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Abstract

Unstable government debt dynamics can typically be stabilized around a certain target level of debt by adjustments in various fiscal instruments, like government spending, transfers, or taxes. This paper investigates properties of debt stabilizing rules which link the needed budgetary adjustments to the state of the economy. The paper establishes that the magnitude of the target level of long-run debt is a key determinant of whether it is possible to find a rule of this type that can be implemented under all available fiscal instruments. Specifically, considering linear feedback rules, the paper demonstrates that there may well exist a critical target level of debt beyond which this is no longer possible. From an applied perspective, this finding is of particular relevance in the context of a monetary union with decentralized fiscal

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policies. Depending on the target level of debt, there might be a conflict between a common fiscal framework that tracks deficit developments as a function of the state of the economy and the unrestricted choice of fiscal policy instruments at the national level.

1. Introduction

Unstable government debt dynamics can typically be stabilized around a certain target level of debt by appropriate budgetary adjustments. To achieve the needed budgetary corrections a government can normally adjust a broad range of fiscal instruments, like government spending, taxes or transfers. Yet, depending on the timing of actions and the particular instrument (or subset of instruments) that gets adjusted, successful stabilizations can be associated with a broad range of possible sequences of the budgetary balance during the adjustment period.

This paper starts out from the observation that in the political process it is often desirable to conduct stabilizations within a framework which imposes certain restrictions on the admissible sequences of the budgetary balance, irrespective of the instrument that is used for the adjustment. In line with this idea, one can think of debt stabilizing fiscal rules which specify a direct link between the state of the economy and the required budgetary adjustments. If it can be ensured that such rules are implementable under all instruments, this would leave, *a priori*, the choice of the instrument unrestricted. This line of reasoning is of particular interest for the design of fiscal rules in monetary unions in which member states remain responsible for their national budgetary policies, subject to binding provisions of a common fiscal framework. Specifically, we envisage a common fiscal framework that is anchored around target levels of government debt and monitors national deficit developments as a function of the underlying state of the economy.[1] Whenever stabilizing budgetary adjustments are needed, such a framework should ideally leave the choice of the instruments unrestricted. To take core principles of EMU as an example, such a recommendation would be in line with the subsidiarity principle, that is, the common fiscal framework, while imposing stable debt dynamics in all member states, should not interfere with the choice of fiscal instruments at the national level.

Motivated by these stylized features, this paper offers a tractable general equilibrium model which can be used to address the question under which circumstances tensions may arise between the provisions of a common fiscal framework and the unrestricted choice of fiscal policy instruments. As we show in detail, the answer to this question depends critically on the

[1] For recent discussions of fiscal frameworks in monetary unions, see, among others, Fatas et al. (2003), Uhlig (2003), Calmfors (2005), and Chari and Kehoe (2007).

target level of long-run debt that anchors the fiscal framework. As long as this target level is sufficiently low, no such tensions can occur, that is, it will be possible to find state-contingent prescription of stabilizing budgetary adjustments that can be implemented under all available instruments. Yet, beyond a threshold level of debt this feature may well disappear.

To make this reasoning precise, this paper considers an OLG model in the spirit of Diamond (1965). To operationalize the notion of unstable government debt dynamics, we consider steady states which are locally unstable under the assumption of a permanently balanced primary budget. However, the economy can be stabilized at these steady states if one allows for appropriate budgetary adjustments. For tractability, we consider debt stabilizing rules that specify these adjustments as a linear function of the two state variables of the model (physical capital and real government debt). Moreover, we assume that such adjustments can be brought about by two different instruments (government consumption or a lump-sum tax on young agents). For given feedback coefficients associated with the two states of the economy, any such "debt stabilizing rule" specifies for each of the two instruments a particular sequence of adjustments towards the steady state.

Why may it not always be possible to find a debt stabilizing rule that, assuming *common* feedback coefficients, can be implemented under both instruments? Intuitively, this finding reflects that the model economy consists of two parts: the budget constraint of the government and a block which summarizes the private sector activities in the economy. By construction, the source of instability is confined to the first part, while the instruments which can be used to achieve the required budgetary adjustments affect the second part through different margins. The target level of steady-state debt determines the relative importance of these margins within the set of intertemporal equilibrium conditions. At a zero level of steady-state debt, these margins carry zero weights, ensuring thereby that there always exists a debt stabilizing rule which can be implemented under both instruments. For positive and rising levels of steady-state debt, however, these margins gain importance, implying that for any particular debt stabilizing rule the instrument-specific stabilization profiles become increasingly distinct. Exploiting this feature, we show that there may well exist a critical target level of debt beyond which it is no longer possible to find a debt stabilizing rule that can be implemented under both instruments.

Our results relate to the literature on fiscal closure rules, as typically used in large scale macroeconomic models. This literature confirms that different instruments, when residually used to enforce the intertemporal budget constraint of the government, lead to different dynamic outcomes which preclude simple comparisons across simulations (Bryant and Zhang 1996, Mitchell, Sault, and Wallis 2000, Perez and Hiebert 2004). Yet, this literature offers few explicit analytical findings and no systematic discussion of the role of government debt. As concerns tractable small-scale models, stability features of Diamond-models have been discussed in a number of studies, but not with the intention to compare stabilization properties of different fiscal instruments.[2] Considering Ramsey economies with infinitely lived agents, Schmitt-Grohe and Uribe (1997), Guo and Harrison (2004), and Giannitsarou (2007) show that for a given fiscal rule (in their context: a balanced-budget rule) equilibria can be locally unique or indeterminate, depending on whether budget balance is achieved by income taxes, consumption taxes or government spending adjustments.[3] Our paper shares with these three papers the descriptive nature of the fiscal rule, but our focus is not on balanced budget dynamics, and the role of debt—as an informative state variable—is substantially different.[4]

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the notion of debt stabilizing rules and derives all the results. Section 4 offers a robustness check and confirms that all results can be established independently of whether the rule is expressed in terms of adjustments of the primary balance or the overall balance (including interest payments). Section 5 offers conclusions.

2. The Model

For simple tractability, consider a Diamond-type overlapping generations economy with exogenous labor supply and lump-sum taxes and transfers.[5]

In period t, the economy is populated by a large number N_t of young agents and N_{t-1} of old agents. Each agent lives for two periods and has a fixed labor supply $l = 1$ when being young and a zero labor supply when being old. The population grows at the constant rate $n > 0$, that is, $N_t = (1 + n) \cdot N_{t-1}$. Let preferences of the representative agent born in period t be given by

$$U(c_t,\ d_{t+1}) = \phi \ln c_t + \beta \ln d_{t+1}, \quad \text{with } \phi > 0, \beta > 0,$$

[2]For a discussion of dynamic equilibria in Diamond-models with production, but without government debt, see Galor and Ryder (1989). For surveys of the dynamics with government debt, see Azariadis (1993) and de la Croix and Michel (2002). Special features of constant deficit rules are discussed by Farmer (1986), with a focus on cyclical adjustment patterns, and by Chalk (2000), with a focus on sustainability issues.

[3]Related to this literature, see also the dynamic analysis of tax changes in Judd (1987), Turnovsky (1990), and Mankiw and Weinzierl (2006). However, these papers do not explicitly focus on the stabilization properties of different fiscal instruments, but rather compare between short- and long-run features of equilibria that are characterized by different tax structures.

[4]The importance of government debt as a relevant state variable for the classification of dynamic equilibria is also stressed by Leith and von Thadden (2008), focusing on monetary policy rules.

[5]A more detailed analysis, covering more general functional forms (including an endogenous labor supply decision) and a more comprehensive analysis of additional fiscal instruments is carried out in the longer working paper version of this paper (see Michel, Von Thadden, and Vidal 2006).

where c_t and d_{t+1} denote first-period and second-period consumption, respectively. In any period t, agents take the wage rate (w_t) and the return factor R_{t+1} on savings (s_t) as given. There exists a tax-transfer-system such that young agents pay lump-sum taxes $\eta_t > 0$, while they receive constant lump-sum transfers θ when being old.[6] This leads to the pair of budget constraints $w_t - \eta_t = c_t + s_t$ and $d_{t+1} = R_{t+1}s_t + \theta$, and savings will be given by

$$s_t = \frac{\beta}{\phi + \beta}(w_t - \eta_t) - \frac{\phi}{\phi + \beta}\frac{\theta}{R_{t+1}}.$$

There exists a large number of competitive firms with access to a standard neoclassical technology $F(K_t, L_t)$, where K_t and L_t denote the aggregate levels of physical capital and labor, respectively. Let $F(K, L) = zK^{\alpha}L^{1-\alpha}$, with $z > 0$ and $\alpha \in (0, 1)$. Firms are price takers in input and output markets. In a competitive equilibrium, labor market clearing requires $L_t = N_t$. Let $k_t = K_t/N_t$ denote the capital stock per young agent, giving rise to the pair of first-order conditions

$$R_t = R(k_t) = z\alpha k_t^{\alpha-1} > 0 \tag{1}$$

$$w_t = w(k_t) = (1-\alpha)zk_t^{\alpha} > 0, \tag{2}$$

where it is assumed that capital fully depreciates between periods. According to (1) and (2), the equilibrium return rates change along the factor price frontier with $R'(k_t) < 0$ and $w'(k_t) > 0$.

In period t, the government consumes an amount g_t of aggregate output per young agent. Let π_t denote the primary surplus per young agent, that is,

$$\pi_t = \eta_t - \frac{\theta}{1+n} - g_t.$$

Agents perceive investments in physical capital and government bonds (in real terms) as perfect substitutes with identical return factor R_t. Then, the flow budget constraint of the government, expressed per young agent, reads as

$$(1+n)b_{t+1} = R(k_t)b_t - \pi_t.$$

In sum, we obtain a simple version of the intertemporal equilibrium conditions of the Diamond-model, modified for the existence of a simple tax-transfer system and the possibility of a primary balance that does not have to be balanced in every period:

$$(1+n)(k_{t+1} + b_{t+1}) = s_t = \frac{\beta}{\phi + \beta}(w(k_t) - \eta_t) - \frac{\phi}{\phi + \beta}\frac{\theta}{R(k_{t+1})} \tag{3}$$

[6] We do not make any explicit sign restriction regarding the second-period lump-sum payment θ. Strictly speaking, the term "tax-transfer"-system would refer only to a scenario with $\theta > 0$.

$$(1 + n)b_{t+1} = R(k_t)b_t - \pi_t \tag{4}$$

$$\pi_t = \eta_t - \frac{\theta}{1+n} - g_t. \tag{5}$$

In each period t, the state of the economy is summarized by the pair (b_t, k_t), denoting the beginning-of-period values of the capital stock and of real government bond holdings which are predetermined by past investment decisions undertaken in period $t - 1$. Hence, when we subsequently classify the dynamic behaviour of the system (3)–(5) under various fiscal closures, it is natural to assume that dynamics are characterized by two initial conditions, b_0 and k_0.[7]

Two benchmark steady states ($\pi = 0$)

To start out with, we consider benchmark steady states of (3)–(5) which are dynamically efficient and which, assuming $b_0 \neq b$ and $k_0 \neq k$, are locally unstable if the primary budget is *permanently* balanced, that is, if $\pi_t \equiv 0$ for all t. Specifically, consider a stationary tax-transfer system with $g \equiv \eta - \frac{\theta}{1+n} > 0$. Using a first-order approximation and assuming $\pi_t \equiv 0$ for all t, local dynamics around steady states evolve according to

$$A_1 \cdot dk_{t+1} + (1 + n) \cdot db_{t+1} = A_2 \cdot dk_t \tag{6}$$

$$(1 + n) \cdot db_{t+1} = R'(k)b \cdot dk_t + R(k) \cdot db_t, \text{ with:} \tag{7}$$

$$A_1 = 1 + n - R'(k)\frac{\phi}{\phi + \beta}\frac{\theta}{R(k)^2} > 0 \tag{8}$$

$$A_2 = \frac{\beta}{\phi + \beta}w'(k) > 0. \tag{9}$$

Under mild assumptions there exist two distinct types of steady states which are unstable under a permanently balanced primary budget: (i) *steady states with zero debt and underaccumulation* ($k > 0, b = 0, R(k) > 1 + n$) and (ii) *golden rule steady states with positive debt* ($k > 0, b_{gr} > 0, R(k) = 1 + n$).[8]

To briefly substantiate this claim, assume first $g = \eta = \theta = 0$. Then, under logarithmic utility and Cobb-Douglas production, there exists a unique steady state with $k > 0$ and $b = 0$. Moreover, this steady state satisfies $A_2 < A_1$, implying that the one-dimensional dynamics in the absence of government

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[7]Note, however, that there is also a branch of the literature that stresses the role of bubbles in related models and treats real government debt as a jumping variable, see Tirole (1985).

[8]For a detailed discussion of the existence and stability of dynamic equilibria in Diamond-models with zero primary deficits, see the references quoted in footnote 2. Specifically, de la Croix and Michel (2002) focus in detail on aspects of lump-sum tax and transfer systems, see p. 195 ff.

debt are stable.[9] Furthermore, if at this steady state $R(k) < 1 + n$, there exists a golden rule steady state with $k > 0$, $b_{gr} > 0$ and $R(k) = 1 + n$, also satisfying $A_2 < A_1$. If $g = \eta > 0$ and $\theta = 0$, this reasoning can be extended as long as g is smaller than some positive bound $\bar{g}$. If $g \equiv \eta - \frac{\theta}{1+n} > 0$ and $\theta \neq 0$, by continuity, such steady states continue to exist as long as θ is sufficiently small.

Assuming $\pi_t \equiv 0$ for all t, both types of steady states are unstable in the presence of government debt dynamics because of two partial effects that can be summarized by means of Equation (7): First, for any given level of the interest rate, interest payments induce a *snowball effect* on debt, as captured by the expression $\frac{R(k)}{1+n} \cdot db_t$. This snowball effect is unstable whenever the interest rate is higher than the growth rate of the economy, that is, whenever $R(k) > 1 + n$. Second, outside the steady state the interest rate will not be constant, implying that, for any given level of debt, there is an additional *interest rate effect* on debt, as captured by the expression $\frac{R'(k)}{1+n} b \cdot dk_t$. The interest rate effect reflects that the interest rate is endogenously determined via investments in physical capital. Variations in the capital stock trigger changes in the interest rate, creating thereby interactions between government debt and capital stock dynamics. In particular, the crowding out of capital leads over time to a higher interest rate which tends to reinforce unstable snowball effects. Importantly, the strength of the interest rate effect is parameterized by the steady-state level of debt, that is, the effect rises in the level of b, while it will be zero in the special case of $b = 0$. It is this feature which implies that there is scope for threshold values of steady-state debt that affect the dynamic properties of fiscal rules designed to offset the just summarized instabilities.

3. Debt Stabilizing Rules: A Common Framework with Two Instruments

This section embeds the previous analysis in a more encompassing framework which allows for adjustments in the primary balance with the idea to stabilize the benchmark steady states, that is, $\pi_t \neq 0$ is admitted for the off-steady-state dynamics. For a generic discussion of such stabilizing adjustments it seems natural to think of state-contingent fiscal rules which link the variations in π_t to deviations of the two state variables b_t and k_t from their steady-state values. In combination with the flow budget constraint of the government, rules of this type give rise to the expression

$$(1 + n) \cdot b_{t+1} = R(k_t) b_t - \pi(k_t - k, b_t - b), \text{with: } \pi(0, 0) = 0. \quad (10)$$

[9] If $F(K_t, L_t)$ is of the more general CES-type this reasoning extends to the case of an elasticity of substitution larger than one. If the elasticity is less than one, there are zero or two steady states with $k > 0$ and $b = 0$. In the latter case, the high activity steady state satisfies $A_2 < A_1$.

which, in contrast to the budget *identity* (4), describes a generic *debt stabilizing rule* that aims to stabilize the economy at the benchmark steady states. To operationalize (10), the use of at least one of the fiscal instruments needs to be linked to the states of the economy. In the following, we consider two different fiscal instruments (g_t and η_t) and distinguish between two scenarios in which adjustments are achieved by variations of one of these instruments, while holding the other one constant at its steady-state value. For simplicity, we assume in both scenarios that the instruments are set as a linear function of the states. Hence, (10) turns into

$$(1+n)\cdot b_{t+1} = R(k_t)b_t - \pi_k(k_t - k) - \pi_b(b_t - b), \tag{11}$$

where (11) describes a broad class of debt stabilizing rules, parametrized by the pair of linear feedback coefficients π_k and π_b. For further reference, we summarize the two scenarios, both of them being consistent with (11):

Scenario 1: Variations in government spending g_t

Assume the government satisfies (11) by adjusting government spending, according to $g_t - g = -\pi_t$.[10] Then, the intertemporal equilibrium conditions read as

$$(1+n)(k_{t+1} + b_{t+1}) = \frac{\beta}{\phi+\beta}(w(k_t) - \eta_t) - \frac{\phi}{\phi+\beta}\frac{\theta}{R(k_{t+1})} \tag{12}$$

$$(1+n)\cdot b_{t+1} = R(k_t)b_t - \pi_k(k_t - k) - \pi_b(b_t - b) \tag{13}$$

$$g_t = \widetilde{g}(k_t, b_t) = g - \pi_k(k_t - k) - \pi_b(b_t - b). \tag{14}$$

Importantly, adjustments in the primary balance via variations in g_t do not affect the accumulation equation, that is, (12) is identical to (3). In other words, variations in g_t offer a particularly convenient, non-distortionary channel to stabilize the benchmark steady states. Since (14) does not feed back into the first equation the linearized dynamics can be assessed from

$$A_1 \cdot dk_{t+1} + (1+n)\cdot db_{t+1} = A_2 \cdot dk_t \tag{15}$$

$$(1+n)\cdot db_{t+1} = (R'(k)b - \pi_k)\cdot dk_t + (R(k) - \pi_b)\cdot db_t. \tag{16}$$

Scenario 2: Variations in lump-sum taxes η_t

Assume, alternatively, that the government satisfies (11) by adjusting first-period taxes such that $\eta_t - \eta = \pi_t$. Then, the intertemporal equilibrium satisfies

$$(1+n)(k_{t+1} + b_{t+1}) = \frac{\beta}{\phi+\beta}(w(k_t) - \widetilde{\eta}(k_t, b_t)) - \frac{\phi}{\phi+\beta}\frac{\theta}{R(k_{t+1})} \tag{17}$$

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[10]Primary surpluses require $g_t < g$. Recall the assumption of $g > 0$. In the following, we assume that g is sufficiently positive such that for the local dynamics around the steady state the inequality $g_t > 0$ is always satisfied.

$$(1+n)b_{t+1} = R(k_t)b_t - \pi_k(k_t - k) - \pi_b(b_t - b)$$
$$\eta_t = \widetilde{\eta}(k_t, b_t) = \eta + \pi_k(k_t - k) + \pi_b(b_t - b). \qquad (18)$$

Again, dynamics are two-dimensional in k_t and b_t but adjustments in the primary balance via variations in η_t do affect the disposable income of young agents and, hence, the accumulation equation. Linearization of (17) and (18) yields

$$A_1 \cdot dk_{t+1} + (1+n) \cdot db_{t+1} = \left(A_2 - \frac{\beta}{\phi+\beta}\pi_k\right) \cdot dk_t - \frac{\beta}{\phi+\beta}\pi_b \cdot db_t \qquad (19)$$

$$(1+n) \cdot db_{t+1} = (R'(k)b - \pi_k) \cdot dk_t + (R(k) - \pi_b) \cdot db_t, \qquad (20)$$

where the link between the instrument and the accumulation equation is captured by the use of the partial derivatives $\widetilde{\eta}_k = \pi_k$ and $\widetilde{\eta}_b = \pi_b$.

Three comments are worth making. First, under the special assumption of $\pi_k = \pi_b = 0$, the dynamics for *both* scenarios are identical to the benchmark scenario of a permanently balanced primary budget with $\pi_t \equiv 0$ for all t, that is, the two systems (15), (16) and (19), (20) are identical to (6) and (7). Second, for any of the two benchmark steady states one can derive two instrument-specific sets of feedback coefficients π_k and π_b ensuring locally stable dynamics. A simple geometric representation of these sets can be achieved if one recognizes that for each of the two instruments the local dynamics around any steady state are two-dimensional in k_t and b_t, giving rise to characteristic polynomials $p(\lambda)_i$, $i = g, \eta$, and that stability requires

$$p(1)|_i > 0, \quad p(-1)|_i > 0, \quad p(0)|_i < 1, \quad \text{with } i = g, \eta.$$

As illustrated later, for each of the two instruments these constraints (at equality) have at any steady state a linear representation in $\pi_b - \pi_k$–space, giving rise to instrument-specific stability regions. Intuitively, it is clear that these two regions are not identical, since variations in g_t leave the intertemporal accumulation Equation (12) unaffected, while variations in η_t affect this equation, as to be inferred from (17). Third, it can be investigated whether there exists a particular debt stabilizing rule, characterized by a particular pair of π_k and π_b, which can be implemented under both instruments. This amounts to check whether the regions of stabilizing feedback coefficients associated with the two instruments do overlap. Whenever this is the case such a debt stabilizing rule may be considered as the basis of a common fiscal framework. Loosely speaking, under such a framework there are no tensions between stabilization issues and the unrestricted choice of fiscal instruments. As shown in the following two subsections, such a common framework may not always exist, depending on the characteristics of the particular steady state under consideration.

3.1. Underaccumulation Steady States

Consider underaccumulation steady states, satisfying $k > 0, b = 0, R(k) > 1 + n$.

3.1.1. Instability of the Benchmark Dynamics with $\pi_t \equiv 0$ for All t

To start out with, we confirm the instability of underaccumulation steady states if $\pi_t \equiv 0$ for all t. Because of $b = 0$, the interest rate effect on debt is zero. Technically speaking, the absence of the interest rate effect implies that government debt dynamics in (7) are independent of (6). This feature ensures that the two eigenvalues of the system (6) and (7) are given by $\lambda_1 = A_2/A_1 \in (0, 1)$ and $\lambda_2 = R(k)/1 + n > 1$, where the latter eigenvalue corresponds directly to the instability of the snowball effect. In sum, this pattern of eigenvalues confirms that dynamics are locally unstable if the primary budget is *permanently* balanced.

3.1.2. Stabilizing Adjustments via π_t

If one allows temporarily for $\pi_t \neq 0$, underaccumulation steady states can be stabilized under either of the two instruments. Moreover, the two instrument-specific sets of stabilizing feedback coefficients have a common intersection. This result is directly linked to the fact that the instability of debt dynamics is driven by the snowball effect, while the interest rate effect on debt at this type of steady state is zero. Because of this feature, it is easy to verify that under either of the two instruments, if the debt stabilizing rule is characterized by the particular value $\pi_k = 0$, government debt dynamics are independent of the accumulation equation and for both systems, (15), (16) and (19), (20), the two eigenvalues are identically given by

$$\lambda_1 = A_2/A_1 \in (0, 1), \ \lambda_2 = \frac{R(k) - \pi_b}{1 + n}.$$

Evidently, if $\pi_k = 0$ and $\pi_b \in (R(k) - (1 + n), R(k) + 1 + n)$ dynamics will be locally stable under both instruments. Intuitively, this reasoning reflects that either of the two instruments can be freely used to generate the necessary revenues to stabilize the snowball effect. Of course, as stressed above, via the accumulation equation the two instruments affect the capital stock and, hence, the interest rate differently. Yet, these differences do not feed back into the dynamics of government debt. In other words, the absence of the interest rate effect on debt ensures that it is always possible to isolate government debt dynamics from instrument-specific adjustments in the private sector equilibrium equations (which in our economy are just given by the single accumulation equation). In sum, this reasoning leads to:

PROPOSITION 1: *Consider the two instrument-specific sets of feedback coefficients π_k and π_b which ensure local stability at the underaccumulation steady state under the debt stabilizing rule (11). The two sets have a joint intersection, that is, there exist* *values for π_k and π_b such that the debt stabilizing rule can be implemented under both instruments with a common set of feedback coefficients.*

The assumption of $\pi_k = 0$ makes it particularly easy to proof Proposition 1. Yet, the proposition is not confined to this special assumption. Generally speaking, whenever $\pi_k \neq 0$ this creates a policy-induced interdependence between capital stock dynamics and government debt dynamics. This interdependence differs between the two instruments. Yet, a debt stabilizing rule with $\pi_k \neq 0$ may nevertheless be implementable under either of the two instruments. To illustrate this, consider

Example 1: $\alpha = 0.4, \phi = 1, \beta = 0.5, z = 15, \eta = -\theta = 2.91, g = 4.1, 1 + n = 2.43$. Assuming a period length of 30 years, this implies an annual population growth of 0.03. If $b = 0$, one obtains $R = 6.29$ which corresponds to an annual real interest rate of 0.06, consistent with an underaccumulation steady state, that is, $R(k) > 1 + n$. Moreover, $k = 0.92$, $y = 14.53$, where y denotes per capita output, leading to $g/y = 0.28$, $\eta/y = 0.2$, $\theta/y = -0.2$, that is, agents have a similar steady-state tax burden in both periods, measured in terms of per capita output.

Figure 1 illustrates Example 1 and plots the stability regions in $\pi_b - \pi_k$−space for adjustments in g_t and, alternatively, in η_t. Points inside the two triangles in bold lines are associated with two stable eigenvalues.[11] The coordinates $\pi_b = \pi_k = 0$ correspond to the dynamics of a permanently balanced primary budget and lie, by construction, outside the two stability triangles. As discussed above, if $\pi_k = 0$ the interval of π_b−values consistent with locally stable dynamics is identical for both instruments. If $\pi_k \neq 0$, however, the two instrument-specific stability regions, while having a common intersection, also have clear idiosyncratic components. These idiosyncratic components reflect that stabilizations via η_t affect the intertemporal accumulation Equation (12), while this is not the case for stabilizations via g_t. With $\pi_k \neq 0$, these differences feed back into the dynamics of government debt, imposing different stability requirements in terms of π_b. Complementing Proposition 1, this ensures, as established formally in Michel, Von Thadden, and Vidal (2006), that for each instrument there exists a stability region which does *not* lead to stability under the other instrument.

PROPOSITION 2: *For each instrument there exist stabilizing feedback coefficients π_k and π_b which lie outside the stability regions of the other instrument.*

[11]Stable pairs of feedback coefficients inside the stability triangles of Figure 1 are associated with a wide range of possible adjustment patterns. Only a small subset of points in either triangle is associated with two positive eigenvalues, ensuring monotone adjustment dynamics.

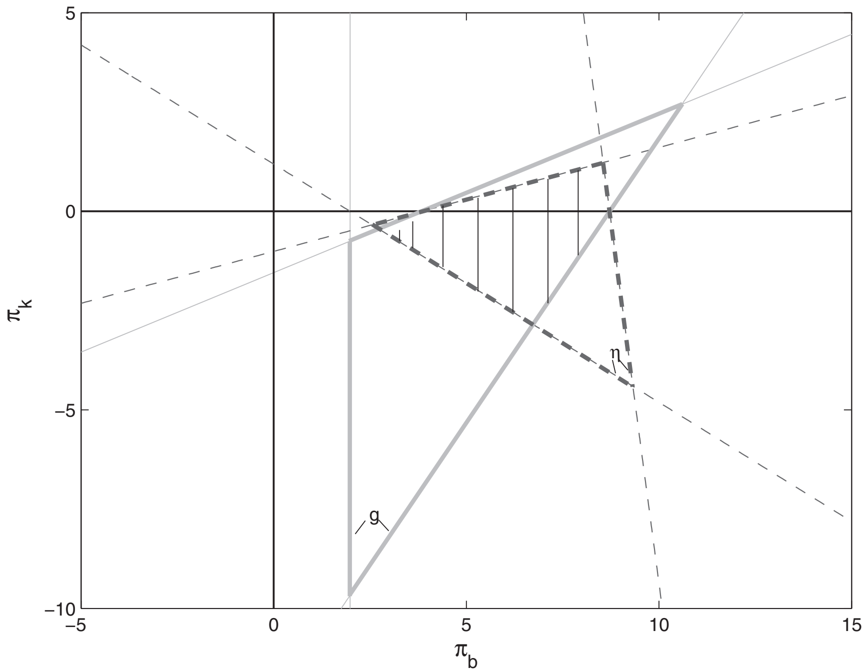

Figure 1: Underaccumulation steady state. Stability regions for both instruments (example 1). Shaded area: common intersection, g_t-regime: solid line; η_t-regime: dashed line.

As the following subsection shows, at steady states with positive debt the idiosyncratic components may become the dominating force, precluding the existence of a common intersection of the two instrument-specific stability regions.

3.2. Golden Rule Steady States

Assume now that savings are sufficiently high such that the economy settles down at a golden rule steady state with a lower interest rate (such that $R(k) = 1 + n$) and a positive debt level, satisfying

$$b_{gr} = \frac{s}{1+n} - k = \frac{\frac{\beta}{\phi+\beta}(w(k)-\eta) - \frac{\phi}{\phi+\beta}\frac{\theta}{R(k)}}{1+n} - k > 0.$$

Compared with Section 3.1, higher savings may reflect structural reasons (like a higher propensity to save because of different preferences) or the response to different government policies (like lower first-period taxes or lower second-period transfers).

3.2.1. *Instability of the Benchmark Dynamics with* $\pi_t \equiv 0$ *for All t*

Because of $R(k) = 1 + n$, the snowball effect is associated with a unit root, and strict instability is ensured by the additionally operating interest rate effect on debt. This can be confirmed by inspecting the characteristic polynomial associated with (6) and (7), evaluated at the golden rule steady state:

$$p(\lambda) = \lambda^2 - \left[1 + \frac{A_2}{A_1} - \frac{R'(k) \cdot b_{gr}}{A_1}\right] \cdot \lambda + \frac{A_2}{A_1}.$$

Then, $p(0) = A_2/A_1 \in (0, 1)$ and $p(1) = R'(k)b_{gr}/A_1 < 0$, implying $0 < \lambda_1 < 1 < \lambda_2$. Hence, dynamics are locally unstable if the primary budget is *permanently* balanced.

3.2.2. *Stabilizing Adjustments via* π_t

Differently from Section 3.1, $b_{gr} > 0$ ensures that the interest rate effect on debt is a key margin of instability. It is precisely this margin which makes it difficult to find for such steady states a debt stabilizing rule that can be implemented under both instruments. Technically speaking, the presence of the interest rate effect on debt makes it impossible to address this source of instability directly by means of adjustments via π_k and, at the same time, to maintain a recursive dynamic structure which isolates government debt dynamics from instrument-specific adjustments in the accumulation equation. Recall from above that the strength of the interest rate effect on debt rises in the level of debt, leading to increasingly distinct stabilization profiles under the two instruments. Hence, the implementability problem (which requires a common set of feedback coefficients) becomes increasingly severe as the level of debt rises.

To support this reasoning, consider the following two examples. First, varying Example 1, we choose a parametrization which leads to a golden rule steady state with a "small" debt ratio of 0.02.[12] To this end, by lowering α, Example 2 chooses a higher wage income share that raises the propensity to save. This structural variation is sufficient to shift the economy to a golden rule steady state:[13]

Example 2: Consider Example 1, but let $\alpha = 0.2$, $\eta = -\theta = 3.16$, $g = 4.46$. Assuming $b = 0$, one obtains $R = 1.76 < 1 + n = 2.43$. At the golden rule steady state, $R = 1 + n = 2.43$, yielding an annual real interest rate of 0.03, $b_{gr} = 0.36 > 0$, $k = 1.3$, $y = 15.8$ and a debt ratio of $b_{gr}/y = 0.02$. Moreover, $g/y = 0.28$, $\eta/y = 0.2$, $\theta/y = -0.2$, that is, agents have a similar tax burden in both periods as in Example 1.

[12]To put this number into perspective it should be stressed that throughout the paper government debt is expressed as *net debt*.

[13]To allow for comparability with Example 1, we adjust the levels of η, θ and g such that the corresponding ratios in terms of income are the same as in Example 1.

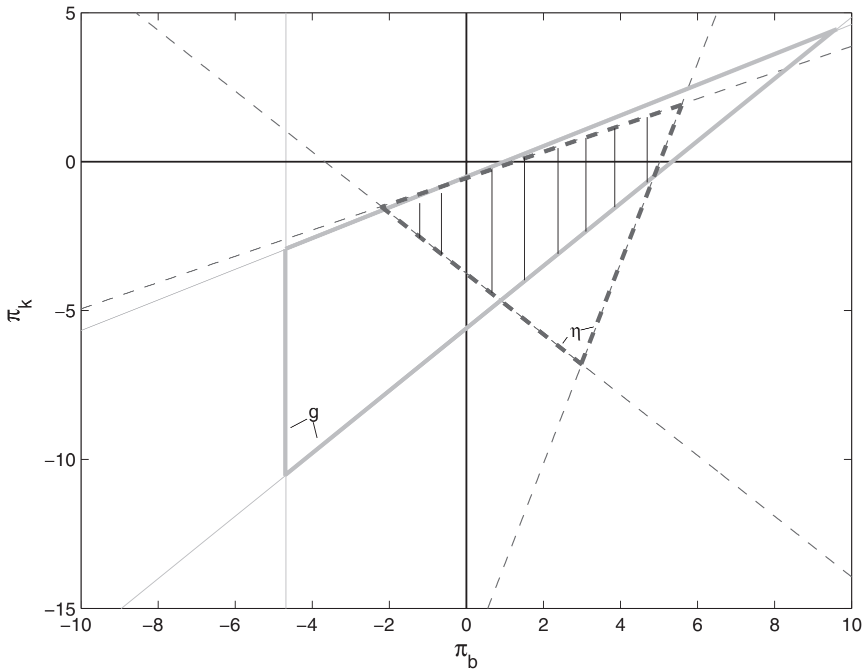

Figure 2: Golden rule steady state. Stability regions for both instruments (example 2). Shaded area: common intersection, g_t-regime: solid line; η_t-regime: dashed line.

Figure 2 illustrates the two instrument-specific stability triangles associated with Example 2. The coordinates $\pi_b = \pi_k = 0$ lie, once again, outside the two stability triangles. Moreover, reflecting the sufficiently low level of debt, Figure 2 shares with Figure 1 the feature that the two regions have a common intersection.

Alternatively, Example 3 varies Example 1 by allowing for a more substantial increase in savings through structural factors (by further lowering α and by raising the savings rate via a higher value of β) as well as policy-related factors (by considering a tax-transfer system that shifts the tax burden more strongly to second period income). In sum, this leads to a substantially higher steady-state debt ratio of 0.14.

Example 3: Consider Example 1, but let $\alpha = 0.15$ and $\beta = 1$. Moreover, $\eta = 0.74$, $\theta = -6.63$, $g = 3.47$. Assuming $b = 0$, one obtains $R = 0.32 < 1 + n = 2.43$. At the golden rule steady state, $R = 1 + n = 2.43$, yielding $b_{gr} = 2.11 > 0$, $k = 0.89$, $y = 14.74$ and a debt ratio of $b_{gr}/y = 0.14$. Moreover, $g/y = 0.24$, $\eta/y = 0.05$, $\theta/y = -0.45$, that is, the tax burden in the second period is now substantially higher than in the first period.

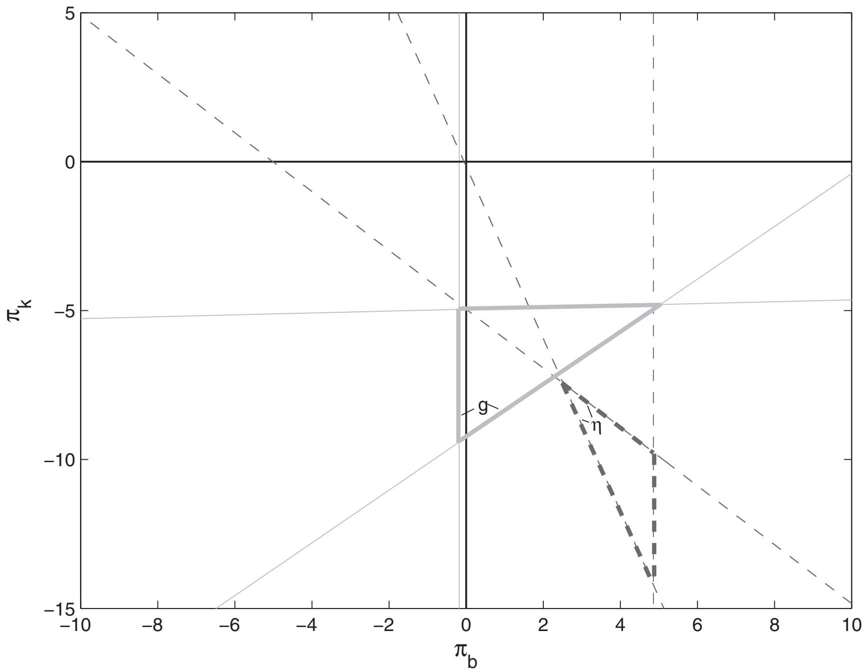

Figure 3: Golden rule steady state. Stability regions for both instruments (example 3). g_t-regime: solid line; η_t-regime: dashed line.

Figure 3 illustrates the stability triangles associated with Example 3. The key result to be inferred from Figure 3 is that the two stability regions associated with variations in g_t and η_t cease to have a common intersection, reflecting that b_{gr} exceeds some threshold value. Intuitively, this finding reflects that it is still possible to use either of the two instruments to generate the necessary revenues to stabilize the economy at the golden rule steady state. However, via the accumulation equation the two instruments affect the capital stock and, hence, the interest rate differently. The interest rate effect ensures that these instrument-specific adjustments feed back into the dynamics of government debt. Hence, depending on the strength of the interest rate effect (which rises in b), it may no longer be possible to find a common pair of $\pi_k$andπ_b which stabilizes the economy under both instruments. In sum, this reasoning gives rise to the general result:[14]

[14]Michel, Von Thadden, and Vidal (2006) offers a more precise discussion of the role of the threshold value of b_{gr} in Proposition 3. To this end, a distinction is made between structural parameters $S = \{\alpha, \beta, \phi, n, z\}$ and policy parameters $P = \{\eta, \theta, g\}$. In particular, it is shown that the maximum amount of steady state debt that can be achieved via changes in P depends on S, and this debt level may not under all calibrations be high enough to rule out the existence of common feedback coefficients.

PROPOSITION 3: *Consider the two instrument-specific sets of feedback coefficients* π_k *and* π_b *which ensure local stability at the golden rule steady state under the debt stabilizing rule (11). These sets do not necessarily have a joint intersection, that is, it is possible that the debt stabilizing rule cannot be implemented under both instruments with a common set of feedback coefficients. In particular, it may well become impossible to implement the debt stabilizing rule under both instruments if the golden rule debt level* b_{gr} *exceeds some threshold value* $b^*_{gr} > 0$.

4. An Alternative Representation of the Fiscal Rule

It is worth pointing out that none of the results established in this paper depends on the assumption to consider stabilizing adjustments via a rule for the primary balance. Specifically, there exists a fully equivalent representation of our analysis which is expressed in terms of a rule for the overall deficit (i.e., inclusive interest payments). This observation is important, since rules of the latter type are more widely observed in reality.

To make this point precise, we repeat the flow budget constraint of the government $(1 + n) \cdot b_{t+1} = R(k_t) \cdot b_t - \pi_t$ and maintain the assumption that at the steady states under consideration the primary balance is zero, that is, $\pi_t = 0$. Yet, let us assume that the debt stabilizing rule is now expressed in terms of stabilizing reactions of the overall deficit Δ_t (inclusive interest payments), using

$$(1 + n) \cdot b_{t+1} = b_t + \Delta_t,$$

$$\Delta_t = \Delta(k_t,\ b_t) = (R(k_t) - 1) \cdot b_t - \pi_t.$$

Note that Δ_t, at any moment in time, consists of a predetermined component linked to interest payments on debt, and a policy component linked to the primary balance. Only the latter part can react to the two predetermined states of the economy, b_t and k_t. Accordingly, a debt stabilizing rule which specifies reactions of the overall deficit to the states of the economy, in linearized form, needs to be established from

$$(1 + n) \cdot db_{t+1} = \Delta_k \cdot dk_t + (1 + \Delta_b) \cdot db_t, \tag{21}$$

$$\text{with} \ :\ \Delta_k = R'(k) \cdot b - \pi_k, \ \text{ and } \ \Delta_b = R(k) - 1 - \pi_b. \tag{22}$$

Consider the two dynamic systems (15), (16) and (19), (20) derived above for the two instruments. Using (21) as the second equation in these two systems and replacing π_k and π_b by the terms $R'(k) \cdot b - \Delta_k$ and $R(k) - 1 - \Delta_b$ in the first equation of the two systems, respectively, this leads to two new systems, both exhibiting two-dimensional dynamics in k_t and b_t. Importantly, since $R(k)$, $R'(k)$, and b are all evaluated at constant steady-state values, the switch from the representation in $\pi_b - \pi_k$−space to a representation in $\Delta_b - \Delta_k$−space amounts to an affine transformation, not affecting the results of Propositions 1–3. In other words, (22) maps every pair of (π_k, π_b), with properties as listed in Propositions 1–3, into a unique pair (Δ_k, Δ_b) with identical properties if stabilizations were done via (21).

Finally, we stress that the reasoning of this paper can also be used to address the stabilization of steady states with non-zero primary surpluses ($\pi \neq 0$), satisfying $\pi_t = \pi + \pi_t^s(k_t, b_t)$, where π_t^s denotes the stabilization component of π_t. Yet, for this extension, prior to the derivation of stability regions, one needs to address that the benchmark steady states themselves change, that is, the focus on the steady states established in Section 2 would only be appropriate in the limit case of $\pi \to 0$.

5. Conclusion

This paper studies the stabilization of government debt dynamics under a number of different fiscal instruments from a comparative perspective. Specifically, the paper addresses the question of whether a debt stabilizing rule, which links the stabilization of long-run debt to the underlying state of the economy, can be implemented under all available fiscal instruments with a common set of feedback coefficients. Using a tractable overlapping generations model, the main analytical result of the paper says that the answer to this question cannot be given without reference to the level of long-run debt at which the economy is stabilized. This finding reflects that different fiscal instruments (like government spending, public transfers, and the menu of available taxes) affect the economy through instrument-specific margins which are associated with different distortions (related, e.g., to the labor-leisure decision, investment decisions, or consumption decisions). The target level of debt to be stabilized determines the weight of these margins within the set of intertemporal equilibrium conditions. As the target level of debt rises the importance of these distinct margins increases, implying that for any particular debt stabilizing rule the instrument-specific adjustment paths become increasingly diverse. Exploiting this feature, the paper shows that there can easily exist a threshold value of long-run debt beyond which the instrument-specific adjustment paths become sufficiently diverse such that there no longer exists a debt stabilizing rule that can be implemented under all instruments with common feedback coefficients.

As the paper stands, these results are derived in a deliberately small and fully tractable model of a closed economy. Yet, the policy implications can probably best be seen in the context of a monetary union with decentralized fiscal policies, subject to certain provisions of a common fiscal framework. The analysis of this paper does not add any new arguments why such a framework is necessary. Instead, it indicates that within any such framework high levels of average debt are likely to create tensions between the necessary provisions of a common framework (ensuring stable debt dynamics in all member states) and the unrestricted choice of fiscal instruments at the national level. This paper implicitly assumes that the latter feature is by itself of considerable value. Therefore, if one wishes to preserve this value under the conditions of a monetary union the results of this paper indicate that the union's fiscal framework should be organized around a sufficiently ambitious target level of debt. We leave it for future work to further explore this mechanism, also with a focus on quantitative issues, within a modeling framework that explicitly allows for features which are characteristic of a set-up with multiple countries.

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