Vanderbilt University Department of Economics Working Paper Series
Nashville, TN 37235-1819

Vanderbilt University Department of Economics Working Papers 14-00001

The optimal supply of liquidity and the regulations of money substitutes: a Baumol-Tobin approach

Benjamin Eden
Vanderbilt University

Abstract

I use the Baumol-Tobin approach to examine the following propositions: (a) The optimal supply of liquidity requires a government loan program in addition to paying interest on reserves held by banks, (b) The adoption of the optimal policy will crowd out private credit arrangement and will thus shrink the financial sector and (c) regulations aimed at eliminating money substitutes may be redundant if the optimal policy is adopted but otherwise may improve welfare.

Citation: Benjamin Eden, (2014) "The optimal supply of liquidity and the regulations of money substitutes: a Baumol-Tobin approach", *Vanderbilt University Department of Economics Working Papers*, VUECON-14-00001.
Contact: Benjamin Eden - ben.eden@vanderbilt.edu.
Submitted: January 10, 2014. **Published:** January 10, 2014.
URL:http://www.accessecon.com/Pubs/VUECON/VUECON-14-00001.pdf

THE OPTIMAL SUPPLY OF LIQUIDITY AND THE REGULATIONS OF MONEY SUBSTITUTES: A BAUMOL-TOBIN APPROACH

Benjamin Eden[2]

Vanderbilt University

January 2014

I use the Baumol-Tobin approach to examine the following propositions: (a) The optimal supply of liquidity requires a government loan program in addition to paying interest on reserves held by banks, (b) The adoption of the optimal policy will crowd out private credit arrangement and will thus shrink the financial sector and (c) regulations aimed at eliminating money substitutes may be redundant if the optimal policy is adopted but otherwise may improve welfare.

Key words: The role of government, Liquidity, Regulations of money substitutes, Implementation of the Friedman Rule, The optimal size of the financial sector.

---

[2] I would like to thank Maya Eden for helpful suggestions.

Monetary theory is like a Japanese garden. It has esthetic unity born of variety; an apparent simplicity that conceals a sophisticated reality; a surface view that dissolves in ever deeper perspectives. Both can be fully appreciated only if examined from many different angles, only if studied leisurely but in depth. Both have elements that can be enjoyed independently of the whole, yet attain their full realizations only as part of the whole.

Milton Friedman (1969, Preface).

1. INTRODUCTION

I use the Friedman rule as a benchmark to evaluate some regulations and policy proposals. In his original article Friedman (1969) argued that creating money is costless from the social point of view. A society that wants to increase the amount of real balances will do it without giving up real resources: The increase in the demand for money will lead to a decline in the price level with no other real effects. However, the cost of accumulating real balances to the individual may be positive if the real rate of return on money is less than the real rate of return on other illiquid assets: An individual who wants to accumulate money must give up consumption.

The difference between the social and the private point of view can be eliminated by a policy that equates the real rate of return on money to the real rate of return on other assets. When cash is not important this can be achieved by paying interest on money. When cash is important it can be achieved by a policy that leads to deflation and zero nominal interest rate. Here I assume that cash is not important and paying interest on money is possible.

There are however many unresolved questions about the optimal policy. Should the government increase liquidity by issuing credit? Will the optimal policy crowd out money substitutes? Should we adopt regulations aimed at eliminating money substitutes, as was advocated by Simons (1948)?

Here I attempt these questions using the Baumol-Tobin approach to modeling money. I consider an economy with two assets: money and riskless nominal bonds. The use of bonds requires trips to the bank and therefore bonds will not be used when the government pays interest on money.

To pay interest on money, the government must raise taxes. These taxes can be viewed as a loss of seigniorage revenues, where seigniorage is defined by the amount that the government saves from taking a loan at below market rate. My reading of the literature is that consumption tax usually dominates seigniorage taxation.[3] Therefore paying interest on money is a good policy for a government that can administer consumption taxes.

Paying interest on money will reduce the need to regulate money substitutes because there will be less incentives to create such substitutes. I also consider the question of regulations under the assumption that the interest paid on money is less than the optimal rate. I find that the model tends to supports Simons' type regulations for eliminating money substitutes, with some qualifications.

Section 2 is an informal discussion of related issues. Section 3 is a general model that can be used to characterize the optimal policy. Section 4 and 5 adopt the Lagos-Wright structure and focus on the comparison of alternative steady states. In the body of the paper I assume two assets: money and government bonds. In the Appendix I allow for private bonds.

[3] See, for example, Kimbrough (1986), Chari, Christiano and Kehoe (1996), Correia and Pedro (1996) and da Costa and Werning (2008).

2. RELATED ISSUES AND LITERATURE

The main points of the paper are: (a) money substitutes are a waste from the social point of view and at the optimal policy they will not be used and (b) in a second best world in which the interest paid on money is below market, regulations that prohibit the use of money substitutes may improve welfare. There is a well-known problem in defining money and as a result a problem in defining money substitutes. Here money is balances in checking and debit accounts and money substitutes are activities and assets aimed at economizing on money. Unfortunately it is not easy to make the connection between this definition and the real world. In this section I attempt to do that by discussing the relevant literature and the current policy debate. The reader may want to read it after reading the model sections that follows.

<u>Modeling aspects</u>:

Patinkin (1965) distinguished between the two by assuming that within period loans (with a term that is less than the length of the period) are prohibitively costly but between periods loans (with a term longer than the length of the period) are costless. Similarly, Lagos and Wright (2005) divide each period into two sub-periods. In the first credit arrangements are costless but in the second the cost of credit arrangements is prohibitive.

An alternative view is in Friedman (1977) who argues that money replaces a variety of credit arrangements and therefore the cost of holding money depends on the entire yield curve.

It seems that the evidence about the origin of money supports the alternative view in Friedman (1977). Benes and Kumhof (2012) argue against the hypothesis that money was created in the private sector to overcome the double coincidence of wants problem. They cite anthropological and historical evidence that support the view that money was typically introduced by government or religious institutions and replaced private credit arrangements. Their view is consistent with the views of Knapp (1924) and Lerner (1947) who argue that general acceptability of almost anything can be established by the state if it is willing to accept it as tax payments. And the state can destroy a particular type of money if it declares that it will not accept it as tax payment.[4]

In this paper I use both the Patinkin-Lagos-Wright approach and the Friedman (1977) approach. The first approach is more tractable but less general. In the first money crowds out short-term bonds while in the second it crowds out bonds of all maturities.

Banks:

Banks intermediate between savers and investors and economize on cash. The first role involves the choice of good investment projects and good collaterals and does not depend much on whether interest is paid on money or not.

Lucas and Stokey (2011) describe the second role using the analysis in Diamond and Dybvig (1983). They consider an economy in which bills for purchases arrive with unpredictable lags and bills must be paid exactly when due. They think of a bank "as an institution that pools payment risks, making all of its client better off than they would be acting on their own".

The pooling of payment risks is beneficial to the clients of a particular bank only if there is no interest on money. Otherwise, there is no need to economize on cash and there is no need to pool payment risks. Here I argue that even if the Friedman rule is not implemented economizing on cash may reduce social welfare in spite of the fact that from the point of view of the clients of an individual bank the pooling of payment risks is welfare improving.

---

[4] The Bitcoin is a direct challenge to this view. It is too soon to tell how successful it will be and whether it will survive in the long run.

Lucas (2013) analysis of 100% reserve requirement also depends critically on the assumption that the nominal interest is positive. Lucas (2013) sees the benefit of 100% reserve requirement in terms of eliminating the risk of bank runs without creating a moral hazard problem. The cost is in terms of trips to the banks that, according to the Baumol-Tobin model, occur more often when the cost of holding money is high. The cost of holding money is the difference between the nominal interest (paid by borrowers) and the interest paid on deposits. In the absence of regulations this difference is equal to the nominal interest rate times the reserve requirement ratio. Thus if the reserve requirement is 15% and the nominal interest is 3.5% the interest on demand deposits will be about 3% and the cost of holding demand deposits is 0.5%. If the reserve requirement is 100% there will be no interest paid on demand deposits and the cost of holding these deposits is 3.5%. As a result adopting 100% reserve requirement will lead to more trips to the bank and loss in output.

The trips to the bank cost of adopting 100% reserve requirement is zero when the nominal interest rate is zero. It is also zero if the Fed pays interest on reserves. In the above example, if the Fed pays 3.5% interest on deposits kept at the Fed to satisfy the 100% reserve requirement the banks will pay 3.5% on demand deposits and charge fees for its services. If, for example, there is a fixed cost for running a checking account, the bank will charge a fixed monthly payment and pay full interest on the balances in the account. This will eliminate the wasteful trips to the bank.

Furthermore, when interest is paid on reserves there is less of a need to impose a 100% reserve requirements. Banks may hold it voluntarily because there is no cost for holding reserves and there is the benefit of eliminating the risk of bank runs.

Nominal bonds:

From the private point of view, nominal bonds are used to economize on real balances: If an agent does not plan to use the funds in his checking account in the next year, he can hold bonds for a year. But as in the case of banks, from the social point of view resources spent on economizing on real balances are a waste. This waste will occur even when a 100% reserve requirement is imposed and will be eliminated only if full interest is paid on money.

The view that using bonds to economize on real balances is a waste from the social point of view, applies to bonds of all maturities. This is different from Simons (1948) who advocate the prohibition of short-term nominal bonds but not of long-term nominal bonds (consuls).

Gorton and Pennacchi (1990) define liquid assets as assets that can be traded with minimal risk of asymmetric information. Stein (2012) adopts this definition but focus on riskless assets. Should the government have a complete monopoly on creating liquidity in this sense? The answer must be in the negative because a firm should be able to finance a riskless project. But the firm can issue stocks to finance projects. Therefore a government monopoly on money and regulations aimed at eliminating nominal bonds (claims on money) will not reduce the ability of firms to finance riskless projects.

Will the elimination of government bonds impair the ability of the government to smooth taxes by running deficits and surpluses? The answer is in the negative because the government can use a checking account held at the central bank.

Money substitutes:

Simons advocated regulations aimed at eliminating substitutes for money. Clearly he did not mean physical capital or other productive assets that may be used as an alternative to money.[5] But distinguishing between money substitutes and productive assets or claims on the output of productive assets is not straightforward.

In Eden (2012) I consider an overlapping generations economy in which agents who are uncertain about their lifetime try to eliminate accidental bequest by annuity type contracts. This activity is perfectly rational from the individual point of view in spite of the fact that it requires resources. But from a social point of view economizing on real balances by attempting to eliminate accidental bequests is a waste.

Abstracting from annuity type contracts that economize on accidental bequest, we may define money substitutes as assets that are traded only when money does not earn interest.

An alternative definition is from the point of view of a planner who can tell agents the amount of real balances that they must hold on average in any given period (say a month or a year) but has no other powers. Money substitutes will be used when the planner set a low average requirement. When the planner raises the required average, there will be less trade in money substitutes and money substitutes will disappear when the required average is sufficiently high. Thus, assets that do not survive a high required-average are money substitutes.[6]

These definitions are not straightforward because they require hypothetical thought experiments that can be performed only in the model economy. Government nominal bonds are an exception. It seems that they will fit the definition of a money substitutes in a wide variety of models in which using bonds to execute any given set of

[5] Unlike nominal bonds, there is no clear relationship between inflation and the level of physical capital. In Friedman (1969) lower inflation does not change the capital stock. In Jovanovic (1982) the effect of inflation on the capital stock is ambiguous.

[6] To fit this definition, bank deposits may be viewed as a portfolio of contingent claims, where each claim specifies the state of nature and the time at which money will be received. Annuity contracts will survive a high required-average because one can always benefit by selling claims on accidental bequest. To incorporate annuity contracts into the definition of money substitutes we may assume a planner who imposes average net money holdings that exclude obligations that will be paid after death.

transactions requires more real resources than using money to execute the same transactions.

.

The role of government in the consumer loans market:

Friedman (1960) argued that a government loan program is needed because of the inability of the private sector to enforce uncollateralized loan contracts. They advocated a student loan program, but their argument applies also to loans that are used for consumption smoothing or for liquidity.

In Eden (2012) I use the money in the utility function approach and focus on liquidity loans. I argue that satiating agents with real balances requires such loans. The argument holds in Friedman original set up with infinitely lived agents but it receives its full force in an overlapping generations model. In the infinitely lived consumer case there is a transition period between the announcement of the optimal policy and the time the economy reaches the steady state. During this transition period agents who cannot lend and borrow are not satiated with money. To elaborate, let us assume that after the announcement of the optimal policy the price level drops to a level that *on average* satiate agents with real balances. After the drop in the price level, some agents will have more money than they want to hold and some will have less money than they want to hold. Those with “too much” money will de-cumulate real balances by consuming more than their income while those who do not have enough money will accumulate money by consuming less than their income. This process will continue until they reach the steady state. See Bhattacharya, Haslag and Martin (2005) for a detailed analysis.

The problem is more severe in an overlapping generations model because young agents that start with no money will have to reduce their consumption until they accumulate the satiating level of real balances. Indeed they may never get to the satiation level even when the nominal interest rate is zero, because they cannot get uncollateralized loans.

The above problem can be solved if the government supplies “liquidity loans”. This is not a new idea. For example, Williamson (2012) allow the government to be a net creditor and note that one way of implementing the Friedman rule is by central bank lending. In this case the central bank lends out the entire stock of currency and then retires currency over time using the net interest on the central bank loans. Combining this idea with Friedman’s education loans idea will lead to the conclusion that government loans are desirable for the initial introduction of money and for education. In Eden (2012) I go beyond that limited role and argue for unrestricted loans to young individuals allowing them to use the loans in any way they want, including loans to obtain liquidity and loans to smooth consumption. The analysis here and in Eden (2012) may be relevant to the debate between the real bills doctrine and the quantity theory as discussed by Sargent and Wallace (1982). It maybe of some interest that implementing the Friedman rule requires direct intervention in the consumer loans market that is not unlike the intervention proposed by the real bills doctrine.

<u>The policy debate</u>:

Friedman (1967) and Friedman and Schwartz (1986) see a role for government in money but are against financial regulations.[7] Their position requires a sharp distinction between money and its substitutes, and is more in line with the Patinkin-Lagos-Wright approach than with Friedman (1977).

Gorton and Metrick (2010) propose to add regulations to the Dodd-Frank Act in three areas: money-market mutual funds (MMMF), securitization, and repurchase agreements (repos). They propose a new form of narrow banks for MMMFs and securitization combined with strict guidelines on collateral for both securitizations and

[7] They seem to struggle with the Austrian school that advocates no role of government in money (see for example, Hayek [1970]) and the empirical evidence they accumulated about the importance of controlling the money supply. Their definition of money is empirically based (they use M2 because it works well empirically and there is better data on this particular aggregate).

repos. Here I argue that the need to regulate money substitutes will diminish if the optimal policy is adopted and consider the welfare implications of tough regulations that will eliminate the "shadow banking" sector.

The role of the central bank in crisis situation is a more difficult subject. I agree with Lucas and Stokey (2011) that the Fed's lending in a crisis should be targeted toward preserving market liquidity, not particular institutions. I think that this will be easier if a government loan program is in place. In this case, the government can simply announce that because of the crisis agents can get additional uncollateralized loans. The alternative supported by Lucas and Stokey (2011) is to follow the Bagehot rule: In a crisis, the central bank should lend at good collateral at a penalty rate. I do not understand the need for collateral and the penalty rate aspect of the rule.[8]

3. A BAUMOL-TOBIN TYPE MODEL

In Friedman (1977), money and bonds are used in all types of transactions but using bonds requires "trips to the bank" as in the Baumol-Tobin model. It is difficult to use this approach for comparing alternative steady states. But it is embarrassingly simple when using it to discuss the Friedman rule. When money earns the same rate of return as bonds, money will completely crowd out bonds and this will eliminate the wasteful trips to the bank.

I assume a single good (corn) endowment economy that is populated by infinitely lived agents. The endowment varies over agents and time and there are two assets: money and government bonds. The endowment of agent h at time t is $\bar{Y}_t^h$ units. To simplify, I assume that the dollar price of corn is constant over time and is equal to \$1 per unit.

[8] The task of finding good collaterals seems particularly difficult in crisis situations. Should we take the fire sale value of the asset or the pre-crisis value? It seems that the Fed policy in the current crisis was to buy troubled assets. This policy can breed corruption. Injecting money directly to individuals is more transparent.

The government has a bank that makes consumer loans at $t=0$ and offers savings and checking accounts. The gross interest on savings account is $R_t = 1 + r_t$ and the gross interest on checking account is $R_{mt} = 1 + r_{mt} \le R_t$. As in the Baumol-Tobin model the agent can vary the amount in the checking account at no cost but a trip to the bank is required to vary the amount in the savings account. The cost of a trip to the bank is α^h and occurs whenever the amount in the savings account, b_t^h is different from the amount that will be passively accumulated, Rb_{t-1}^h. The transaction cost of using funds in the savings account is thus $\alpha^h I_t^h$, where

(1) $$I_t^h = \{1 \text{ if } b_t^h \neq Rb_{t-1}^h \text{ and zero otherwise}\}$$

At $t=0$, the agent takes a loan of d_0^h dollars (since the dollar price of a unit of corn is unity, the loan is in real - units or corn - terms). The loan has no maturity: The borrower pays interest but does not pay the principle. The agent chooses the initial portfolio out of the following budget constraint:

(2) $$m_0^h + b_0^h = m_{-1}^h + d_0^h \quad \text{and} \quad 0 \le d_0^h \le B$$

where m_{-1}^h is the amount of real balances carried from $t=-1$, m_0^h is the amount he puts in his checking account, b_0^h is the amount he puts in his savings account and B is a large but finite limit on the initial borrowing. The asset evolution equation is given by:

(3) $$b_t^h + m_t^h = \bar{Y}_t^h + R_{mt} m_{t-1}^h + R_t b_{t-1}^h + g_t^h - Y_t^h - \alpha^h I_t^h - r_t d_0^h$$

where m_t^h is the end of period balances in the checking account, b_t^h is the end of period balances in the savings account, $\bar{Y}_t^h$ is the endowment of corn, g_t^h is government transfer, Y_t^h is the consumption of corn and the last two terms on the right hand side of (3) are the cost of the trip to the bank and the interest payment on the initial loan.

The agent solves the following problem:

(4) $$\max_{Y_t^h, I_t^h, m_t^h, b_t^h, d_0^h \ge 0} \sum_{t=1}^{\infty} \beta^t U(Y_t^h) \quad \text{s.t. (1)-(3).}$$

Equilibrium is a sequence of interest rates $\{R_t, R_{mt}\}$, a sequence of government transfers $\{g_t^h\}$ and a sequence of endogenous variables $\{Y_t^h, I_t^h, d_0^h, b_t^h, m_t^h; h = 1,...,n\}$ such that (a) given the interest rates $\{R_t, R_{mt}\}$ and the government transfers $\{g_t\}$, the sequence $\{Y_t^h, I_t^h, d_0^h, b_t^h, m_t^h\}$ solves (4) for all h, and (b) the market clearing conditions are satisfied (for all t):

(5) $$\sum_h Y_t^h = \sum_h (\bar{Y}_t^h - \alpha^h I_t^h)$$

The Friedman rule is a policy that satisfies:

(6) $$R_{mt} = R_t \quad \text{for all } t$$

To implement the Friedman rule, the government imposes the following individual specific tax on the initial real balances.

(7) $$g_t^h = -r_t m_{-1}^h$$

Thus, under the Friedman rule the government collects taxes to pay interest on the balances carried from $t = -1$, before the announcement of the Friedman rule policy.[9] The government finances the rest of the interest payments out of the interest it receives on the initial loan.

I now show the following Claim.

Claim 1: Under the policy (6) and (7), the equilibrium outcome is Pareto efficient.

To show the claim note that under (6) there is no reason to go to the bank and the consumer will put all his money in the checking account. We can therefore write the budget constraint (3) as:

(8) $$m_t^h = \bar{Y}_t^h + R_{mt} m_{t-1}^h + g_t^h - Y_t^h - r_t d_0^h$$

[9] It is also possible to eliminate the initial real balances by adopting a new currency or by inducing a jump in the price level that will make the initial balances worthless (i.e., hyper-inflation at the initial phase). But these other ways of dealing with the amount of money held at the time the optimal policy is adopted have an effect on income distribution.

By forward substitution of (8) and using (7) we get:

(9) $$\sum_{t=1}^{\infty} D_t Y_t^h = \sum_{t=1}^{\infty} D_t \bar{Y}_t^h ,$$

where $D_t = (R_1)^{-1} \times (R_2)^{-1} \times ... \times (R_t)^{-1}$ is the discount factor for corn delivered at time t . See the Appendix for the derivation of (9).[10]

Substituting $I_t^h = 0$ in (5) leads to:

(10) $$\sum_h Y_t^h = \sum_h \bar{Y}_t^h$$

The budget constraints (9) and the market clearing conditions (10) are standard for exchange economies with no money. It is well known that there exists equilibrium for this standard exchange economy and the equilibrium outcome is efficient.

We have thus shown that the implementation of the Friedman rule by a government loan leads to an efficient allocation. Note that an initial government loan is necessary because otherwise, agents who start with less than the average amount of real balances will have to accumulate real balances and will therefore consume initially less than their permanent income (or less than the optimal consumption under the budget constraint [9]). See Bhattacharya, Haslag and Martin (2005) and Eden (2012) for a detailed analysis. Note also that at the Friedman rule, money completely crowds out bonds.

[10] To get the intuition for (9) and see how the initial loan can be used to smooth consumption, consider an agent who wants to buy a unit at time $t = \tau$ for R^{Δ} units at time $t = \tau + \Delta$, where $R^{\Delta} = (R_t) \times (R_{t+1}) \times ... \times R_{t+\Delta-1}$ is the price of the unit at time $t = \tau$ in terms of units at time $t = \tau + \Delta$. To do this trade the agent can take a loan of $d_0 \geq 1$ units at $t = 0$ and deposit it in the checking account until $t = \tau$, using the interest on the balances in the checking account to pay the interest on the loan. Following this strategy he will have d_0 units at $t = \tau$. Then, at time $t = \tau$, he buys the unit and reduces his balances to $d_0 - 1$. By time $t = \tau + \Delta$ the balance in the checking account is $d_0 - R^{\Delta}$ units. At that time he sells R^{Δ} units of the good and deposit the revenues in the checking account achieving the initial balances of d_0 units. From that point he uses the interest on the balance in the checking account to pay the interest on the loan.

4. BAUMOL-TOBIN MEETS LAGOS-WRIGHT

The analysis in the previous section did not have enough structure to yield comparative static results: We could not compare equilibria with different R_m. To allow for such comparison and to gain some further insights, I adopt the two sub-periods structure in Lagos and Wright (2005) and some elements from Williamson (2012).

The population consists of two types of infinitely lived agents: buyers and sellers. There is a continuum of buyers with mass one and a continuum of sellers with mass one. Each period is divided into two sub-periods: In the first there is a centralized Walrasian market (CM) and in the second there is a decentralized market (DM) in which bilateral meetings take place. Buyers produce Y in the first sub-period (when the CM is active) and sellers produce X in the second sub-period (when the DM is active).

Both buyers and sellers maximize expected utility. The utility function of the typical buyer is:

$$\sum_{t=1}^{\infty} \beta^t \left(x_t^b - v(L_t^b) \right) \tag{11}$$

where (y_t^b, x_t^b) are the quantities of (Y, X) consumed by the buyer, L_t^b is the amount of labor supplied, $0 < \beta < 1$ is a discount factor and $v(L) = (\frac{1}{2})L^2$ is the utility cost of supplying labor. Using similar notation, the utility function of the seller is:

$$\sum_{t=1}^{\infty} \beta^t (y_t^s - L_t^s) \tag{12}$$

In the CM the buyer produces θ units of Y per unit of labor where the productivity parameter θ is a random variable with a continuous and differentiable distribution function $F(\theta)$. In the DM the seller produces 1 unit of X per unit of labor.

A planner's problem:

To describe the environment it may be useful to consider the following planner's problem.

(13) $\max_{y,x,l^s,l(\theta)\geq 0} y + x - l^s - \int v\left(l^b(\theta)\right)dF(\theta)$ s.t. $y = \int \theta l^b(\theta)dF(\theta)$ and $x = l^s$

Here (y,x) are the aggregate consumption of (Y,X), l^s is the aggregate supply of labor by sellers and $l^b(\theta)$ is the supply of labor by a buyer with productivity θ. Any amount supplied by sellers l^s solves (13). The first order conditions that determine the amount of labor supplied by buyers are:

(14) $$v'\left(l^b(\theta)\right) = \theta$$

Equilibrium:

As before the price of corn is 1 dollar per unit and this price does not change over time. There is a government bank with two types of accounts: checking and savings. (The case of private bonds is in the Appendix.) During the DM buyers and sellers can meet either at a bank or at a non-bank location. Balances in the savings account can be exchanged for goods only at a bank. Balances in the checking account can be exchanged for goods everywhere.[11]

Buyers sell their labor for money in the CM and then choose whether to deposit it in checking or savings. The government pays interest on balances held at the beginning of each sub-period. The interest on balances held at the beginning of the CM is $R' = 1/\beta$ and is the same for both accounts. The interest paid at the beginning of the DM may be different across accounts. The gross interest paid on balances held in the savings account (at the beginning of the DM) is R and the gross interest paid on balances held in the

[11] This is similar to the distinction between cash and credit goods in Lucas and Stokey (1987). We may think of X as a "cash good" that is subject to a cash in advance constraint and of Y as a "credit good" that is not subject to the constraint. At the bank you can buy X with savings because you can literally withdraw cash from savings and use it to pay the seller.

checking account is $R_m \leq R$. The case $R = 1$ makes more intuitive sense because in this case there are no (net) interest payments during the period. In a monetary model in which the price level changes over time, we can still have $R_m < 1$ due to inflation. I will therefore pay special attention to the case $R_m \leq R = 1$.

Note that the interest paid at the beginning of the CM must equal $1/\beta$ because otherwise, the sellers will adopt a corner solution: they will not supply X if $R' < 1/\beta$ and will supply an infinite amount otherwise. The interest at the beginning of the DM may vary because the marginal cost of supplying Y is increasing.

Balances deposited during the CM in the savings account earn a higher interest but using these balances in the following DM sub-period requires a trip to the bank that costs α units of X. The interpretation of the trip to the bank will be discussed later. For now we may think of it as type Q regulations imposed by the government: You cannot write checks on savings accounts.

At the beginning of the CM the government collects a lump sum tax of τ^s dollars from each seller and at the beginning of the DM it collects a lump sum tax of τ^b from each buyer.

The buyer spends his entire CM earnings in the DM. The period utility of a buyer with productivity θ is:

$V_m(\theta, R_m) = \max_L R_m \theta L - v(L)$

if he deposits his revenues in checking and

$V_b(\theta, R) = \max_L R\theta L - v(L) - \alpha$

if he uses savings. The buyer will choose savings if $V_b \geq V_m$.

Let $\rho(R_m, R) = \Pr ob\left(V_m(\theta, R_m) \geq V_b(\theta, R)\right)$ denotes the probability that a buyer will realize a productivity that will make him choose checking. Thus, in equilibrium a fraction $\rho(R_m, R)$ of the buyers will use checking and as will be showed later, checking users are buyers with relatively low θ.

Welfare:

To compare welfare across alternative policies (choice of (R_m, R)) let

$$L(w) = \arg\max_L wL - v(L) \tag{15}$$

be the labor supply of a buyer who faces the wage w. Thus, the labor supply of a money user is $L(R_m\theta)$ and the labor supply of a bond user is $L(R\theta)$. Let

$S(\theta, r) = \theta L(r\theta) - v\big(L(r\theta)\big)$

be the social surplus from the labor supplied by a buyer whose productivity is θ and his real wage is $r\theta$. A buyer who uses money contributes $S(\theta, R_m)$ to social welfare and a buyer who uses bonds contributes $S(\theta, R) - \alpha$ to social welfare, where social welfare is defined by:

$$W(R_m, R) = \int_0^{\rho(R_m,R)} S(\theta, R_m)\,dF(\theta) + \int_{\rho(R_m,R)}^1 \big(S(\theta, R) - \alpha\big)\,dF(\theta) \tag{16}$$

In what follows I assume $R = 1$ and therefore the contribution of a buyer with productivity θ to social welfare can be expressed in terms of the areas A, B and C in Figure 1. A bond user with productivity θ contributes to social welfare

$$A(\theta) + B(\theta) + C(\theta) - \alpha \tag{17}$$

units. A money user contributes:

$$A(\theta) + C(\theta) \tag{18}$$

From the social point of view a buyer should use bonds if:

$$A(\theta) + B(\theta) + C(\theta) - \alpha \geq A(\theta) + C(\theta) \tag{19}$$

From the private point of view the bond option is preferred if:

$$A(\theta) + B(\theta) + C(\theta) - \alpha \geq C(\theta) \tag{20}$$

There is thus a difference between the social and the private points of view. The private calculation does not take into account the area A and therefore there is an excessive use of bonds. Note that the area A is the seigniorage payment obtained by the government.

We can write the inequalities (19) and (20) as:

$$B(\theta) \geq \alpha \tag{21}$$

(22) $$A(\theta) + B(\theta) \geq \alpha$$

Thus from the social point of view a buyer should use bonds if (21) holds. From the private point of view he should use bonds if (22) holds.

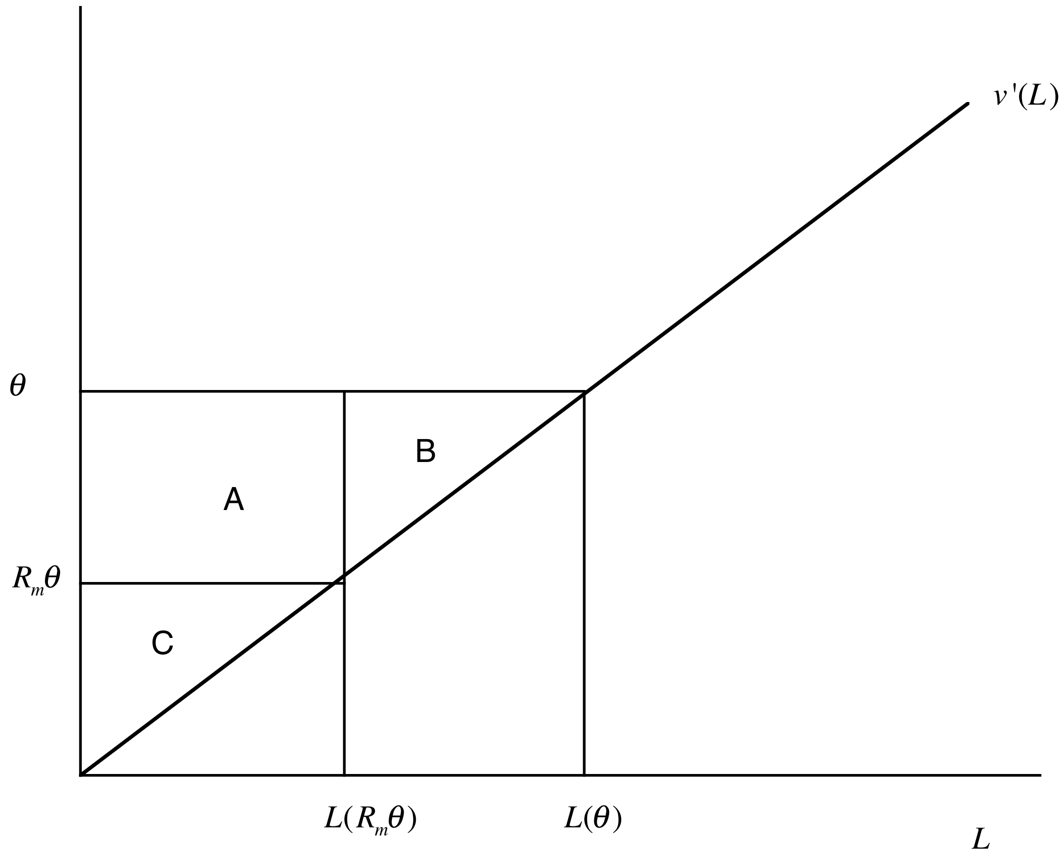


Figure 1

The buyer's choice as a function of R_m holding θ (and $R = 1$) constant:

The difference $\theta(1 - R_m)$ is increasing in θ and $v'(L) = L$, the area $A(\theta) + B(\theta)$ in Figure 1 is increasing in θ. It follows that the buyer will use money when θ is small. I consider the case in which θ is sufficiently large and there is a range of R_m in which the buyer will use bonds.

Since the area $A(\theta)+B(\theta)$ decreases with R_m and goes to zero when $R_m \to 1$, (22) is not satisfied when R_m is close to 1. In the intermediate range in which (22) is satisfied and (21) is not, the buyer will use bonds but his choice is not efficient from the social point of view. When we reduce R_m further so that (21) is satisfied, the buyer will use bonds and his choice is efficient. Figure 2 illustrates.

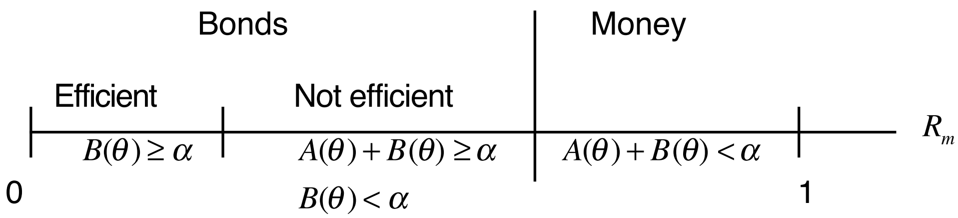


Figure 2: The buyer's choice as a function of R_m holding θ constant at a sufficiently high level (R_m increases from left to right)

I now turn to show the following Claim.

Claim 2: When $R = 1$, an increase in R_m improves welfare.

This is the standard result about the relationship between inflation and welfare in the steady state.

To show this claim I consider a small increase in R_m. When R_m increases some bond users will not change their behavior and some will switch to money. The buyers who switch to money are initially close to being indifferent between money and bonds and for them we may use the approximation that (22) holds with equality. Initially, the contribution of the bond users who are indifferent between the two options to social welfare is $C(\theta)$. (To see this, substitute $\alpha = A(\theta)+B(\theta)$ in (17)). After switching to money their contribution goes up to $A(\theta)+C(\theta)$. It follows that the switchers increase social welfare by the area $A(\theta)$. Money users will increase labor and increase their contribution to social welfare (because the area A+B increases with R_m). It follows that the contribution of all buyers to social welfare either strictly increase or stay the same and therefore welfare goes up when R_m increases.

Maximum welfare is attained at the Friedman rule when $R_m = R = 1$. (This is a special form of the Friedman rule because there is no discounting between sub-periods). At the optimal policy, buyers will choose to invest in checking only and will produce the optimal amount. Thus, at the optimal policy money completely crowds out bonds.

The buyer's choice as a function of θ holding R_m constant:

Since the left hand side of (22) increases with θ, there exist a cut-off point $\theta^{sw}(R_m)$ for which buyers with $\theta \geq \theta^{sw}$ will use bonds and buyers with $\theta < \theta^{sw}$ will use money. The superscript sw is for "switchers" because buyers with productivity θ^{sw} will switch to money when R_m increases. Since , $B(\theta)$ is increasing in θ only the choice of bond users with high θ is justified from the social point of view. Figure 3 illustrates.

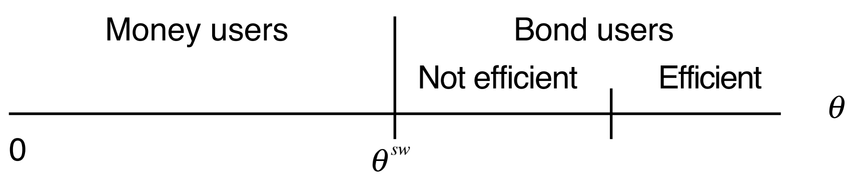


Figure 3: The buyer's choice as a function of θ holding $R_m < R = 1$ constant (θ increases from left to right)

Figure 4 combines Figures 2 and 3. Buyers who face the parameter pair (R_m, θ) in areas C+ F+H+I use money. Buyers who face the parameter pair in the area B+E+G use bonds but their use of bonds is not efficient from the social point of view. Buyers who face the parameter pair in the area A+D use bonds and their choice is efficient from the social point of view.

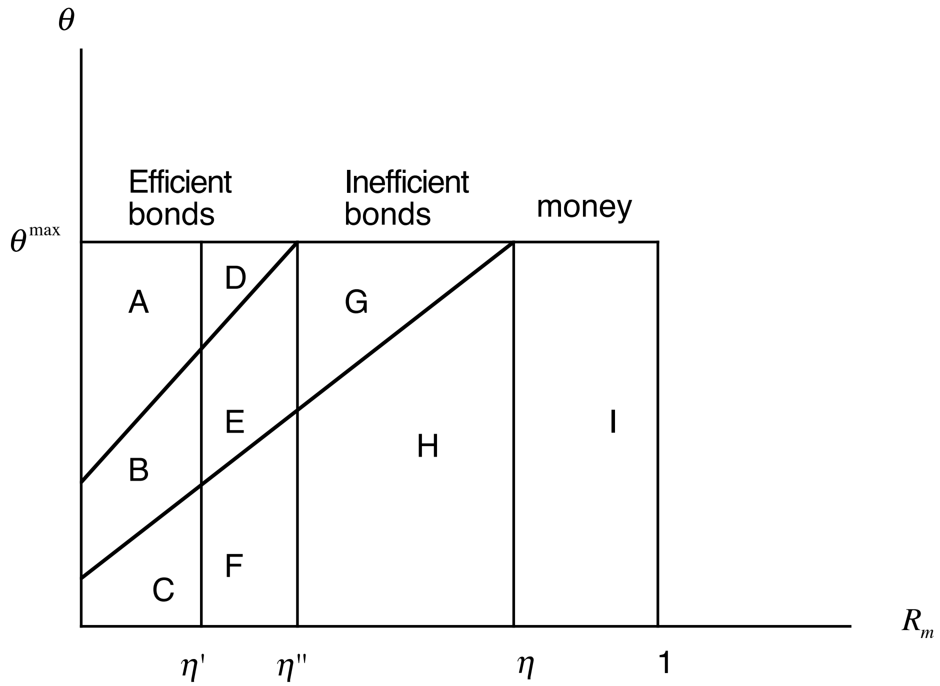


Figure 4: The buyer's choice as a function of the parameter pair (θ, R_m). He will choose money if the pair is in the area CFHI. His choice of bonds is not efficient in the area BEG and efficient in the area AD.

The effect of Simons' type regulations:

Simons (1948) supported regulations aimed at discouraging money substitutes. Here bonds are the only money substitute and I therefore consider the effect of regulations that effectively prohibit the use of bonds. (In our setup this can be accomplished simply by not offering savings accounts, but in the case of private bonds this requires some additional regulations).

The effect of the regulation depends on R_m. Using Figure 4, the regulation will have no effect when $\eta \leq R_m \leq 1$ because in this range no one use bonds. Welfare goes up as a result of the regulation when $\eta'' \leq R_m \leq \eta$ because in this range all bonds users increase social welfare when they switch to money. Welfare also goes up if we reduce R_m slightly below η'' because the welfare gain from switching bond users in E is larger than the welfare loss from switching bond users in D. Figure 5 describes the welfare gain from imposing the regulation as a function of R_m.

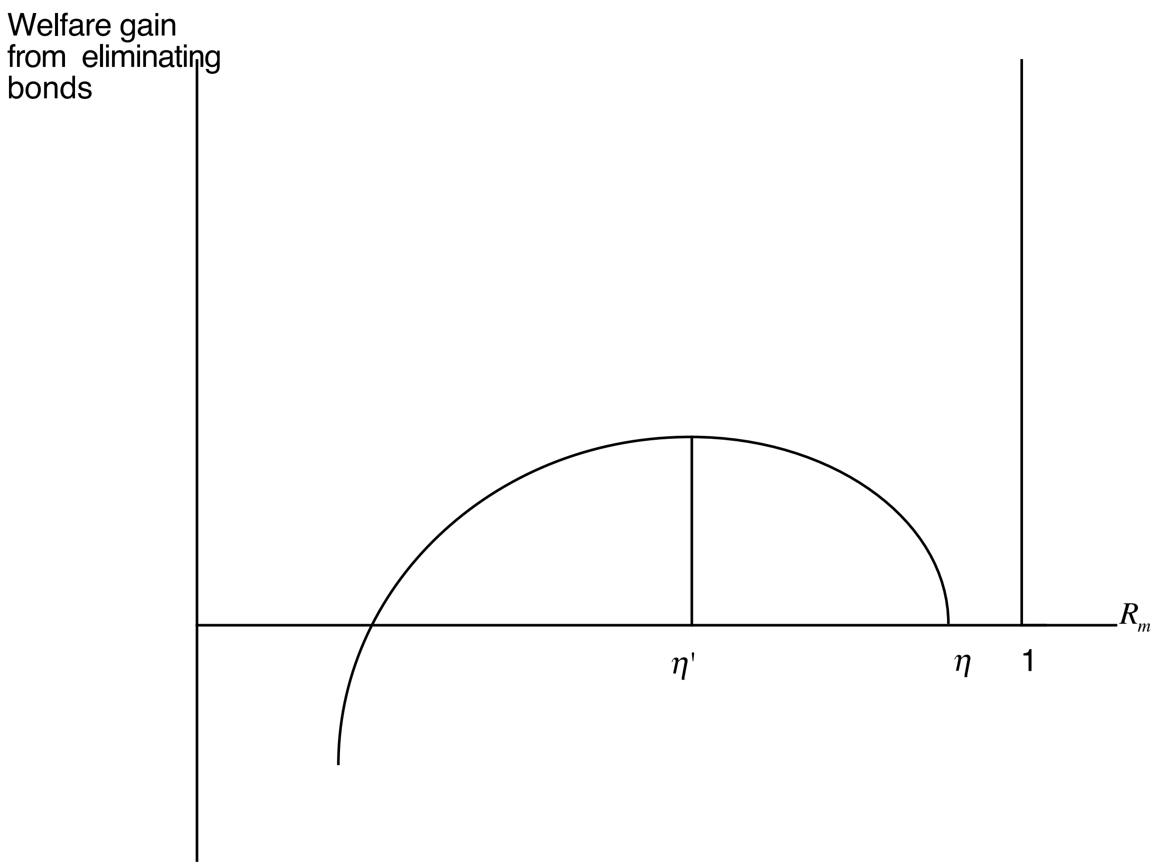
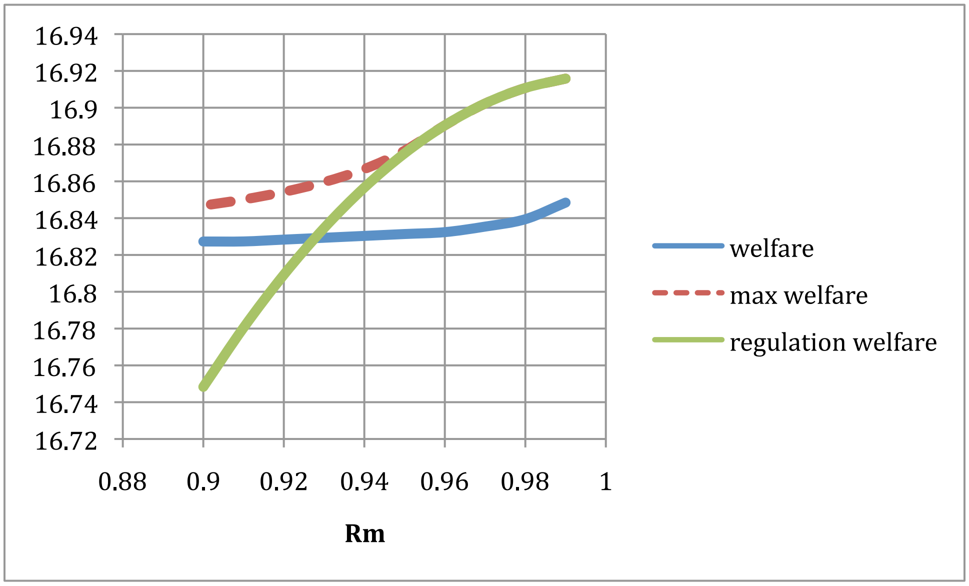


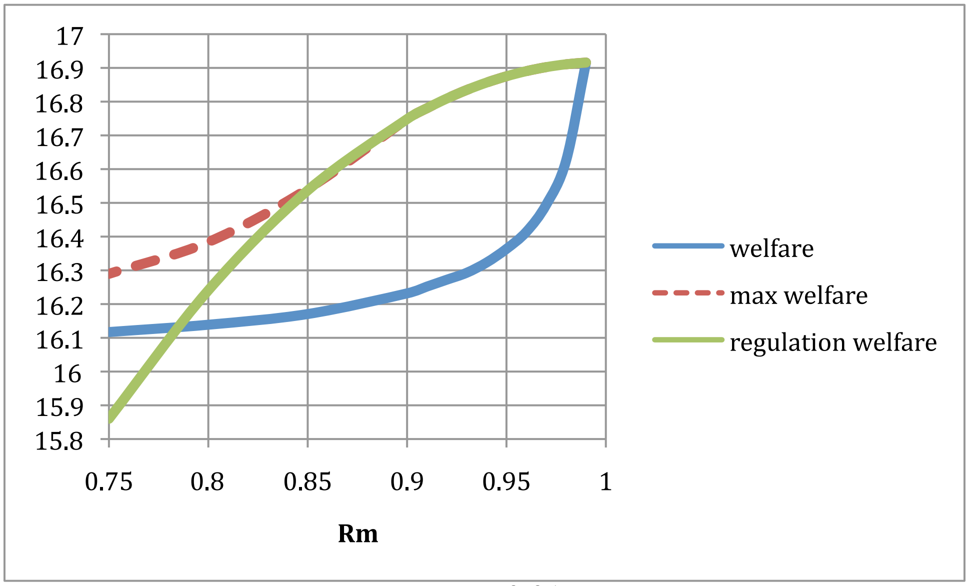
Figure 5: Welfare gain from eliminating bonds as a function of the real rate of return on money ($R_m \le R = 1$).

Numerical examples:

The numerical examples in Figure 6 assume that θ is uniformly distributed in the range $0.01 \le \theta \le 1$. I use two values for α: $\alpha = 0.001$ and $\alpha = 0.01$. There are three measures of welfare: The curve labeled "welfare" is the social welfare $W(R_m, 1)$, when buyers make the decision whether to use bonds or money. The curve labeled "max welfare" is the social welfare when a planner makes the decision and the curve labeled "regulation welfare" is social welfare when bonds are prohibited. These three measures of welfare are plotted as a function of R_m assuming $R = 1$. When R_m is sufficiently large, the "max welfare" curve coincides with the "regulation welfare" curve. In this range the choice of the planner is not to use bonds for all θ in the above range. Not surprisingly the "max welfare" curve is above the equilibrium outcome labeled as "welfare". The "regulation welfare" is above the "welfare" curve when R_m is large and below the "welfare" curve when R_m is low. The cutoff point depends on α: It is about 0.93 when $\alpha = 0.001$ and 0.78 when $\alpha = 0.01$. Note that the point in which the curve "max welfare" splits from the curve "regulation welfare" is the point in which the planner starts using bonds (for high θ). The split occurs at around $R_m = 0.95$ when $\alpha = 0.001$ and at $R_m = 0.85$ when $\alpha = 0.01$. Thus the planner starts using bonds at a relatively high rate of return on money when the fixed cost is relatively low.



A. $\alpha = 0.001$



B. $\alpha = 0.01$

Figure 6: Welfare measures as a function of R_m

5. SEIGNIORAGE

Some governments have limited ability to collect explicit taxes and are forced to use inflation tax. From the point of view of these governments, bond users are tax evaders: They pay a fixed cost for avoiding the tax on holding money. We may thus think of bonds as a loophole in the tax system. Does this loophole improve matters?

In an attempt to answer this question I consider the problem of maximizing welfare under the constraint that the seigniorage tax revenues must be greater than k, where k is a constant that is not too large and does not exceed the maximum seigniorage possible. I consider this problem under two regimes: The Friedman regime that allows the use of bonds and the Simons regime that does not allow the use of bonds.

Under the Friedman regime, the problem of the policy maker is:

(23)

$$H(k) = \max_{R_m} W(R_m,1) = \int_0^{\rho(R_m,1)} \left(\theta(R_m\theta) - (1/2)(R_m\theta)^2\right) dF(\theta) + \int_{\rho(R_m,1)}^1 \left((1/2)\theta^2 - \alpha\right) dF(\theta)$$

s.t.

$$(24) \qquad SE(R_m) = \int_0^{\rho(R_m,1)} \theta(1-R_m)L(R_m\theta)dF(\theta) \geq k$$

The objective function (23) is (16) under the quadratic cost function and $R=1$. Note that under these assumptions, labor supply for money users is $L(R_m\theta) = R_m\theta$ and labor supply for bond users is $L(\theta) = \theta$. The constraint says that the seigniorage tax revenues, $SE(R_m)$, are greater than k. Note that seigniorage revenues are defined by the amount of interest payment that the government "saves" relative to the bond alternative.

Under the Simons regime, the problem of the policy maker is:

$$(25) \qquad h(k) = \max_{R_m} \overline{W}(R_m) = \int_0^1 \left(\theta(R_m\theta) - (½)(R_m\theta)^2\right) dF(\theta)$$

s.t.

$$(26) \qquad se(R_m) = \int_0^1 \theta(1-R_m)L(R_m\theta)dF(\theta) \geq k$$

I now turn to a special case that allows for the comparison of welfare across the two regimes.

<u>An Example</u>:

I consider a special case in which θ has two possible realizations: half of the buyers' population gets the realization θ_1 and the remaining half get the realization $\theta_2 < \theta_1$.

In the Friedman regime we can have a solution in which all buyers use money. In this case, welfare is the same across the two regimes. I consider now the case in which the high productivity buyers use bonds; the gross rate of return on money is $\hat{R}_m$; the supply of labor is $\hat{L}_1 = \theta_1$ for the high productivity buyers and $\hat{L}_2 = \theta_2\hat{R}_m$ for the low productivity buyers. And tax revenues are $\hat{\Omega}$, where:

$$(27) \qquad 2\hat{\Omega} = \hat{L}_2\theta_2(1-\hat{R}_m) = \hat{R}_m(1-\hat{R}_m)(\theta_2)^2$$

In the Simons regime the gross rate of return on money are $\overline{R}_m$ and the labor supplies are: $\overline{L}_1 = \theta_1\overline{R}_m$ and $\overline{L}_2 = \theta_2\overline{R}_m$. Tax revenues in the Simons regime are $\overline{\Omega}$ where:

(28) $$2\overline{\Omega} = \theta_1 \overline{L}_1 (1 - \overline{R}_m) + \theta_2 \overline{L}_2 (1 - \overline{R}_m) = \overline{R}_m (1 - \overline{R}_m)\left((\theta_1)^2 + (\theta_2)^2\right)$$

At the solutions to the problems (23) and (25) the constraints are binding and revenues are the same in both cases. Thus,

(29) $$\hat{R}_m (1 - \hat{R}_m)(\theta_2)^2 = \overline{R}_m (1 - \overline{R}_m)\left((\theta_1)^2 + (\theta_2)^2\right) = (½)k$$

There are 2 solutions for each of the quadratic equation in (29). We pick the high solution that maximizes welfare. Figure 7 illustrates. As can be seen the rate of return on money under the Simons regime is higher than the rate of return under the Friedman regime. This is intuitive.

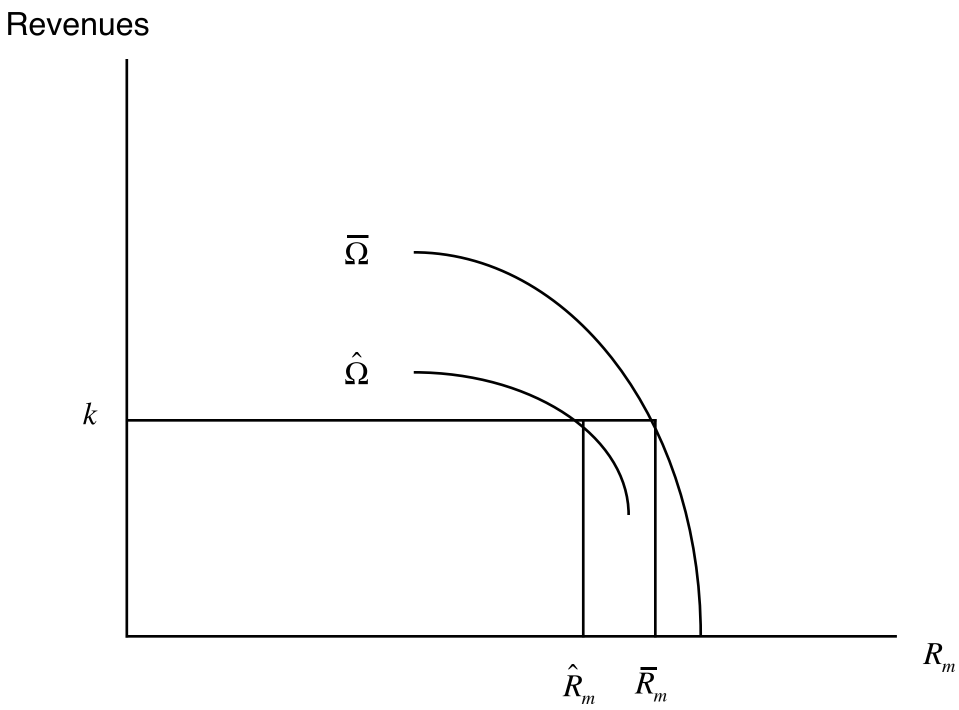


Figure 7

I now show the following claim[12].

[12] The Claim can be generalized to the case in which the fraction of agents with the high productivity is different from half. The proof should go through as long as the two groups of buyers can each be represented by a single agent. Adding concavity to the utility from consumption will only strengthen the case for Simons' type regulations.

Claim 3: $h(k) \geq H(k)$ for all feasible k.

Proof: I show that by imposing Simons' regulations it is possible to improve on the optimal policy under the Friedman regime. Starting from the Friedman regime with $R_m = \hat{R}_m$, I impose the regulations and increase R_m from $\hat{R}_m$ to $\bar{R}_m > \hat{R}_m$, where here $\bar{R}_m$ is a variable rather than the solution to (29). As a result of the increase in R_m, the real wage of the high productivity buyers drops by $\theta_1(1-\bar{R}_m)$ and the real wage of the low productivity buyers increases by $\theta_2(\bar{R}_m - \hat{R}_m)$. I choose $\bar{R}_m$ so that the absolute value of the change in real wage is the same for both groups. Thus,

(30) $$\theta_2(\bar{R}_m - \hat{R}_m) = \theta_1(1-\bar{R}_m) \text{ and } \hat{R}_m < \bar{R}_m = \frac{\theta_1 + \theta_2\hat{R}_m}{\theta_2 + \theta_1} < 1$$

The per buyer contribution of the high productivity buyers to social welfare went down from $(½)(\theta_1)^2 - \alpha$ to $(\theta_1)(\theta_1\bar{R}_m) - (½)(\theta_1\bar{R}_m)^2$. In terms of the areas in Figure 8 the drop in the per-buyer contribution is $g - \alpha$.

The per buyer contribution of the low productivity buyers went up from $(\theta_2)(\theta_2\hat{R}_m) - (½)(\theta_2\hat{R}_m)^2$ to $(\theta_2)(\theta_2\bar{R}_m) - (½)(\theta_2\bar{R}_m)^2$. This is the area m in Figure 8. Since by construction: $m = g$, it follows that welfare increases by $(½)\alpha$ as a result of imposing the regulations.

I now turn to examine the change in revenues. Under the Friedman regime the revenues in terms of the areas in Figure 8 were $(½)(h+k)$. After imposing the regulations the revenues are: $(½)(h+i)+(½)(d+e+f)$. Since by construction $k = d$, it follows that as a result of the regulation revenues went up by: $(½)(i+e+f)$.

Thus, by imposing the regulation we can improve welfare and increase revenues. Therefore the optimal regulation improves welfare without violating the government's budget constraint. □

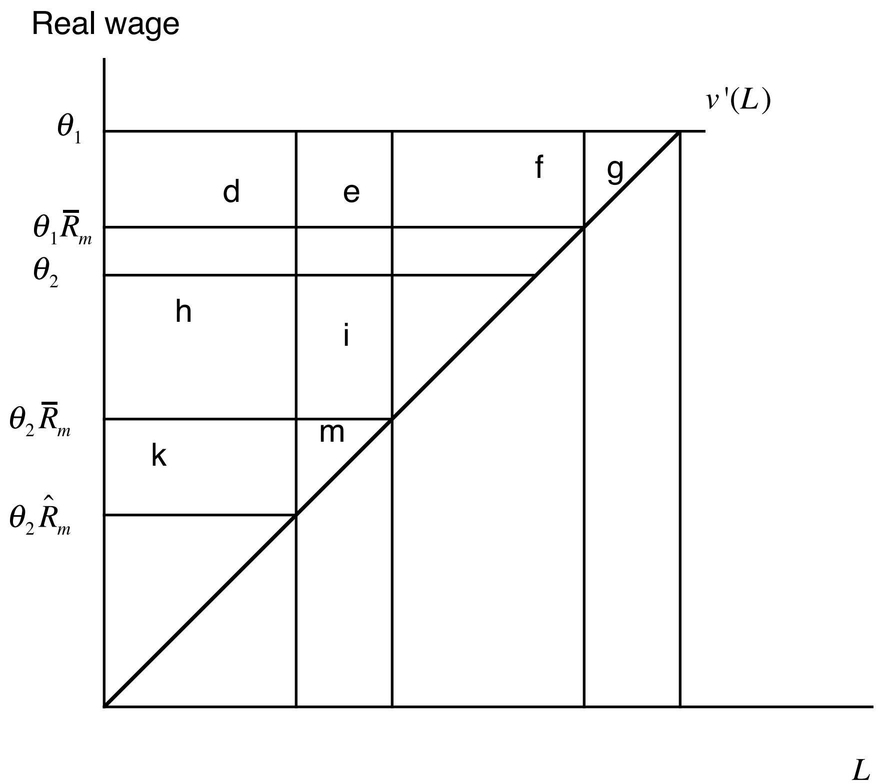
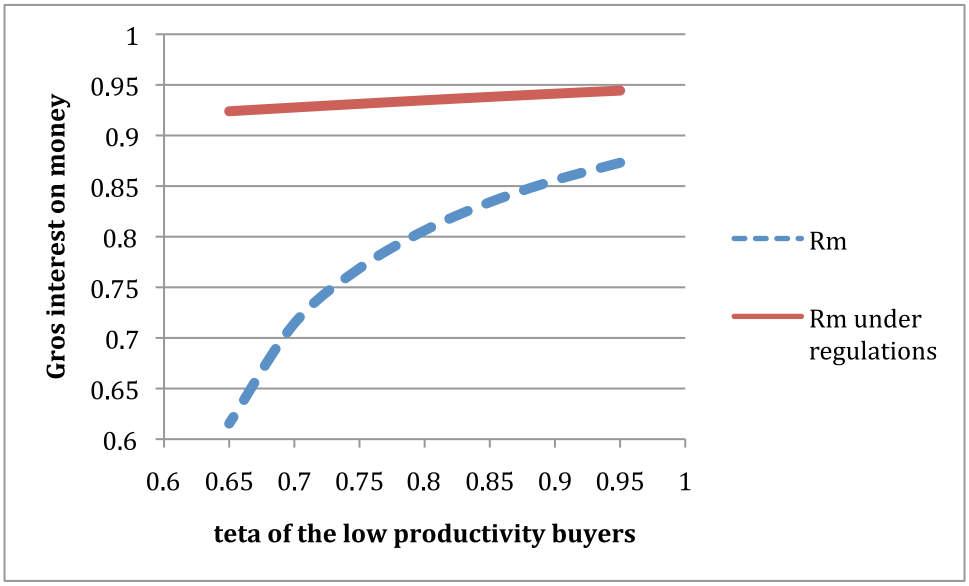


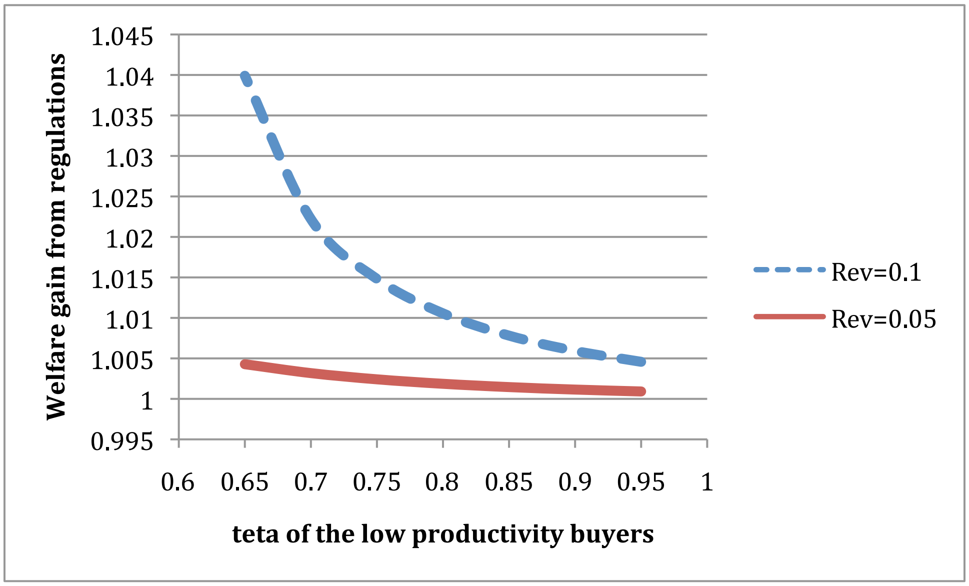
Figure 8

Numerical examples:

Figure 9 computes the welfare gains from regulations for two different levels of revenues, holding $\theta_1 = 1$ constant and varying θ_2. Figure 9A computes the interest rate on money when holding revenues constant at the level of 0.1 and varying θ_2. The upper curve is the gross interest under regulations and the lower curve is the gross interest in the absence of regulations. Figure 9B computes the ratio of welfare under regulations to welfare in the absence of regulations. This is done for two levels of revenues: 0.1 and 0.05. As can be seen the gain in welfare from regulations is higher when the level of tax revenues is higher.



A. Gross interest on money with and without regulations, holding revenues constant



B. The ratio of welfare under regulations to welfare without regulations for two levels of revenues

Figure 9: The effect of regulations on the interest rate and on welfare holding $\theta_1 = 1$ constant and varying θ_2

6. CONCLUDING REMARKS

Money crowds out bonds and at the optimal policy the crowding out is complete. This is not surprising. What may be somewhat of a surprise is that when using bonds requires real resources, as in the Baumol-Tobin model, regulations that prohibit the use of bonds may improve matters.

Regulations may improve matters because of the difference between the social and the individual point of view: When agents choose bonds they do not take into account the loss of seigniorage revenues implied by their choice. When the rate of return on money is exogenous and the government is indifferent to the amount of seigniorage revenues raised, regulations increase welfare when the rate of return on money is relatively high and the inflation tax is relatively low. When the government cares about seigniorage revenues and operates under the constraint that seigniorage revenues must be higher than a given amount, regulations tend to improve welfare regardless of the rate of return on money that is now an endogenous variable.

REFERENCES

Baumol, W.J., 1952: "The Transaction Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics, 66, 545–56.

Bhattacharya Joydeep, Joseph H. Haslag, and Antoine Matin., "Heterogeneity, Redistribution, And the Friedman Rule" International Economic Review, Vol. 46, No. 2, May 2005.

Benes, Jaromir and Michael Kumhof., 2012: "**The Chicago Plan Revisited**," IMF Working Papers 12/202, International Monetary Fund.

Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe, 1996: "Optimality of the Friedman Rule in Economies With Distorting Taxes," Journal of Monetary Economics, 37 (2), 203–23.

Correia, Isabel and Teles Pedro, 1996: "Is the Friedman Rule Optimal When Money Is an Intermediate Good?," Journal of Monetary Economics, 38, 223–44.

da Costa, Carlos E. and Ivan Werning, 2008: "On the Optimality of the Friedman Rule with Heterogeneous Agents and Non-Linear Income Taxation" The Journal of Political Economy, Vol. 116, No. 1, 82-112.

Diamond, Douglas W & Dybvig, Philip H, 1983. "**Bank Runs, Deposit Insurance, and Liquidity**," Journal of Political Economy, University of Chicago Press, vol. 91(3), pages 401-19, June.

Eden, Benjamin, 2012, "Costly Intermediation and the Friedman rule", *Vanderbilt University Department of Economics Working Papers*, VUECON-12-00003.

Friedman, Milton., 1960, Capitalism and Freedom, Fortieth Anniversary Edition, The University of Chicago Press.

——, 1969: "The Optimum Quantity of Money." In The Optimum Quantity of Money and Other Essays. Chicago: Aldine, pp. 1–50.

____, 1977: "Time Perspective in Demand for Money" The Scandinavian Journal of Economics, Vol. 79, No. 4, pp. 397-416.

____ and Anna J. Schwartz, 1986: "Has government any role in money?" Journal of Monetary Economics, Volume 17, Issue 1, January 1986, Pages 37-62

Gorton, Gary B. and George Pennacchi, 1990, "Financial intermediaries and liquidity creation". Journal of Finance 45, 49-72.

Gorton, Gary & Andrew Metrick, 2010. "**Regulating the Shadow Banking System**," Brookings Papers on Economic Activity, Economic Studies Program, The Brookings Institution, vol. 41(2 (Fall)), pages 261-312.

Hayek F. 1976, Denationalization of Money (Institute of Economic Affairs, London). 1978, second extended edition.

Jovanovic, Boyan., 1982: "Inflation and Welfare in the Steady State" Journal of Political Economy, Vol. 90, No. 3 (Jun., 1982), pp. 561-577.

Kimbrough Kent, P., 1986: "The Optimum Quantity of Money Rule in the Theory of Public Finance," Journal of Monetary Economics, 18, 277– 84.

Knapp, G.F. (1973 [1924]), The State Theory of Money, Clifton, NY: Augustus M. Kelley.

Lerner, A.P., 1947: "Money as a Creature of the State" The American Economic Review, 37, May: 312-317.

Lucas, Robert E., 2013: "**Glass-Steagall: A Requiem**," American Economic Review, American Economic Association, vol. 103(3), pages 43-47, May.

______ and Nancy L. Stokey, 2011: "**Liquidity crises**," Economic Policy Paper 11-3, Federal Reserve Bank of Minneapolis.

________ 1987: "**Money and Interest in a Cash-in-Advance Economy**," Econometrica, Econometric Society, vol. 55(3), pages 491-513, May.

Patinkin, Don., 1965: Money, Interest and Prices, Harper and Row, New York. (Second Edition).

Sargent, Thomas J. and Wallace, Neil, 1982: "The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration," Journal of Political Economy, 90 (6), 1212–36.

Simons, H., 1948: Economic Policy for a Free Society, Chicago: University of Chicago Press.

Stein Jeremy C., 2012: "Monetary Policy as Financial Stability Regulation" The Quarterly Journal of Economics, Vol. 127, issue 1, pp. 57-95.

Tobin, James, 1956: "The Interest Elasticity of the Transactions Demand for Cash," Review of Economics and Statistics, 38 (3), 241–7.

Williamson, Stephen D., 2012: "**Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach**," American Economic Review, American Economic Association, vol. 102(6), pages 2570-2605, October.

APPENDIX A: A MONETARY VERSION WITH PRIVATE LENDING AND BORROWING

In the paper the alternative asset (money substitute) was government bond. Here I consider the case in which the alternative asset is private bond. I also assume that money does not earn explicit interest and allow the price level to vary over time.

At the CM all agents are in the same geographical location and buyers can sell Y for money or for a contract. At the DM agents are distributed over many locations in a random manner. Money is generally accepted and buyers can always find a seller in their DM location that will sell them X for money. But contracts are between a specific buyer and a specific seller. During the DM, the buyer who holds a contract on the output of a specific seller is likely to be in a different location than the specific seller. Therefore, there is a delivery cost that covers the transportation of the good from the seller's location to the buyer's location.

I assume that the buyer pays the delivery cost and the typical contract offers to deliver $Ry-\alpha$ units of X to the buyer's location at the DM for y units of Y received at the CM. Thus the seller borrows from the buyer at the gross real interest R and the buyer pays the delivery fee that covers on average the cost of transportation.

At the beginning of the period t CM the seller holds M_t dollars. The seller gets a lump sum transfer of μM_t at the beginning of period $t+1$ and the (post transfer) money supply at $t+1$ is: $M_{t+1} = M_t(1+\mu)$.

The dollar price of the two goods at time t is proportional to the beginning of period (post transfer) money supply and is given by:

(A1) $$P_{xt} = p_x M_t \text{ and } P_{yt} = p_y M_t$$

The proportionality constants (p_y, p_x) are called normalized prices and unlike dollar prices they do not change over time.

The problem of a buyer with productivity θ who uses money is:

(A2) $$\max_{x,L} x - v(L) \text{ s.t. } p_x x = p_y \theta L$$

We can write the problem (A2) as:

(A3) $$V_m(\theta, R_m) = \max_L R_m \theta L - v(L)$$

where $R_m = \frac{p_y}{p_x}$ is the real rate of return on money. The real wage for a money user is thus θR_m and his labor supply is: $L(\theta R_m)$.

The problem of a buyer who sells for a contract is:

(A4) $$V_b(\theta, R) = \max_L R\theta L - v(L) - \alpha$$

As before the buyer will use a contract if $V_b(\theta, R) \geq V_m(\theta, R)$ and a fraction $1 - \rho(R_m, R)$ of the buyer population will sell for contracts.

The money market clearing condition at the CM is:

(A5) $$1 = p_y \int_0^{\rho(R_m, R)} \theta L(\theta R_m)$$

On the left hand side of (A5) is the amount of money held by the sellers (in terms of normalized dollars). On the right hand side of (A5) is the nominal supply of money users (again in normalized dollars).

In the CM, the seller can sell X for current Y by contract or he can sell X for money in the DM. If he uses a contract he will get $1/R$ units of current Y per unit of X. If he sells his output for money it is $\beta R'$ where

(A6) $$R' = \frac{P_{xt}}{P_{yt+1}} = \frac{p_x M_t}{p_y M_{t+1}} = \frac{p_x M_t}{p_y M_t (1+\mu)} = \frac{p_x}{p_y (1+\mu)} = \frac{1}{R_m (1+\mu)}$$

In equilibrium the seller must be indifferent between borrowing and lending (or between selling on a contract to selling for money) and between producing and not producing. We therefore have the following arbitrage condition:

(A7) $$1/R = \beta R' = 1$$

A steady state equilibrium is thus a vector (R_m, R, R', p_x, p_y) such that

(A8) $$R_m = \frac{p_y}{p_x}$$

(A9) $$R' = \frac{1}{R_m(1+\mu)}$$

(A10) $$1 = p_y \int_0^{\rho(R_m,R)} \theta L(\theta R_m)$$

(A11) $$1/_R = \beta R' = 1$$

Solving for the steady state equilibrium:

Condition (A11) leads to: $R' = 1/_\beta$ and $R = 1$. Substituting $R' = 1/_\beta$ in (A9) leads to:

(A12) $$R_m = \frac{\beta}{1+\mu}$$

Substituting $R = 1$ in (A10) yields:

(A13) $$p_y = \left(\int_0^{\rho(R_m,1)} \theta L(\theta R_m)\right)^{-1} = \left(\int_0^{\rho(\frac{\beta}{1+\mu},1)} \theta L\left(\frac{\beta\theta}{1+\mu}\right)\right)^{-1}$$

Substituting (A12) and (A13) in (A8) leads to:

(A14) $$p_x = \beta^{-1}(1+\mu)\left(\int_0^{\rho(\frac{\beta}{1+\mu},1)} \theta L\left(\frac{\beta\theta}{1+\mu}\right)\right)^{-1}$$

Welfare analysis is the same as in the previous section. The optimal policy can be obtained when $1+\mu = \beta$ which leads to: $R_m = 1$.

APPENDIX B: THE DERIVATION OF THE BUDGET CONSTRAINT (9)

We start with the asset accumulation equation (8). Omitting the superscript (8) can be written as:

(B1) $m_1 = \bar{Y}_1 + R_1 m_0 + g_1 - Y_1 - r_1 d_0$

Using $m_2 = \bar{Y}_2 + R_2 m_1 + g_2 - Y_2 - r_2 d_0$ to get:

(B2) $m_1 = \left(m_2 - \bar{Y}_2 - g_2 + Y_2 + r_2 d_0\right) / R_2$

Substituting (B2) in (B1) yields:

(B3) $m_0 = \left(m_2 - \bar{Y}_2 - g_2 + Y_2 + r_2 d_0\right) / R_1 R_2 - \left(\bar{Y}_1 + g_1 - Y_1 - r_1 d_0\right) / R_1$

Using $m_3 = \bar{Y}_3 + R_3 m_2 + g_3 - Y_3 - r_3 d_0$ to get:

(B4) $m_2 = \left(m_3 - \bar{Y}_3 - g_3 + Y_3 + r_3 d_0\right) / R_3$

Substituting (B4) in (B3) yields:

(B5) $m_0 = \left(m_3 - \bar{Y}_3 - g_3 + Y_3 + r_3 d_0\right) / R_3 R_2 R_1$

$-\left(\bar{Y}_2 + g_2 - Y_2 - r_2 d_0\right) / R_2 R_1 - \left(\bar{Y}_1 + g_1 - Y_1 - r_1 d_0\right) / R_1$

Repeating this leads to:

(B6) $m_0 = \sum_{t=1}^{T} D_t \left(Y_t + r_t d_0 - \bar{Y}_t - g_t\right) + D_{T+1}\left(m_{T+1} - \bar{Y}_{T+1} - g_{T+1} + Y_{T+1} + r_{T+1} d_0\right)$

where $D_t = (R_1)^{-1} \times (R_2)^{-1} \times \bullet\bullet\bullet \times (R_t)^{-1}$.

When $T \to \infty$ we can write (B6) as:

(B7) $m_0 = \sum_{t=1}^{\infty} D_t \left(Y_t + r_t d_0 - \bar{Y}_t - g_t\right)$

Since $m_0 = m_{-1} + d_0$; $d_0 = \sum_{t=1}^{\infty} D_t r_t d_0$ and $m_{-1} = -\sum_{t=1}^{\infty} D_t g_t$, we can write (B7) as:

(B8) $\sum_{t=1}^{\infty} D_t \left(Y_t^h - \bar{Y}_t^h\right) = 0$

This is the same as (9).