E C O N O M I C S B U L L E T I N

Price−induced technical progress and comparative statics

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Abstract

The conjecture of a price−induced technical progress was formulated by Hicks in 1932. It acquired prominence during the seventies with the work of Hayami and Ruttan who dealt with the agricultural sectors in the United States and Japan. A novel specification of this hypothesis is that output and input prices enter the production function as shift parameters of the technology frontier. In this paper, it is found that empirically verifiable hypotheses under price−induced technical progress can be expressed in the form of estimable combinations of partial derivatives of the input demand functions with the partial derivatives of the production function. In principle, these novel comparative statics relations are observable but their measurement requires the simultaneous estimation of the input demand and of the production functions. The price−induced technical progress specification presented in this paper includes a generalization of the Hotelling lemma.

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1. Introduction

In *The Theory of Wages* (1932, 1957, p. 124), John Hicks formulated the conjecture that a change in the relative prices of factors of production could influence the direction and the magnitude of technical progress (TP). This is probably the original statement that inspired specifications of price-induced technical progress. One of the first empirical studies in support of this idea is the work of Rothbarth (1946) who compared United States and British industries. This analysis was continued by Habbakuk (1962) who concluded that the faster pace of technical progress in the United States as compared to Great Britain was to be attributed to a higher capital intensity of the American industries. In the sixties, a lively debate took place among economists with the purpose of sharpening the identification of the process of innovation and technical progress. Ahmad (1966) provides a critical survey of that debate. In the seventies, the price-induced hypothesis of technical progress was re-formulated by Hayami and Ruttan (1971, p. 82) in the analysis of United States and Japanese agricultural sectors. More recently, Mitchell (1999) surveyed the economic models for studying price-induced technical change. He concluded (Mitchell, 1999, p. 5) that the neoclassical production function approach is difficult to implement empirically and that there are few testable hypotheses generated by such a model without a clear specification of the inputs to and outputs of innovation.

A fundamental deficiency of many studies dealing with technical progress is the lack of testable hypotheses that can be used to either support or invalidate the specified model. The possibility that technical progress may be induced by a set of complex forces such as entrepreneurial motivation (of Schumpeterian inspiration) and waves of R&D expenditures has received widespread consideration. Prices, on the other hand, have been tantalizingly mentioned from time to time as capable of inducing technical progress without a formal specification of models that articulate this hypothesis. This aspect of the analysis, therefore, will be the focus of this paper.

Moreover, the comparative statics implications of a price-induced-technical-progress hypothesis have never been analyzed. In this paper, therefore, we explore the comparative statics of price-induced technical progress from a microeconomic perspective. More specifically, we incorporate output and input prices directly into the production function where they act as shift parameters of the technological frontier.

The double role assumed by prices, namely as scarcity signals and as shift parameters, destroys the traditional comparative statics relations of the competitive firm as derived by Hicks. In order to recover refutable qualitative conditions of empirically verifiable hypotheses for the competitive firm operating under price-induced technical progress, it is necessary to consider a more complex set of comparative statics relations. The major result of this novel analysis, therefore, is a set of comparative statics relations that depend upon dual and primal functions and comes in the form of a symmetric and positive semidefinite matrix of observable terms. Hence, the empirical implementation of the comparative statics conditions developed in this paper requires the concomitant measurement of the input derived demand functions and of the production function.

In section 2 we show the impossibility of signing the direct partial derivatives of output supply and input derived demand functions. In section 3 we develop the framework for deriving comparative statics conditions for the price-induced TP model that are observable and testable. In section 4 we state a generalization of the Hotelling lemma. In section 5 we discuss some estimation issues associated with the novel TP model. Section 6 concludes the paper.

2. Initial Analysis

In this section, we show that the profit maximizing specification under price-induced technical progress does not produce the familiar Hicksian comparative statics conditions involving the individual direct partial derivatives of the output supply and input demand functions. Let $y = f(\mathbf{x}, p, \mathbf{r})$ be a twice continuously differentiable production function for a single output $y > 0$, where $\mathbf{x} \in \Re^N_{++}$ is a vector of inputs, $\mathbf{r} \in \Re^N_{++}$ is a vector of input prices, and $p > 0$ is output price. Throughout the paper, we assume that there exists a locally once continuously differentiable solution to the profit maximizing problem of the competitive firm. Furthermore, and in this section only, we assume that the matrix of second partial derivatives of the production function with respect to input quantities, $f_{\mathbf{xx}'}$, is nonsingular at the optimum solution. We do not assume anything further about the production function $f(\cdot)$.

The profit maximization problem for a competitive firm is given by

$$\max_{\mathbf{x}} \{F(\mathbf{x}, p, \mathbf{r}) \stackrel{def}{=} pf(\mathbf{x}, p, \mathbf{r}) - \mathbf{r}'\mathbf{x}\}. \tag{1}$$

The first-order necessary conditions are

$$\frac{\partial F}{\partial \mathbf{x}} = pf_{\mathbf{x}}(\mathbf{x}, p, \mathbf{r}) - \mathbf{r} = \mathbf{0}. \tag{2}$$

Under the postulated assumptions, relations (2) can be solved, in principle, for the vector of input derived demand functions $\mathbf{x} = \mathbf{x}(p, \mathbf{r})$ while the supply function is $y(p, \mathbf{r}) = f[\mathbf{x}(p, \mathbf{r}), p, \mathbf{r}]$. These functions are not, in general, homogeneous of degree zero in prices because the production function is not assumed to be homogeneous of degree zero in $(p, \mathbf{r})$.

We now are interested in demonstrating that the traditional Hicksian comparative statics relations do not hold. These relations are obtained by inserting the solution vector $\mathbf{x} = \mathbf{x}(p, \mathbf{r})$ into Eq. (2) and differentiating the resulting identity with respect to p and $\mathbf{r}$ to obtain

$$f_{\mathbf{x}} + pf_{\mathbf{xx}'}\frac{\partial \mathbf{x}}{\partial p} + pf_{\mathbf{x}p} = \mathbf{0}, \tag{3}$$

$$pf_{\mathbf{xx}'}\frac{\partial \mathbf{x}}{\partial \mathbf{r}'} + pf_{\mathbf{xr}'} - I = \mathbf{0}. \tag{4}$$

Solving Eqs. (3) and (4) for the price slopes of the input demand functions, we obtain

$$\frac{\partial \mathbf{x}}{\partial \mathbf{r}'} = \frac{f_{\mathbf{xx}'}^{-1}}{p} - f_{\mathbf{xx}'}^{-1}f_{\mathbf{xr}'}, \tag{5}$$

$$\frac{\partial \mathbf{x}}{\partial p} = -\frac{f_{\mathbf{xx}'}^{-1}f_{\mathbf{x}}}{p} - f_{\mathbf{xx}'}^{-1}f_{\mathbf{x}p}. \tag{6}$$

Similarly, by differentiating the supply function with respect to p and $\mathbf{r}$ we find

$$\frac{\partial y}{\partial p} = f_{\mathbf{x}}'\frac{\partial \mathbf{x}}{\partial p} + f_p = -\frac{f_{\mathbf{x}}'f_{\mathbf{xx}'}^{-1}f_{\mathbf{x}}}{p} - f_{\mathbf{x}}'f_{\mathbf{xx}'}^{-1}f_{\mathbf{x}p} + f_p, \tag{7}$$

$$\frac{\partial y}{\partial \mathbf{r}'} = f_{\mathbf{x}}'\frac{\partial \mathbf{x}}{\partial \mathbf{r}'} + f_{\mathbf{r}'} = \frac{f_{\mathbf{x}}'f_{\mathbf{xx}'}^{-1}}{p} - f_{\mathbf{x}}'f_{\mathbf{xx}'}^{-1}f_{\mathbf{xr}'} + f_{\mathbf{r}'}. \tag{8}$$

None of the derivatives in Eqs. (5), (6), (7) and (8) are signable because of the presence of cross-derivatives of the production function involving the quantity and the prices of all commodities. The traditional Hicksian relations are special cases of the above conditions and are obtained precisely when prices do not enter the production function.

3. Comparative Statics of the Competitive Firm Under Price Induced Technical Progress

This section establishes, for the first time, that testable hypotheses of producer's behavior, consistent with the specification of a price-dependent production function, can be obtained as an estimable combination of partial derivatives of the production function and input demand functions. To this purpose, we re-state the problem of the competitive firm analyzed in section 2 using the primal-dual methodology of Silberberg (1974).

First, let us define the indirect or maximized profit function $\phi(\cdot)$ of the competitive firm under price-induced technical progress for problem (1) as

$$\phi(p,\mathbf{r}) \overset{def}{=} \max_{\mathbf{x}}\{F(\mathbf{x},p,\mathbf{r}) \overset{def}{=} pf(\mathbf{x},p,\mathbf{r}) - \mathbf{r}'\mathbf{x}\}. \tag{9}$$

It follows from inspection of Eq. (9) that, in general, $\phi(\cdot)$ is not convex in output and input prices, it is not homogeneous of any degree in those prices, it is not nondecreasing in p, and it is not nonincreasing in $\mathbf{r}$. These conclusions are all a result of the appearance of the prices in the production function and the fact that we have not assumed anything about the partial derivatives of $f(\cdot)$ with respect to $(p,\mathbf{r})$. The only things that can be said about the profit function $\phi(\cdot)$ is that it is locally C^2 and $\phi(p,\mathbf{r}) \geq 0$. A major consequence of these statements is that the traditional Hotelling lemma does not apply to the indirect profit function $\phi(\cdot)$. Further on, we will present a Generalized Hotelling lemma that re-establishes the familiar derivative property to recover the output supply and the input derived demand functions.

The conclusion of the preceding section found that the direct partial derivatives of the choice functions $y(p,\mathbf{r})$ and $\mathbf{x}(p,\mathbf{r})$ cannot be signed. We now re-state the optimization problem of the competitive firm using Silberberg's (1974) primal-dual methodology. The ensuing analysis will result in comparative statics conditions that can form the scaffolding for a measurable test of the price-induced TP hypothesis. This methodology begins with the statement of the primal-dual problem

$$\max_{p,\mathbf{r}} D(\mathbf{x},p,\mathbf{r}) \overset{def}{=} F(\mathbf{x},p,\mathbf{r}) - \phi(p,\mathbf{r}) = pf(\mathbf{x},p,\mathbf{r}) - \mathbf{r}'\mathbf{x} - \phi(p,\mathbf{r}). \tag{10}$$

Given the assumptions made in section 2 about the existence of a locally once continuously differentiable solution to the profit maximizing behavior of the competitive producer, problem (10) implies the first-order necessary conditions (or envelope relations)

$$D_p = pf_p + f - \phi_p = 0, \tag{11}$$

$$D_{\mathbf{r}} = pf_{\mathbf{r}} - \mathbf{x} - \phi_{\mathbf{r}} = \mathbf{0}, \tag{12}$$

and the second-order necessary conditions

$$\mathbf{u}'D_{\alpha\alpha'}\mathbf{u} = \mathbf{u}'[F_{\alpha\alpha'} - \phi_{\alpha\alpha'}]\mathbf{u} \leq 0 \quad \text{for all } \mathbf{u} \in \Re^{N+1}, \tag{13}$$

where $\alpha \overset{def}{=} (p,\mathbf{r})$. This condition means that the matrix $D_{\alpha\alpha'}$, and its equivalent $[F_{\alpha\alpha'} - \phi_{\alpha\alpha'}]$, must be symmetric negative semidefinite.

In order to attribute economic significance to each element of the matrix $[F_{\alpha\alpha'} - \phi_{\alpha\alpha'}]$ it is convenient to first compute the Hessian matrix of the primal-dual objective function $D(\cdot)$ with respect to p and $\mathbf{r}$. Doing just that using Eqs. (11) and (12) yields

$$\begin{bmatrix} D_{pp} & D_{p\mathbf{r}'} \\ D_{\mathbf{r}p} & D_{\mathbf{r}\mathbf{r}'} \end{bmatrix} = \begin{bmatrix} 2f_p + pf_{pp} - \phi_{pp} & f_{\mathbf{r}'} + pf_{p\mathbf{r}'} - \phi_{p\mathbf{r}'} \\ f_{\mathbf{r}} + pf_{\mathbf{r}p} - \phi_{\mathbf{r}p} & pf_{\mathbf{r}\mathbf{r}'} - \phi_{\mathbf{r}\mathbf{r}'} \end{bmatrix}. \tag{14}$$

This matrix is symmetric by Young's theorem and negative semidefinite by the requirement of the second-order necessary conditions of problem (10) given in Eq. (13). Using the second term of Eq. (13), we see that these conditions can be written in the somewhat more informative form according to which $\mathbf{u}'F_{\alpha\alpha}\mathbf{u} \leq \mathbf{u}'\phi_{\alpha\alpha}\mathbf{u}$ for all $\mathbf{u} \in \Re^{N+1}$. This means that the indirect profit function $\phi(\cdot)$ is not, in general, convex in prices since $F(\cdot)$ is not either, as noted above.

The final step is the differentiation of the envelope relations (11) and (12) after they have been transformed into identities such as

$$pf_p[\mathbf{x}(p,\mathbf{r}),p,\mathbf{r}] + f[\mathbf{x}(p,\mathbf{r}),p,\mathbf{r}] - \phi_p(p,\mathbf{r}) \equiv 0, \tag{15}$$

$$pf_{\mathbf{r}}[\mathbf{x}(p,\mathbf{r}),p,\mathbf{r}] - \mathbf{x}(p,\mathbf{r}) - \phi_{\mathbf{r}}(p,\mathbf{r}) \equiv \mathbf{0}. \tag{16}$$

The differentiation of Eqs. (15) and (16) with respect to p and $\mathbf{r}$ then produces the desired result, namely

$$S(\alpha) \stackrel{def}{=} \begin{bmatrix} \big(f_{\mathbf{x}'} + pf_{p\mathbf{x}'}\big)\dfrac{\partial \mathbf{x}}{\partial p} & \big(f_{\mathbf{x}'} + pf_{p\mathbf{x}'}\big)\dfrac{\partial \mathbf{x}}{\partial \mathbf{r}'} \\ & \\ -\big(I - pf_{\mathbf{r}\mathbf{x}'}\big)\dfrac{\partial \mathbf{x}}{\partial p} & -\big(I - pf_{\mathbf{r}\mathbf{x}'}\big)\dfrac{\partial \mathbf{x}}{\partial \mathbf{r}'} \end{bmatrix} = - \begin{bmatrix} 2f_p + pf_{pp} - \phi_{pp} & f_{\mathbf{r}'} + pf_{p\mathbf{r}'} - \phi_{p\mathbf{r}'} \\ & \\ f_{\mathbf{r}} + pf_{\mathbf{r}p} - \phi_{\mathbf{r}p} & pf_{\mathbf{r}\mathbf{r}'} - \phi_{\mathbf{r}\mathbf{r}'} \end{bmatrix}. \quad (17)$$

Since the right-hand-side matrix of Eq. (17) is symmetric positive semidefinite so is the left-hand-side matrix $S(\alpha)$. This latter matrix is measurable and constitutes the only test of the price-induced technical progress hypothesis. Notice that the measurement of the matrix $S(\alpha)$ requires the estimation of the production function together with the derived demand functions for inputs. A more detailed examination of condition (17) allows us to state the following three relations that constitute verifiable restrictions of the model:

$$\big(f_{\mathbf{x}'} + pf_{p\mathbf{x}'}\big)\frac{\partial \mathbf{x}}{\partial p} \geq 0, \quad (18)$$

$$\Sigma_{k=1}^{N}(\delta_{jk} - pf_{r_j x_k})\frac{\partial x_k}{\partial r_j} \leq 0, \qquad j = 1, \ldots, N, \quad (19)$$

$$-\big(I - pf_{\mathbf{r}\mathbf{x}'}\big)\frac{\partial \mathbf{x}}{\partial p} = \Big[\big(f_{\mathbf{x}'} + pf_{p\mathbf{x}'}\big)\frac{\partial \mathbf{x}}{\partial \mathbf{r}'}\Big]', \quad (20)$$

where δ_{jk} is the Kronecker delta. Relation (19) provides more insight into the price-induced TP model when it is restated as

$$\frac{\partial x_j}{\partial r_j} - p\Sigma_{k=1}^{N} f_{r_j x_k}\frac{\partial x_k}{\partial r_j} \leq 0, \qquad j = 1, \ldots, N. \quad (21)$$

The first term is the standard response of the j-th input demand function to a variation of its own price, while the second term must be regarded as the effect of the price-induced technical progress when the j-th input price is varied. The latter effect involves the product of all the inputs' responses to a variation of the j-th input price and the second cross derivatives of the production function with respect to the j-th input price and all the input quantities.

It is surprising to notice that the slopes of the supply function do not enter into the elements of the matrix $S(\alpha)$. Using the output supply function in the form of $y(p, \mathbf{r}) = f[\mathbf{x}(p, \mathbf{r}), p, \mathbf{r}]$ and the results in Eqs. (18) and (20), however, it is possible to obtain the following relations involving the slopes of this function:

$$\frac{\partial y}{\partial p} = f_p + f_{\mathbf{x}'}\frac{\partial \mathbf{x}}{\partial p}, \quad (22)$$

$$\frac{\partial y}{\partial p} - f_p + pf_{p\mathbf{x}'}\frac{\partial \mathbf{x}}{\partial p} \geq 0. \quad (23)$$

Using $\partial y/\partial \mathbf{r}' = f_{\mathbf{x}'}\partial \mathbf{x}/\partial \mathbf{r}' + f_{\mathbf{r}'}$ and Eq. (20), the standard Hicksian cross symmetry condition involving the input price slopes of the output supply function and the output price slopes of the input demand functions take on the following more complex specification

$$\Big[\frac{\partial y}{\partial \mathbf{r}'} - f_{\mathbf{r}'} + pf_{p\mathbf{x}'}\frac{\partial \mathbf{x}}{\partial \mathbf{r}'}\Big]' = \Big[pf_{\mathbf{r}\mathbf{x}'} - I\Big]\frac{\partial \mathbf{x}}{\partial p}. \quad (24)$$

If prices do not enter the production function, relation (24) reduces to the familiar Hicksian reciprocity condition.

4. Generalized Hotelling Lemma

Within the model of price-induced technical progress analyzed in the previous section, the partial derivatives of the indirect profit function $\phi(\cdot)$ with respect to output and input prices do not produce the supply and derived demand functions as in the standard Hotelling lemma. In the present case, the failure of this celebrated lemma is due to the fact that the production function $f(\cdot)$ depends explicitly on the prices, in contrast to the standard profit maximization model of the competitive producer. The structure of the envelope relations (11) and (12) provides the basis for a generalization of the standard Hotelling lemma to include the partial derivatives of the production function with respect to prices. Hence, the output supply function can be recovered as

$$y(p, \mathbf{r}) = \phi_p - p f_p, \tag{25}$$

while the vector of input derived demand functions is recovered by

$$\mathbf{x}(p, \mathbf{r}) = -[\phi_{\mathbf{r}} - p f_{\mathbf{r}}]. \tag{26}$$

The profit maximizing output supply and input demand functions under the price-induced TP hypothesis can be obtained from a pair of consistent production and profit functions by combining their derivatives with respect to prices as indicated by the right-hand-side of Eqs. (25) and (26). The results of the traditional Hotelling lemma are obtained as special cases of Eqs. (25) and (26) when prices do not enter the production function. Nothing can be said *a priori* about the slopes of either the supply or the input derived demand functions presented in Eqs. (25) and (26). Also, these functions are no longer homogeneous of degree zero in prices.

5. Estimation Considerations

Although the focus of this paper is on developing the theoretical framework for testing the price-induced TP hypothesis, we wish also to consider a few aspects of the collateral estimation process. To this purpose, we will contrast the estimation of the standard model with the challenge of testing the novel specification.

In the standard model, where the profit function is convex in prices and the Hotelling lemma recovers the supply and demand functions directly, the testing of the profit-maximizing hypothesis entails the estimation of the system of relations that is constituted by the output supply and input derived demand functions (and possibly the profit function itself). In this context, there are so many conditions to verify, or properties to impose, on the model that the likelihood of such a model fitting the sample data ought to be drastically reduced. These output supply and input demand functions must be homogeneous of degree zero in prices; the matrix of own price slopes of the input demand function must be symmetric negative semidefinite; the own price slope of the output supply function must be nonnegative; the vector of output price derivatives of the input demand functions must be equal to the negative vector of input price derivatives of the output supply function. These are formidable requirements and it is not surprising that often the empirical data did not fit the model.

Testing of the price-induced TP model developed in this paper requires the estimation of the production function and of the system of input derived demand functions. In other words, the output supply function of the standard model is replaced by the production function. This novel mixture of primal and dual relations characterizes a model specification where there is no separation hyperplane between the primal space of technology and the dual space of prices. The empirical specification of the model must satisfy only two conditions or properties: the concavity of the production function in the input quantities, and the symmetry and positive semidefinitenes of the matrix $S(\alpha)$ in Eq. (17). This condition can be imposed by means of the Cholesky factorization technique. Since none of the relations are homogeneous of any degree, attention should be paid to avoid any price normalization which is a typical step of the estimation process in the standard model. As the standard model's specification is nested within the price-induced TP model, it provides an alternative specification for improving the power of the test.

A potentially difficult aspect of any empirical specification of the price-induced TP model based on exact flexible functional forms is that these forms do not have an explicit dual representation. In this case it would be impossible to specify a consistent output supply function and input demand functions following the Generalized Hotelling lemma. A way around this crucial aspect is to view the primal and dual specifications as second order approximations to the true functions.

6. Conclusion

The literature on technical progress has neglected the development of comparative statics conditions as the basis for empirically verifiable hypotheses associated with the postulated process of technical progress. In this paper, we have explored the most general specification of technical progress induced by a change in the relative prices of commodities within the context of a competitive firm. The crucial assumption of this type of technical process is that prices of outputs and inputs enter the firm's production function and act as shift parameters of the technological frontier.

The comparative statics analysis of the price-induced TP model reveals that the traditional Hicks conditions for the competitive firms no longer hold. In order to derive unambiguous qualitative relations that form the basis for empirically verifiable hypotheses of the model, it is necessary to assemble a precise combination of the partial derivatives of the demand functions by means of partial derivatives of the production function. The final result of this analysis is a symmetric positive semidefinite matrix that contains all the comparative statics relations of the model. Importantly, all the components of this matrix are observable.

The principal novelty of this analysis is that the observability of these relations requires the concomitant estimation of the production function and of the input demand functions. This is unusual because, traditionally, the domain of the production function is the quantity space of inputs whereas the domain of the demand functions is the price space of commodities. In the analysis presented here, however, the domain of the production function is extended to include the price space. This specification produces second cross partial derivatives of the production function involving both commodity quantities and their prices which are, at the same time, the source of the breakdown of all the traditional Hicksian properties and the generator of the technical progress effect.

The results of this paper are rather general. They have been obtained with minimal assumptions. To the standard assumptions of the neoclassical model of the competitive firm we added only the assumption that prices enter the production function as shift parameters of the technological frontier. This assumption is testable against the verifiable hypotheses of the standard model.

7. References

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