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Appendix and Supplemental material not intended for publication-Round 2

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Supplemental material we cite in the text.

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Supplemental material

1 Illustration from Chen and Rey (2016)

We focus on the simple example (page 6) and transform it slightly in order to demonstrate our point more clearly within their setting.

Consumers wish to buy two goods, A and B, which can both be supplied by two firms, 1 and 2. Let v_1^A and v_1^B denote *market values* (MVs) for A and B from firm 1, and v_2^A and v_2^B MVs for A and B from firm 2. We assume that firms are symmetric such that $v_1^A + v_1^B = v_2^A + v_2^B$; however, firm 1 enjoys a larger MV for A ($v_1^A > v_2^A$) whereas firm 2 enjoys a larger MV for B ($v_2^B > v_1^B$): $v_1^A = v_2^B > v_2^A = v_1^B$.

Consumers face a shopping cost, reflecting the opportunity cost of the time spent in traffic, selecting products and so on. Some consumers face a "low" shopping cost, $\underline{s}$, such that they will adopt multi-stop shopping behavior, purchasing each product at the lowest available price. Let α denote the proportion of these consumers. While some consumers incur a low shopping cost, other consumers face a "high" shopping cost, that is, $\overline{s}$, and $(1-\alpha)$ denotes the proportion of these consumers.

Let r_1^A, r_1^B and r_1 denote firm 1's margins for A and B, and the total margin, such that $r_1 = r_1^A + r_1^B$ and r_2^A, r_2^B and r_2 firm 2's margins for A and B, and the total margin, that is $r_2 = r_2^A + r_2^B$.[1]

Suppose first, that consumers face a high shopping cost (smaller than $v_1^A + v_1^B = v_2^A + v_2^B$). In equilibrium, consumers behave as one-stop shoppers and buy both products from the same firm, and thus only the total margin, r_1 and r_2 matter. As the firms deliver the same consumer value, Bertrand-like competition drives the basket margin down to zero: $r_1 = r_2 = 0$.

Suppose instead that all consumers face a low shopping cost such that, in equilibrium, consumers behave as multi-stop shoppers and purchase each product at the lowest available price. Asymmetric Bertrand competition then leads firms to sell weak products at a zero margin, and strong products at a margin equal (or just below) the consumer value gain minus consumers' shopping costs: $r_1^A = v_1^A - v_2^A - \underline{s} = r_2^B = v_2^B - v_1^B - \underline{s}$ (i.e., $v_1^A - r_1^A - \underline{s} = v_2^A$ and $v_2^B - r_2^B - \underline{s} = v_1^B$). Note that $r_1^A = v_1^A - \underline{s}$ and $r_2^B = v_2^B - \underline{s}$ if $v_1^B = v_2^A < 0$.

Next, suppose that a fraction of consumers face a high shopping cost, $\overline{s}$, whereas the others have a low shopping cost, that is, $\underline{s}$. As shown by Chen and Rey (2016), cross-subsidization naturally arises. As before, fierce price competition dissipates profits from one-stop shoppers, and drives basket margins down to zero: $r_1^A + r_1^B = r_2^A + r_2^B = 0$. Then, keeping the total margin constant for one-stop shoppers, it suffices to undercut the rival's weak product by the amount of $\underline{s}$ to attract multi-stop shoppers. It follows that equilibrium margins are given by:

$$v_1^A - r_1^A - \underline{s} = v_2^A - r_2^A,$$
$$v_2^B - r_2^B - \underline{s} = v_1^B - r_1^B.$$

[1] Consumer valuation for a product is the difference between the MV and the firm margin on a product.

Replacing r_1^B and r_2^A by $-r_1^A$ and $-r_2^B$ (as $r_1^A + r_1^B = 0$ and $r_2^A + r_2^B = 0$), we obtain:

$$v_1^A - r_1^A - \underline{s} = v_2^A + r_2^B,$$
$$v_2^B - r_2^B - \underline{s} = v_1^B + r_1^A.$$

By symmetry, $r_1^A = r_2^B$ and $r_1^A = \frac{v_1^A - v_2^A - \underline{s}}{2} = r_2^B = \frac{v_2^B - v_1^B - \underline{s}}{2}$, the result is $r_1^B = -\frac{v_1^A - v_2^A - \underline{s}}{2} = r_2^A = -\frac{v_2^B - v_1^B - \underline{s}}{2}$. This pricing strategy does not affect the shopping behavior of high-cost consumers (who still face a zero margin), but generates a positive profit from multi-stop shoppers, who buy A from firm 1 and B from firm 2, giving each firm a positive margin of $\frac{v_1^A - v_2^A - \underline{s}}{2} = \frac{v_2^B - v_1^B - \underline{s}}{2}$ on these consumers.

We now focus on our point and assume that $v_1^A = v_2^B > \overline{s}$ and $v_1^B = v_2^A < 0$.

Suppose first, that firm 1 were alone (by symmetry, the same analysis applies for firm 2 by replacing good A by good B and good B by good A), as $v_1^B < 0$, firm 1 would only supply good A. Two cases should be distinguished as long as all consumers are served or low-cost consumers only are served, but in any case firm 1 would only supply good A. We can define a threshold in α such that, for low α, firm 1 provides the good A to all consumers and, for high α, firm 1 provides the good A to low-cost consumers.

Next, we suppose that both firms compete (our previous analysis applies) and we can show that firm 1 supplies A and B and firm 2 supplies A and B even if $v_1^B = v_2^A < 0$.

Numerical example: $v_1^A = v_2^B = 26 > \overline{s} = 20$ and $v_1^B = v_2^A = -2 < 0$. We can define consumer utilities and costs as follows: $u_1^A = u_2^B = 36$, $u_1^B = u_2^A = 28$ and $c_1^A = c_2^B = 10$, and $c_1^B = c_2^A = 30$. We also assume for the numerical example that $\underline{s} = 2$.

When firms are monopolists, the threshold in α is given by $\alpha = \frac{1}{4}$, but in any case, each firm only provides its strong product as $v_1^B = v_2^A = -2$.

When the firms compete, firms supply both goods, which generates a profit of $\frac{v_1^A - v_2^A - \underline{s}}{2}\alpha = \frac{v_2^B - v_1^B - \underline{s}}{2}\alpha = 13\alpha$ for each firm, even if $v_1^B = v_2^A = -2$. Q.E.D.

2 Illustration from Johnson (2017)

Following Johnson (2017), we assume asymmetric competition, in which a large retailer L with a full product line competes against a small firm S with a limited product line. We focus on the pricing behavior of the large retailer and we assume that the small firm is not a strategic player: the expected "in-store" utility of shopping at retailer S will be given by $\widehat{U}_S$.

L carries m products. For simplicity, we assume that $m = 3$. Let c_1, c_2 and c_3 denote the retailing costs of the large retailer for these products. Prices are perfectly observed by consumers, who then decide whether or not to go shopping.

A consumer who visits retailer L purchases quantities x_1, x_2 and x_3 to maximize:

$$\sum_i \zeta_i \left[u_i\left(x_i\right) - p_i x_i\right], \qquad i = 1, 2, 3,$$

where $\zeta_i \in (0, 1)$ is a binary random variable after the consumer chooses the large retailer but before final in-store purchasing decisions are made. Hence, for any i that is carried by

L, a consumer has zero demand for it (so that $\zeta_i = 0$) and buys zero units, or instead has a positive demand for it (so that $\zeta_i = 1$) and buys quantity x_i to maximize $u_i(x_i) - p_i x_i$. Let $v_i(p_i)$ denote the indirect utility associated with product i: $v_i(p_i) = \max_{x_i} u_i(x_i) - p_i x_i$; we obtain $\frac{dv_i(p_i)}{dp_i} = -x_i$. The values $\{\zeta_i\}$ are realized independently of each other, and independently and identically across consumers. The true probability that a consumer has positive demand for i is given by θ_i. That is, for any given consumer, $\Pr[\zeta_i = 1] = \theta_i > 0$. While the true probability is θ_i, each consumer believes that he will have positive demand for product i with some probability $\widehat{\theta}_i$ with $\widehat{\theta}_i \neq \theta_i$. Consumers make unplanned purchases such that $\theta_i \geq \widehat{\theta}_i$. Let $\alpha_i = \frac{\widehat{\theta}_i}{\theta_i}$ denote the accuracy ratio with $\alpha_i \leq 1$.

Because consumers believe that they will have a positive demand for i with probability $\widehat{\theta}_i$, each consumer forecasts his expected "in-store" utility of shopping at retailer L to be:

$$\widehat{U}_L = \sum_i \widehat{\theta}_i v_i(p_i).$$

As noticed previously, the expected "in-store" utility of shopping at retailer S is given by $\widehat{U}_S$.

Consumers choose whether to shop at retailer L or at retailer S by considering the values $\left\{\widehat{U}_L, \widehat{U}_S\right\}$. The number of consumers shopping at L is given by $Q\left(\widehat{U}_L, \widehat{U}_S\right)$. Let Q_1 denote the derivative with respect to the first argument; $Q_1 > 0$ so that Q is increasing in $\widehat{U}_L$.

The large retailer knows the true probabilities $\{\theta_i\}$ but also know that consumers forecast their utility values $\left\{\widehat{U}_L, \widehat{U}_S\right\}$ based on the values $\left\{\widehat{\theta}_i\right\}$. The result is L sets prices to maximize:

$$Q\left(\widehat{U}_L, \widehat{U}_S\right)\pi_L,$$

where $\pi_L = \sum_i \theta_i (p_i - c_i) x_i(p_i)$.

Define $L_i(p_i) = \frac{p_i - c_i}{p_i}\epsilon_i(p_i)$, where $\epsilon_i(p_i) = \frac{p_i x_i'(p_i)}{x_i(p_i)}$; $L_i(p_i)$ is the Lerner index of good i multiplied by its elasticity, so that if L were simply maximizing $(p_i - c_i) x_i(p_i)$, it would choose a price $\overline{p}_i$ such that $L_i(\overline{p}_i) = -1$ (by using the first-order condition: $(p_i - c_i) x_i'(p_i) + x_i(p_i) = 0$).

We assume in the following in order to make our point easily, that $x_i(p_i) = a - p_i$. Then, we assume that $c_1 = c_2 = c < a$; however we put no restriction on c_3. We will say that good 3 offers consumers a positive value if $c_3 < a$ and offers consumers a negative value if $c_3 \geq a$. So that, if L were simply maximizing $(p_3 - c_3) x_3(p_3)$, it would choose a price $\overline{p}_3$ such that $L_3(\overline{p}_3) = -1$ if the consumer value of the good 3 were positive and it would not sell the good in case of negative value, that is $c_3 \geq a$.

From the maximization problem of L which is given by $\max_{p_1,p_2,p_3} Q\left(\widehat{U}_L, \widehat{U}_S\right)\pi_L$, we derive first-order conditions ($i = 1, 2, 3$):

$$\frac{\partial \Pi_L}{\partial p_i} = Q\theta_i\left[x_i(p_i) + (p_i - c) x_i'(p_i)\right] + Q_1\left[\widehat{\theta}_i \frac{dv_i(p_i)}{dp_i}\right]\pi_L = 0.$$

Using $\frac{dv_i(p_i)}{dp_i} = -x_i(p_i)$ and $L_i(p_i) = \frac{p_i - c}{p_i}\epsilon_i(p_i)$ leads to:

$$\frac{\partial \Pi_L}{\partial p_i} = \frac{x_i(p_i)}{\widehat{\theta}_i}\left[Q\theta_i\left[1 + L_i(p_i)\right] - Q_1\pi_L\right] = 0.$$

Then, with $\alpha_i = \frac{\widehat{\theta}_i}{\theta_i}$, we obtain:

$$\frac{1}{\alpha_i}\left[1 + L_i\left(p_i\right)\right] = \frac{Q_1}{Q}\pi_L,$$

as it is derived in Johnson (2017) (See page 939).

We assume that $\alpha_1 < \alpha_2 < \alpha_3$ and that $p_2 = c$ at equilibrium. We know from Proposition 1 (page 939) that good 3 is priced below-cost because $\alpha_2 < \alpha_3$. The result is that, assuming $c_3 = a$, good 3 is sold because it is priced below-cost at equilibrium: $p_3 < a$. By continuity, there exists a threshold in c_3, which is larger than a, such that good 3 is sold even if it provides a negative MV (i.e., $c_3 > a$). The result is obtained because good 3 generates traffic to the large retailer. As claimed by Johnson (2017), goods with few unplanned purchases behave in this way (we can think about bread, milk, and so on). While these goods may provide negative MVs at L, they can be sold by L, which corresponds to the point we demonstrate in the present paper. Q.E.D.

References:

Chen, Z. and P. Rey (2016) "Competitive Cross-Subsidization" TSE Working Paper 13-450, revised version.

Johnson, J. P. (2017) "Unplanned Purchases and Retail Competition" *American Economic Review*, **107**(3), 931-65.