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Appendix and Supplemental material not intended for publication-Round 2

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Supplementary proofs

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Supplementary Appendix

Proof of Proposition 3 and 4: First, I explicitly define each function I give in the sketch of the proof. First, the advisor's expected life-time payoff from reporting r given the strategy η and signal received is 1, under no commitment case, in general, is given by

$$E(r,\eta) = 1 - \sum_{\omega} \Pr(\omega|1)(\Gamma(r,\eta) - \omega)^2 + \mathbb{I}_{r,\omega}\delta v(\lambda_2).$$

The additional payoff from truthfully reporting signal 1, i.e., $\eta = 1$ under $\boldsymbol{\phi}$ is

$$\Upsilon(\boldsymbol{\phi}) = \delta[\phi_1 \gamma v(\Lambda_1) + \phi_0(1-\gamma)v(\Lambda_0)].$$

Here, the decision maker needs to form a belief about the advisor's type when she receives report 1, depending on the realized state. Let Λ_0 and Λ_1 be such beliefs given the realized state 0 and 1, respectively. It is clear that $\Lambda_1 > \Lambda_0$.

Finally, the decision maker's life-time expected payoff given the retention rule $\boldsymbol{\phi}$ and the advisor's strategy η is

$$\begin{aligned}
\Pi(\boldsymbol{\phi},\eta) = &\Pr(1,1)\left(-\left(\Gamma(1,\eta)-1\right)^2 + \phi_1 \cdot V(\Lambda_1) + (1-\phi_1)\cdot V(\lambda_1)\right) \\
&+ \Pr(1,0)\left(-\Gamma(1,\eta)^2 + \phi_0 \cdot V(\Lambda_0) + (1-\phi_0)\cdot V(\lambda_1)\right) \\
&+ \Pr(0,1)\left(-\left(\Gamma(0,\eta)-1\right)^2 + V(1)\right) \\
&+ \Pr(0,0)\left(-\Gamma(0,\eta)^2 + V(1)\right),
\end{aligned}$$

It is straightforward to show that, for any η, $\Pi(\boldsymbol{\phi},\eta)$ is a decreasing function in $\boldsymbol{\phi}$.

To characterize the set Φ_R, the condition (1) with specific Υ is

$$\Phi_R \equiv \{\boldsymbol{\phi} : E(0,1) \leq E(1,1) + \delta[\phi_1\gamma v(\Lambda_1) + \phi_0(1-\gamma)v(\Lambda_0)]\}.$$

The right hand side of the condition is clearly increasing in ϕ_0 and ϕ_1, so Φ_R is the upper contour set of the negatively sloped line on the (ϕ_0, ϕ_1) plane, which is give by the equality of the condition, as shown in the left panel of Figure 1. It is important to note that $E(0,1)$ depends on δ while $E(1,1)$ does not.

To characterize the set Φ_{PR}, since $\Pi(\phi,\eta)$ is decreasing in both ϕ_0 and ϕ_1, Φ_{PR} is the lower contour set of the negatively sloped line on the (ϕ_0,ϕ_1) plane, which is given by the equality of the condition $\Pi(\mathbf{0},0) = \Pi(\boldsymbol{\phi},1)$, as shown in the right panel of Figure 1. I call the line representing the equality of these respective conditions the *boundary.*

Computing with the functions defined above, the slope of the boundary of Φ_R is $-\frac{(1-\gamma)v(\Lambda_0)}{\gamma v(\Lambda_1)}$, and the slope of the boundary of Φ_{PR} is $-\frac{(1-\lambda_1\gamma)[V(\lambda_1)-V(\Lambda_0]}{(1-\lambda_1+\lambda_1\gamma)[V(\lambda_1)-V(\Lambda_1]}$. Rearranging terms shows that the boundary of Φ_{PR} is steeper than that of Φ_R if and only if

$$\begin{aligned}
&(1-\gamma)(1-\lambda_1+\lambda_1\gamma)v(\Lambda_0)[V(\lambda_1)-V(\Lambda_1)] \\
&< \gamma(1-\lambda_1\gamma)v(\Lambda_1)[V(\lambda_1)-V(\Lambda_0)]
\end{aligned}$$

But since $\Lambda_1 > \Lambda_0$ and $(1-\gamma)(1-\lambda_1+\lambda_1\gamma) < \gamma(1-\lambda_1\gamma)$ for $\gamma > \frac{1}{2}$, the right hand side is larger. This proves Lemma 2 in the sketch of the proof.

To prove Proposition 3, realizing that $\Pi(\boldsymbol{\phi}, \cdot)$ is decreasing both in ϕ_0 and ϕ_1, the optimal retention rule must be on the boundary of Φ_R. Using this result, I will show that $\Pi(\boldsymbol{\phi}, \cdot)$ is decreasing in ϕ_0 on the boundary, which implies that the decision maker is better off by reducing ϕ_0 while increasing ϕ_1 along the boundary. This implies the statement of the proposition. If I evaluate $\Pi(\boldsymbol{\phi}, \eta)$ at $\eta = 1$,

$$\begin{aligned}\Pi(\boldsymbol{\phi}, 1) = &- \frac{1}{2}\phi_1(1-\lambda_1+\lambda_1\gamma)[V(\lambda_1) - V(\Lambda(1,1)] \\ &- \frac{1}{2}\phi_0(1-\lambda_1\gamma)[V(\lambda_1) - V(\Lambda(1,0)] \\ &+ (1+\frac{1}{2}(2-\lambda_1))V(\lambda_1) + \frac{\lambda_1}{2}V(1).\end{aligned}$$

The boundary of Φ_R is given by

$$\phi_1 = -\frac{(1-\gamma)v(\Lambda_0)}{\gamma v(\Lambda_1)}\phi_0 + \frac{E(0,1)-E(1,1)}{\delta v(\Lambda_1)}.$$

Substituting this equation into $\Pi(\boldsymbol{\phi}, 1)$ and differentiating with respect to ϕ_0 gives

$$\frac{\partial \Pi(\phi_0, 1)}{\partial \phi_0} = \frac{1}{2}\left[\frac{(1-\gamma)v(\Lambda_0)}{\gamma v(\Lambda_1)}(1-\lambda_1+\lambda_1\gamma)[V(\lambda_1)-V(\Lambda_1)] - (1-\lambda_1\gamma)[V(\lambda_1)-V(\Lambda_0]\right].$$

Since the term $\gamma v(\Lambda_1)$ is positive, multiplying the above equation by $\gamma v(\Lambda_1)$, I get

$$\begin{aligned}\text{sign}\frac{\partial \Pi(\phi_0, 1)}{\partial \phi_0} = \text{sign}\Big\{&(1-\gamma)(1-\lambda_1+\lambda_1\gamma)v(\Lambda_0)[V(\lambda_1) - V(\Lambda_1)] \\ &- \gamma(1-\lambda_1\gamma)v(\Lambda_1)[V(\lambda_1) - V(\Lambda_0)]\Big\}.\end{aligned}$$

As shown before, the right hand side is negative, which completes the proof for Proposition 3.

To prove Proposition 4, for the problem to be well-defined, assume that Φ_{PR} is a nonempty proper subset of $[0,1]^2$. Moreover, if $\delta = \bar{\delta}$, $\Phi_R = [0,1]^2$. Therefore for any (λ_1, γ), if δ is sufficiently small, Ω is nonempty. However, as δ increases, the boundary of Φ_R shifts outward, which decreases the size of Ω. To construct the threshold value of δ, let $\underline{\phi}_1$ denote the minimum of 1 and the intercept of the boundary of Φ_R on ϕ_1-axis, that is,

$$\underline{\phi}_1 \equiv \min\left\{\frac{E(0,1)-E(1,1)}{\delta\gamma v(\Lambda_1)}, 1\right\}.$$

and $\underline{\phi}_0$ denote the maximum of 0 and the ϕ_0-coordinate of the boundary evaluated at $\phi_1 = 1$, that is,

$$\underline{\phi}_0 \equiv \max\left\{0, \frac{E(0,1)-E(1,1)-\delta\gamma v(\Lambda_1)}{\delta(1-\gamma)v(\Lambda_0)}\right\}.$$

Similarly, let $\bar{\phi}_1$ denote the minimum of 1 and the intercept of the boundary of Φ_{PR} on ϕ_1-axis, that is,

$$\bar{\phi}_1 \equiv \min \left\{ \frac{\frac{1}{2} + (2 + \lambda_1)V(\lambda_1) - \lambda_1 V(1)}{(1 - \lambda_1 + \lambda_1 \gamma)\left[V(\lambda_1) - V(\Lambda_1)\right]}, 1 \right\},$$

and $\bar{\phi}_0$ denote the maximum of 0 and the ϕ_0-coordinate of the boundary evaluated at $\phi_1 = 1$, that is,

$$\bar{\phi}_0 \equiv \max \left\{ 0, \frac{\frac{1}{2} + (1 - \lambda_1 + \lambda_1 \gamma)V(\Lambda_1) + (1 + 2\lambda_1 - \lambda_1 \gamma)V(\lambda_1) - \lambda_1 V(1)}{(1 - \lambda_1 \gamma)\left[V(\lambda_1) - V(\Lambda_0)\right]} \right\}.$$

There are two cases: one is where $\bar{\phi}_1 < 1$ and $\bar{\phi}_0 = 0$, and the other is where $\bar{\phi}_1 = 1$ and $\bar{\phi}_0 > 0$. In the former, $\delta_{\lambda_1,\gamma}$ is a unique solution to the equation $\bar{\phi}_1 = \underline{\phi}_1$. In the latter, $\delta_{\lambda_1,\gamma}$ is a unique solution to the equation $\bar{\phi}_0 = \underline{\phi}_0$. $\square$