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Proofs and Semi-Pooling Equilibria

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Appendix

Semi-Pooling Equilibria

***Lemma 2:* When $R_l < 0$ and $Pr(h|\neg f) = 0$, $S_I(\neg f) = \neg v$.**

Proof. Given that $Z > 0$ and $D > 0$, the expected utility of $S_I(v|l) < 0$, while the expected utility of $S_I(\neg v|l) = 0$. If $Pr(h|\neg f) = 0$, then $Pr(h|\neg f) = 1$. Thus, $S_I(v|\neg f) < 0$, and $S_I(v|\neg f) < S_I(\neg v|\neg f)$.[1]

***Proposition 9:* When $R_l \leq 0$, $G_l > Y$ and $p \leq \frac{R_l}{R_l - R_h}$, the strategy profiles $S_C(h) = f$, $S_C(l) =$(f with probability q), $S_I(f) =$ (v with probability x), and $S_I(\neg f) = \neg v$ are in equilibrium.**

Proof. Assume that Lemma 1 is true (see proof below). $R_l \leq 0$ must hold true for I to play $\neg v$ with positive probability. I plays a mixed strategy following f, and in order for that strategy to be played in equilibrium, I must be indifferent between v and $\neg v$, $EU_{h|f} + EU_{l|f} = 0$. Let q represent the probability that f is played by C in state l. Using Bayes' Theorem, we derive the following probabilities: $Pr(h|f) = \frac{p}{p+q(1-p)}$ and $Pr(l|f) = \frac{q(1-p)}{p+q(1-p)}$. Therefore, in order for I to be indifferent between v and $\neg v$ when f is played, $\frac{ZR_h p}{p+q(1-p)} + \frac{ZR_l q(1-p)}{p+q(1-p)} = 0$. Solving for q, we find that I in indifferent when $q^* = \frac{-R_h p}{R_l(1-p)}$. Given *Lemma 2*, $R_l \leq 0$ and $S_C(h) = f$, I always plays $\neg v$ in response to $\neg f$. C is indifferent between strategy f and $\neg f$ in state l, when the expected utility of instituting financial reforms equals the expected utility of not instituting financial reforms. Let x represent the probability that I plays v in response to f. C receives a payoff of 0 when playing $\neg f$. Let x represent the probability that I plays v in response to f. In order for C to be indifferent between f and $\neg f$, the following equality must hold: $xG_l - Y = 0$. Solving for x, we find the following indifference condition: $x^* = \frac{Y}{G_l}$. By definition, $Y > 0$, thus, $x^* > 0$. In order for x^* to hold in equilibrium, $x < 1$, which is true when $G_l > Y$. In order for C to prefer strategy f in equilibrium when in state h,

[1] Cases where only when $R_l = 0$ would an equilibrium exist are excluded from the set of semi-pooling equilibria that are analyzed

$EU_{f|h} \geq EU_{\neg f|h}$. Thus, in order for C to choose f the following must be true: $xG_h - Y \geq 0$, or $x \geq \frac{Y}{G_h}$. Given that $G_h > G_l$, $x \geq \frac{Y}{G_h}$ holds true whenever $x^* = \frac{Y}{G_l}$.

Proposition 10: When** $R_l \leq 0$**,** $Y \geq (1-D)G_h$ **and** $p \geq \frac{R_l}{R_l - R_h}$**, the strategy profiles** $S_C(h)$ = **(**f **with probability** q**),** $S_C(l) = \neg f$**,** $S_I(f) = v$**, and** $S_I(\neg f)$ = **(**v **with probability** x**) are in equilibrium.

Proof. I always plays v in response to f since $R_h > 0$, and $Pr[h|f] = 1$. I plays a mixed strategy following $\neg f$, and in order for that strategy to be played in equilibrium, I must be indifferent between v and $\neg v$, $EU_{h|\neg f} + EU_{l|\neg f} = 0$. Using Bayes' Theorem, we derive the following probabilities: $Pr(h|\neg f) = \frac{pq}{pq+(1-p)}$ and $Pr(l|\neg f) = \frac{1-p}{pq+(1-p)}$. Therefore, in order for I to be indifferent between v and $\neg v$ when $\neg f$ is played, $\frac{ZDR_h pq}{pq+(1-p)} + \frac{ZDR_l(1-p)}{pq+(1-p)} = 0$. Solving for q, we find that I in indifferent when $q^* = \frac{-R_l(1-p)}{R_h p}$. Given that $q^* \leq 1$, $\frac{-R_l(1-p)}{R_h p} \leq 1$. This only holds true when $p \geq \frac{R_l}{R_l - R_h}$. In order for C to prefer equilibrium play when in state h, C must be indifferent between f and $\neg f$. C receives a payoff of $G_h - Y$ when playing f, and a payoff of $xDG_h + (1-x)(0)$ when playing $negf$. Thus, $xDG_h + (1-x)(0) = G_h - Y$ in equilibrium. Solving for x, we find the following indifference curve: $x^* = \frac{G_h - Y}{DG_h}$. Since $x \leq 1$, this holds true when $\frac{G_h - Y}{DG_h} \leq 1$. Solving for Y, we find that the following condition must hold: $Y \geq (1-D)G_h$. C must also prefer equilibrium play when in state L. In order for this to be true, $EU_{\neg f|l} \geq EU_{f|l}$. Thus, $xDG_l > G_l - Y$. Solving for x, $x = \frac{G_h - Y}{DG_h}$, and substituting for x^*, we find that the following must hold true $\frac{G_h - Y}{DG_h} \geq \frac{G_h - Y}{DG_h}$. Since $G^h > G^l > 0$ and $D > 0$, this holds for all values of x^*.

Proofs

Proof of Lemma 1. I receives a payoff of 0 when playing $\neg v$ in response to either action by C. Thus, in order for player I to prefer v to $negv$, the expected payoff in equilibrium of playing v must be greater than 0. R_h is by definition greater than 0, and, thus, the payoff from v must be greater than 0. Therefore, when I assigns a probability of 1 to the state of the world being h, she will always play v in equilibrium. Given that $R_h > R_l$, when $R_l > 0$, I will also always play v in response to any action by C and for all beliefs regarding the state of the world.

Proof of Proposition 1. Given Lemma 1, an investor will only play v when assigning a probability of 1 to state l when $R_l > 0$, and will always play v when the of probability of being in state h is 1. $Pr(h|f) = 1$, $Pr(l|\neg f) = 1$. Thus, in order for I to play v in response to f when $Pr(l|\neg f) = 1$, the following condition must be met: $R_l \geq 0$.Given this condition, I will always prefer to play v. Thus, whether C plays f or $\neg f$ hinges on whether the added value financial institutional development justifies the costs associated with development. In order for this separating equilibrium to hold, C must prefer to play f when in state h, and prefer $\neg f$ when in state l. In order for this to hold true, $DG_h \leq G_h - Y$ and $DG_l \geq G_l - Y$. Thus, the strategy set $S_C(h) = f$, $S_C(l) = \neg f$ and $S_I(f) = S_I(\neg f) = v$ is in equilibrium when $\frac{G_l - Y}{G_l} \leq D \leq \frac{G_h - Y}{G_h}$ and $R_l \geq 0$.

Proof of Proposition 2. Given Lemma 1, I will always play v when assigning a probability of 1 to the state of the world being h, and will never play v when the probability of being in state l is 1 if $R_l < 0$. $Pr(h|f) = 1$, $Pr(l|\neg f) = 1$. In order for $S_I(\neg f) = \neg v$ to be played in equilibrium, the following must hold true $R_l \leq 0$. $S_I(f) = v$ will always be played since $Pr(h|f) = 1$. C will always prefer to play f when in state h given I's strategy set since $G_h - Y$ is always greater than 0. In order for C to prefer $\neg f$ when in state l, $G_l - Y$ must be less than or equal to 0. Given that $G_l \geq Y$, this only true when $G_l = Y$. Thus, in order for $S_C(h) = f$, $S_C(l) = \neg f$, $S_I(f) = v$ and $S_I(\neg f) = \neg v$ to be played in equilibrium, $R_l \leq 0$ and $G_l \geq Y$.

Proof of Proposition 3. Assume that I plays v against both f and $\neg f$. In order for C to play f in equilibrium when in state l, $G_l - Y \geq DG_l$. In order for C to play $\neg f$ in equilibrium when in state h, $G_h - Y \leq DG_h$. This means that $D \leq \frac{G_l - Y}{G_l}$ and $D \geq \frac{G_h - Y}{G_h}$. This would imply that $\frac{G_l - Y}{G_l} > \frac{G_h - Y}{G_h}$, which can never hold true since $G_h > G_l$ by definition. Given that $Pr(h|\neg f) = 1$, we know that I will always play v in response to $\neg f$, so the only other strategy that could be played by I is $S_I(\neg f) = v$ and $S_I(f) = \neg v$. Since D, Y and G_l are always positive, C will always prefer the payoff DGl to $-Y$ in equilibrium. Thus, C will never play f when in state l if I will respond with $\neg v$. In fact, C will never play f in response to $\neg v$, whether the state is h or l.

Proof of Proposition 4. Assume that *Lemma 1* holds true, and $R_l \geq 0$. I can never increase her payoff by choosing $\neg v$. When $R_l \leq 0$, the probability that the state of the world is h must be high enough to justify I playing v. Given that C plays f in both states, the utility I derives from playing v is $p(ZR_h) + (1-p)(ZR_l)$, and in order for it to be weakly preferred to $\neg v$, $p(ZR_h) + (1-p)(ZR_l) \geq 0$. This holds true when $p \geq \frac{-R_l}{R_h - R_l}$. I must also prefer v to $\neg v$ in response to $\neg f$ in order for this strategy set to be played in equilibrium. For this to be true, the following inequality must be satisfied: $(Pr[h|\neg f])ZDR_h + (1 - Pr[h|\neg f])\frac{ZDR_l}{D} \geq 0$. Thus, when $Pr[h|\neg f] \geq \frac{-R_l}{R_h D^2 - R_l}$, I will prefer to play v in response to $\neg f$. Given that I will play v in response to either f or $\neg f$, C will only choose f when the growth benefits associated with financial institutions outweigh their costs. Since $G_l < G_h$, $\frac{G_l - Y}{G_l} < \frac{G_h - Y}{G_h}$. In order for C to prefer f in both states of the world, given I's strategy set, the following inequality must hold true: $G_l - Y \geq DG_l$. This will hold true when $D \leq \frac{G_l - Y}{G_l}$.

Proof of Proposition 5. Given *Lemma 1*, I will only choose $\neg v$ as a strategy in equilibrium when $R_l \leq 0$. Thus, this equilibrium can only exist when $R_l \leq 0$. Given that C plays f in both states, the utility I derives from playing v is $p(ZR_h) + (1-p)(ZR_l)$, and in order for it to be weakly preferred to $\neg v$, $p(ZR_h) + (1-p)(ZR_l) \geq 0$. This holds true when $p \geq \frac{-R_l}{R_h - R_l}$. I must also prefer $\neg v$ to v in response to $\neg f$ in order for this strategy set to be played in equilibrium. For this to be true, the following inequality must be satisfied: $Pr[h|\neg f](ZDR_h) + (1 - Pr[h|\neg f])\frac{ZDR_l}{D} \leq 0$. Thus, when $Pr[h|\neg f] \leq \frac{-R_l}{R_h D^2 - R_l}$, I will prefer to play $\neg v$ in response to $\neg f$. Given that I will play v in response to f and $\neg v$ in response to $\neg f$, C will always prefer to play f. This is because D and G are always positive, and

$G_l \geq Y$, and the utility derived from playing $\neg f$ is 0. Thus, f will always be preferred in equilibrium given I's strategy set.

Proof of Proposition 6. Assume that *Lemma 1* holds true, and $R_l \geq 0$. I can never increase her payoff by choosing $\neg v$, which always results in a payoff of 0, and, thus, will always play v. When $R_l \leq 0$, the probability that the state of the world is h must be high enough to justify I playing v. Given that C plays $\neg f$ in both states, the expected utility I derives from playing v is $pZDR_h + (1-p)\frac{ZDR_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZDR_h + (1-p)\frac{ZDR_l}{D} \geq 0$. This holds true when $p \geq \frac{-R_l}{R_h D^2 - R_l}$. I's off-the-equilbrium-path belief must also support playing v for this strategy to hold. Thus, $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \geq 0$, and, therefore, the following off-the-equilibrium-path belief is required:$Pr[h|f] \geq \frac{-R_l}{R_h - R_l}$. Given that I will play v in response to either f or $\neg f$, C will only choose $\neg f$ when the growth benefits associated with financial institutions are overshadowed by their costs. Since $G_l < G_h$, $\frac{G_l - Y}{G_l} < \frac{G_h - Y}{G_h}$. In order for C to prefer $\neg f$ in both states of the world, given I's strategy set, the following inequality must hold true: $G_h - Y \leq DG_h$. This will hold true when $D \geq \frac{G_h - Y}{G_h}$.

Proof of Proposition 7. Given *Lemma 1*, I will only choose $\neg v$ as a strategy in equilibrium when $R_l \leq 0$. Thus, this equilibrium can only exist when $R_l \leq 0$. Given that C plays $\neg f$ in both states, the utility I derives from playing v is $pZDR_h + (1-p)\frac{ZDR_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZDR_h + (1-p)\frac{ZDR_l}{D} \leq 0$. This holds true when $p \leq \frac{-R_l}{R_h D^2 - R_l}$. I's off-the-equilbrium path belief must also support playing $\neg v$ for this strategy to hold. Thus, $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \leq 0$, and, therefore, the following off-the-equilibrium-path belief is required:$Pr[h|f] \leq \frac{-R_l}{R_h - R_l}$. Since I always plays $\neg v$, C will always receive a payoff of $-Y$ when playing f, and 0 when playing $\neg f$. Since $Y > 0$, C will always prefer $\neg f$ in equilibrium.

Proof of Proposition 8. Given *Lemma 1*, I will only choose $\neg v$ as a strategy in equilibrium when $R_l \leq 0$. Thus, this equilibrium can only exist when $R_l \leq 0$. Since C plays $\neg f$ in both states, the utility I derives from playing v is $pZDR_h + (1-p)\frac{ZDR_l}{D}$. Since the utility from $\neg v$ is 0, the following must hold true for this strategy set to be played in equilibrium: $pZDR_h + (1-p)\frac{ZDR_l}{D} \geq 0$. This holds true when $p \geq \frac{-R_l}{R_h D^2 - R_l}$. In order for this equilibrium to hold, I must prefer to play $\neg v$ in response f. Note that weak consistency is not required for off-the-equilibrium-path beliefs. Thus, we can assign any belief that satisfies the following inequality: $Pr[h|f](ZR_h)(1 - Pr[h|f])(ZR_l) \leq 0$, and, therefore, the following off-the-equilibrium-path belief is required:$Pr[h|f] \leq \frac{-R_l}{R_h - R_l}$. Since player C will receive a payoff of $-Y$ for playing f, and receives a positive payoff from playing $\neg f$ when in both state h and l, C will always play $\neg f$ in this equilibrium.