g_i^L \end{cases}$
$\Psi_{2}^{i}(.) < 0$	$\begin{cases} g_i^N > g_i^F \\ g_i^N > g_i^L \end{cases}$	$\begin{cases} g_i^N > g_i^F \\ g_i^N < g_i^L \end{cases}$

(24)

We first consider the case the public goods involve positive externalities and they are complements in both countries: $\forall i \in \{A, B\}, \quad \Psi_{2}^{i}(.) > 0$ and $\Psi_{12}^{i}(.) > 0$. We have:

$$-\frac{\Psi_{12}^{i}(.)}{\Psi_{11}^{i}(.)}\Psi_{2}^{j}(.) > 0 \tag{25}$$

Combining the FOC for three different games and the sign of (25) yields

$$\forall i \in \{A, B\}, \quad \Psi_{1}^{i}\left(g_i^L, g_j^F\right) < \Psi_{1}^{i}\left(g_i^F, g_j^L\right) = \Psi_{1}^{i}\left(g_i^N, g_j^N\right) \tag{26}$$

From (26), it is deduced that

$$g_i^L < g_i^F \Longrightarrow g_j^F < g_j^L \tag{27}$$

From $\Psi_{1}^{i}\left(g_i^L, g_j^F\right) < \Psi_{1}^{i}\left(g_i^F, g_j^L\right)$ and since $\Psi_{11}^{i}(.) < 0$,

$$g_i^L < g_i^F \Leftrightarrow \Psi_{1}^{i}\left(g_i^F, g_j^L\right) < \Psi_{1}^{i}\left(g_i^L, g_j^L\right) \tag{28}$$

Since $\Psi_{12}^{i}(.) > 0$, we may establish that

$$g_j^L < g_j^F \Leftrightarrow \Psi_{1}^{i}\left(g_i^L, g_j^L\right) < \Psi_{1}^{i}\left(g_i^L, g_j^F\right) \tag{29}$$

Combining (28) and (29) involves to contradict inequality (26) since

$$\begin{cases} g_i^L < g_i^F \\ g_j^L < g_j^F \end{cases} \Leftrightarrow \Psi_{1}^{i}\left(g_i^F, g_j^L\right) < \Psi_{1}^{i}\left(g_i^L, g_j^L\right) < \Psi_{1}^{i}\left(g_i^L, g_j^F\right) \tag{30}$$

Thus, we deduce that

$$g_i^L < g_i^F \Longrightarrow g_j^F < g_j^L \tag{31}$$

The case where $g_i^L < g_i^F$ and $g_j^L < g_j^F$ is impossible. We may have only either $g_i^L < g_i^F$ and $g_j^L > g_j^F$ or $g_i^L > g_i^F$ and $g_j^L < g_j^F$.
From Table (24) and (27), we deduce two following possible rankings:

$$\begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^L > g_j^F > g_j^N \end{cases} \quad \text{or} \quad \begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^F > g_j^L > g_j^N \end{cases} \tag{32}$$

We note that in the symmetrical case, only one possible ranking is possible:

$$g^L > g^F > g^N. \tag{33}$$

The other cases can be solved through similar reasoning. The detailed proofs are available in Kempf and Rota-Graziosi (2008).□

A.2 Proof of Proposition 2 (First-mover and second-mover advantages).

We consider the case where the public goods are complements.
For $\forall i \in \{A, B\}$, $\Psi_{12}^i(.) > 0$.

- If $\Psi_2^i(.) > 0$, the provision levels are given by PROPOSITION 1. Using the definition of the first- (second-) mover advantage, we get that, when $g_j^L > g_j^F$,

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^L, g_j^L\right) > U_i\left(g_i^L, g_j^F\right), \tag{34}$$

  where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L > g_j^F$ and $\Psi_2(.) > 0$. Since $g_i^L > g_i^F$ always holds for at least one of the two countries in either one of the two possible rankings given by (32), we deduce that at least one country has a second-mover advantage.

- If $\Psi_2(.) < 0$, we have the rankings given by PROPOSITION 1. In a similar way as the preceding case, we deduce that when $g_j^F > g_j^L$,

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^L, g_j^L\right) > U_i\left(g_i^L, g_j^F\right), \tag{35}$$

  where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L < g_j^F$ and $\Psi_2(.) < 0$.

The other cases can be solved through similar reasoning.□

A.3 Proof of Proposition 3 (Subgame Perfect Equilibria).

From (18) we always have: $U_i\left(g_i^L, g_j^F\right) > U_i\left(g_i^N, g_j^N\right)$, $\forall i \in \{A, B\}$. In order to determine the SPE, we only have to compare the utility levels when the country follows and when it plays simultaneously $\left(U_i\left(g_i^F, g_j^L\right) \lesseqgtr U_i\left(g_i^N, g_j^N\right)\right)$. We consider where the public goods are complements: $\forall i \in \{A, B\}$, $\quad \Psi_{12}^i(.) > 0$:

- If $\Psi_2^i(.) > 0$, we have the rankings (32), and we establish that

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right), \tag{36}$$

  where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L > g_j^N$ and $\Psi_2(.) > 0$. We deduce that the SPE correspond to the two Stackelberg situations.

- If $\Psi_2^i(.) < 0$, the rankings given in Proposition 1 yield

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right), \tag{37}$$

  since $g_j^L < g_j^N$ and $\Psi_2^i(.) < 0$. As in the preceding case, the SPEs are the two Stackelberg situations.

The other cases can be solved through similar reasoning. □

A.4 Proof of Proposition 4.

Using (14), after solving for the various games, we obtain:

$$\begin{array}{ll} U_A^N = w + (1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}\alpha_B^{\frac{\gamma_B}{\delta}}, & U_B^N = w + \alpha_A^{\frac{\gamma_B}{\delta}}(1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}, \\ U_A^L = w + \alpha_A^{\frac{\gamma_A}{\delta}}(1-\Omega_A)\,\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}, & U_B^F = w + (1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}\Omega_A^{\frac{\gamma_B}{\delta}}, \\ U_A^F = w + (1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}\Omega_B^{\frac{\gamma_A}{\delta}}, & U_B^L = w + \alpha_A^{\frac{\gamma_B}{\delta}}(1-\Omega_B)\,\Omega_B^{-1+\frac{1-\alpha_A}{\delta}}. \end{array} \tag{38}$$

and therefore:

$$\begin{array}{c} \left(U_A^L - U_A^N\right)\left(U_B^F - U_B^N\right) = (1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\left(\alpha_A^{\frac{\gamma_B}{\delta}} - \Omega_A^{\frac{\gamma_B}{\delta}}\right) \\ \left[(1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B}{\delta}} - (1-\Omega_A)\,\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}\right] \end{array} \tag{39}$$

and

$$\begin{array}{c} \left(U_A^F - U_A^N\right)\left(U_B^L - U_B^N\right) = (1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\left(\alpha_B^{\frac{\gamma_A}{\delta}} - \Omega_B^{\frac{\gamma_A}{\delta}}\right) \\ \left[(1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A}{\delta}} - (1-\Omega_B)\,\Omega_B^{-1+\frac{1-\alpha_A}{\delta}}\right] \end{array} \tag{40}$$

We focus on the particular case where $\alpha_A = \alpha_B = \alpha$. We have:

$$\begin{array}{c} \left(U_A^L - U_A^N\right)\left(U_B^F - U_B^N\right) - \left(U_A^F - U_A^N\right)\left(U_B^L - U_B^N\right) \\ = (1-\alpha)\,\alpha^{-1+\frac{1-\alpha+\gamma_A}{\delta}}\left(\alpha^{\frac{\gamma_B}{\delta}} - \Omega^{\frac{\gamma_B}{\delta}}\right)\left[(1-\alpha)\,\alpha^{-1+\frac{1-\alpha}{\delta}} - (1-\Omega)\,\Omega^{-1+\frac{1-\alpha}{\delta}}\right] \\ -(1-\alpha)\,\alpha^{-1+\frac{1-\alpha+\gamma_B}{\delta}}\left(\alpha^{\frac{\gamma_A}{\delta}} - \Omega^{\frac{\gamma_A}{\delta}}\right)\left[(1-\alpha)\,\alpha^{-1+\frac{1-\alpha}{\delta}} - (1-\Omega)\,\Omega^{-1+\frac{1-\alpha}{\delta}}\right], \end{array} \tag{41}$$

which yields

$$\begin{aligned}&\left(U_A^L-U_A^N\right)\left(U_B^F-U_B^N\right)-\left(U_A^F-U_A^N\right)\left(U_B^L-U_B^N\right)\\ =\,&(1-\alpha)^2\,\alpha^{-2+\frac{2-2\alpha-\gamma_A-\gamma_B}{\delta}}\left[1-\frac{1-\Omega}{1-\alpha}\left(\frac{\Omega}{\alpha}\right)^{-1+\frac{1-\alpha}{\delta}}\right]\left[\left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_A}{\delta}}-\left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_B}{\delta}}\right].\end{aligned} \tag{42}$$

Since $\frac{\Omega}{\alpha}=1+\frac{\gamma_A\gamma_B}{\alpha(1-\alpha)}>1$, $-1+\frac{1-\alpha}{\delta}=\frac{-(1-\alpha)^2+\gamma_A\gamma_B+1-\alpha}{\delta}=\frac{\alpha(1-\alpha)+\gamma_A\gamma_B}{\delta}>0$ and $\frac{1-\Omega}{1-\alpha}=\frac{1-\alpha+\frac{\gamma_A\gamma_B}{1-\alpha}}{1-\alpha}=1+\frac{\gamma_A\gamma_B}{(1-\alpha)^2}>1$, we have

$$\Pi\left(\alpha_A,\alpha_B,\gamma_A,\gamma_B\right)>0\Longleftrightarrow\gamma_A<\gamma_B. \tag{43}$$

□

A.5 Proof of Proposition 5.

- **Risk-dominance:**
  It is obvious that $\Pi\left(\theta_A,\beta\right)>0$, $\forall\beta<0$ and $\theta_A<1$ since $(\beta^2+\left(1-\beta^2\right)\log\left(1-\beta^2\right))$ is always negative on $[-1,0]$.
  For $\theta_A<1$, (*A Leads, B Follows*) Risk-dominates (*A Follows, B Leads*).
  For $\theta_A>1$, (*A Follows, B Leads*) Risk-dominates (*A Leads, B Follows*).

- **Pareto-dominance:**
  We have:

$$\begin{aligned}U_A^L-U_A^F&=\frac{\beta^2(\beta+\theta_A)}{1-\beta^2}+\theta_A\log\left(1-\beta^2\right).\\ U_B^F-U_B^L&=-\frac{\beta^2(1+\beta\theta_A)}{1-\beta^2}-\log\left(1-\beta^2\right).\end{aligned} \tag{44}$$

  which yield

$$\begin{aligned}\forall\theta_A>0,\quad U_A^L<U_A^F\\ \forall\theta_A>1>\theta^*=-\frac{\beta^2+\left(1-\beta^2\right)\log\left(1-\beta^2\right)}{\beta^3},\quad U_B^F>U_B^L.\end{aligned} \tag{45}$$

  Country A always has a second-mover-advantage, while country B has a first-mover advantage as soon as $\theta_A>1$. Thus, we have without ambiguity: the SPE (*A Follows, B Leads*) Pareto-dominates (*A Leads, B Follows*).□