E C O N O M I C S   B U L L E T I N

Economic solubility of the agency problem

John Quiggin
Australian Research Council Senior Fellow, Australian National University

Abstract

Conditions are derived under which the solution to the Grossman−Hart formulation of the agency problem involves zero or non−zero effort

I thank the Australian Research Council for support provided through a Fellowship and Large Grant
Citation: Quiggin, John, (2001) "Economic solubility of the agency problem." *Economics Bulletin,* Vol. 3, No. 4 pp. 1−6
Submitted: January 31, 2001. **Accepted:** May 17, 2001.
URL: http://www.economicsbulletin.com/2001/volume3/EB−01C70001A.pdf

The agency problem formulated and solved by Grossman and Hart (1983) has been used as a basis for analysis of many problems involving the design of incentive structures under asymmetric information. Although Grossman and Hart demonstrated the existence of a solution to the mathematical problem of deriving the optimal contract, there has been relatively little analysis of the conditions under which the agency problem is soluble in an economic sense. A situation in which the principal can do no better than to guarantee the agent a constant payment in return for zero effort is one in which the agency problem is economically insoluble, since the cost of inducing the agent to put forth positive effort exceeds the benefits of any feasible contract. Jensen's (1989) argument in favor of leveraged buyouts may be interpreted as a claim that the agency problems associated with corporate structures are economically insoluble.

In this paper, conditions are derived under which the Grossman-Hart principal-agent problem will be economically soluble or insoluble. No (non-trivial) contract can arise if the agency problem is insoluble in this sense.

1 Solubility

The analysis of the parametrized distribution formulation of the agency problem is largely due to the work of Grossman and Hart (1983). We take the simplest version of the Grossman-Hart model, with two outcomes $z_1 \leq z_2$ and scalar effort $x \geq 0$. Effort level x corresponds to probability $\pi(x)$ of the good outcome z_2. We assume that $\pi(x)$ is smooth with $\pi'(x) \geq 0$ and $\pi'(x)$ bounded as x approaches zero from above. The fixed support assumption, essential to the existence of a non-trivial equilibrium, requires that

$$0 \leq \pi(0) \leq \pi(x) < 1 \forall x$$

with strict inequality for $x > 0$.

As was first observed by Mirrlees (1974) if there is an economically relevant effort level x such that $\pi(x) = 1$, this effort level can always be elicited by specifying an arbitrary large penalty if the bad outcome is observed, while paying the agent reservation utility if the good output is observed. As will be shown below, the value of $\pi(0)$ is not relevant to the solution of the problem, since the set of contractible effort levels has a lower bound strictly greater than zero. Nevertheless, to set the problem of insolubility in an economic context, it may be helpful to consider the case $\pi(0) = z_1 = 0$, where an effort level of zero leads to output zero with probability 1.

The agent has preferences $w(x, y)$ with $w_x < 0, w_y > 0$ and $w_{yy} < 0$ all bounded away from zero. The principal must offer a payment schedule (y_1, y_2) conditional on the observed output y. Given the agent's preferences $w(x, y)$, if effort x is to be achieved, the contract offered by the principal must satisfy the participation constraint

$$(1 - \pi(x))w(x, y_1) + \pi(x)w(x, y_2) \geq \bar{w} \tag{1}$$

where $\bar{w}$ is reservation utility.

The contract must also satisfy the truth-telling constraint

$$(1-\pi(x))w(x,y_1)+\pi(x)w(x,y_2)\geq(1-\pi(x'))w(x',y_1)+\pi(x')w(x',y_2)\quad\forall x' \tag{2}$$

ensuring that the choice of the specified effort level x is optimal for the agent. We say that an effort level x is contractible if there exist y_1, y_2 satisfying (2), and the positive profit requirement:

$$(1-\pi(x))(z_1-y_1)+\pi(x)(z_2-y_2)\geq 0. \tag{3}$$

Consider first the full information case, in which the contract is required to satisfy only the participation and positive profit constraints. We define

$$\bar{z}(x)=(1-\pi(x))z_1+\pi(x)z_2.$$

For any given x, define $\bar{y}(x)$ implicitly by $w(x,\bar{y}(x))=\bar{w}$. The full information problem is economically soluble if and only if

$$x^*=\text{argmax}[\bar{z}(x)-\bar{y}(x)]>0$$

and

$$\bar{z}(x^*)-\bar{y}(x^*)\geq 0.$$

We will confine attention to cases where these conditions is satisfied. Further, we will impose the condition

$$\bar{z}(0)-\bar{y}(0)=0.$$

This condition ensures that the contract associated with zero effort yields zero profit. Thus the principal is indifferent between the zero effort contract and no contract at all. We therefore say that the moral hazard problem is economically soluble if there exists a contractible effort level $x>0$. We define the cost-benefit ratio associated with effort x as

$$\beta(x)=\frac{(\bar{y}(x)-\bar{y}(0))}{(\pi(x)-\pi(0))(z_2-z_1)}$$

Thus the full-information contract associated with effort x yields positive profit if and only if $\beta(x)<1$. Turn now to the second-best case and define $y_1(x), y_2(x)$ as the solution to the problem:

$$\min_{y_1,y_2}\{\pi(x)y_2+(1-\pi(x))y_1\}$$

subject to 1and 2. It is straightforward to show (Grossman and Hart Proposition 1) that a solution exists. Uniqueness will be of no concern in what follows. Define

$$\kappa(x)=\pi(x)y_2(x)+(1-\pi(x))\,y_1(x)-\bar{y}(x)$$

and

$$\sigma(x)=\frac{\kappa(x)}{(\pi(x)-\pi(0))\,(z_2-z_1)}$$

In the case where the agent receives reservation utility, $\kappa(x)$ may be interpreted as her risk premium. (As is shown by Grossman and Hart, if the agent's preferences may be represented by an additively or multiplicatively separable function, the least-cost contract eliciting effort x will be such that 1 is satisfied exactly.) More generally, $\kappa(x)$ is a measure of the excess cost to the principal associated with the constraint . Risk aversion implies that $\kappa(x) > 0$ except in the full-information case when $y_1 = y_2 = y(x)$. We have

Proposition 1 *Assume $\bar{z}(0) - \bar{y}(0) = 0$. An effort level $x > 0$ is contractible if and only if $\beta(x) + \sigma(x) < 1$.*

Proof: The maximum achievable profit associated with the effort level x is

$$\begin{aligned}
\bar{z}(x) - \pi(x)y_2(x) + (1-\pi(x))y_1(x) &= \bar{z}(x) - \bar{y}(x) - \kappa(x) \\
&= \bar{z}(0) - \bar{y}(0) + (\pi(x) - \pi(0))(z_2 - z_1) - (\bar{y}(x) - \bar{y}(0)) - \kappa(x) \\
&= (\pi(x) - \pi(0))(z_2 - z_1) - (\bar{y}(x) - \bar{y}(0)) - \kappa(x) \\
&= (\pi(x) - \pi(0))(z_2 - z_1)\left[1 - \frac{(\bar{y}(x) - \bar{y}(0))}{(\pi(x) - \pi(0))(z_2 - z_1)} - \sigma\right] \\
&= (\pi(x) - \pi(0))(z_2 - z_1)[1 - \beta - \sigma] \\
&> 0 \text{ if and only if } \beta(x) + \sigma(x) < 1
\end{aligned}$$

QED :

Proposition 2 *For any values of z_1, z_2, any function $\pi(x)$ satisfying the conditions set out above, and any w such that $w(x, y)$ is strictly concave in $y \forall x$, there exists $x > 0$ such that no effort level $x', 0 < x' < x$, is contractible.*

Proof: It is sufficient to show that, under the stated conditions, $\kappa(x)$ is bounded away from zero and hence

$$\lim_{x \to 0^+} \sigma(x) = \infty$$

. Choose x, x' such that $x' < x$. Then

$$(1-\pi(x))w(x, y_1(x)) + \pi(x)w(x, y_2(x)) \geq (1-\pi(x'))w(x', y_1(x)) + \pi(x')w(x', y_2(x))$$

$$= (1-\pi(x))w(x', y_1(x)) + \pi(x)w(x', y_2(x)) - (\pi(x) - \pi(x'))(w(x', y_2(x)) - w(x', y_1(x)))$$

Rearranging yields

$$w(x', y_2(x)) - w(x', y_1(x)) \geq \frac{(1-\pi(x))[w(x, y_1(x)) - w(x', y_1(x))] + \pi(x)\left(w(x, y_2(x)) - w(x', y_2(x))\right)}{\pi(x) - \pi(x')}$$

Let the upper bound on $\pi'(x)$ be denoted M, and the lower bound on w_x be denoted m. Then

$$\begin{aligned}
(w(x', y_2(x)) - w(x', y_1(x))) &\geq \frac{(1-\pi(x))m(x - x') + \pi(x)m(x - x')}{M(x - x')} \\
&= m/M \\
&> 0
\end{aligned}$$

Hence, $(w(x', y_2(x)) - w(x', y_1(x)))$ is bounded away from zero for given x and $x' < x$. The continuity of w now implies that $(w(x, y_2(x)) - w(x, y_1(x)))$ is bounded away from zero. Now the fact that w is strictly concave in y implies that $\kappa(x)$ is bounded away from zero. QED

Proposition 2 shows that for sufficiently low effort levels, there will exist no positive profit contract consistent with truth-telling. The basic problem is that the risk premium associated with the contract is larger than the net benefit. An immediate corollary is that the moral hazard problem will be economically insoluble whenever the range of non-contractible effort levels includes all effort levels with a positive benefit-cost ratio.

Corollary 3 *Let π, z_1, z_2, w be such that $\beta(x) \geq 1$, where x is defined as in Proposition 2. Then the moral hazard problem is economically insoluble.*

The problem of insolubility is most likely to arise when z_1 and z_2 are far apart and $d\pi/dx$ is small. In situations of this kind, a positive effort level may be optimal in the absence of information asymmetries, but the risk premium associated with eliciting that effort level in the second-best case may exceed the benefits of the agent's effort. The likelihood of insolubility depends on the agent's risk attitudes. For any given z_1, z_2, π , the moral hazard problem will be insoluble for all agents who are sufficiently risk-averse (with risk-aversion measured by w_{yy}/w_y).

A central feature of problems of this kind is that $\pi(x^*) - \pi(0)$ is small relative to $\pi(0) + 1 - \pi(x^*)$. The actions of the agent may be viewed as having no effect with probability $\pi(0) + 1 - \pi(x^*)$ and raising output from z_1 to z_2 with probability $\pi(x^*) - \pi(0)$. When $\pi(x^*) - \pi(0)$ is close to zero, this means that (within the range of actions that yield an improvement in the first-best) the agent's actions have no effect on the payment received with probability close to 1. It follows that the penalty associated with the observation of the low outcome must be large in order to deter the agent from shirking. Since this large penalty is imposed with probability $1 - \pi(x) > 0$, the agent will demand a substantial risk premium.

Economically insoluble agency problems are likely to arise for plausible parametrizations of principal-agent relationships modelled in the Grossman-Hart framework. For example, Haubrich's (1994) analysis of pay-performance ratios for CEOs yields a range of cases that are economically insoluble in our terminology. An important issue for future work is whether the apparent insolubility is real or an artefact of the Grossman-Hart representation of the problem. It will be useful to consider alternative representations such as those of Holmstrom and Milgrom (1987) or Quiggin and Chambers (1998).

1.1 References

Grossman, Sanford J. and Hart, Oliver D (1983). "An Analysis of the Principal-Agent Problem." Econometrica 51, 7-45.

Haubrich, Joseph (1994). "Risk Aversion, Performance Pay and the Principal-agent Problem." Journal of Political Economy 102, 258-76.

Holmstrom, B. and Milgrom, P. (1987), 'Aggregation and linearity in the provision of intertemporal incentives', Econometrica 55(2), 303-328.

Jensen, Michael C (1989). "Eclipse of the Public Corporation." Harvard Business Review, 61-74.

Mirrlees, James (1974) "Notes on Welfare Economics, Information and Uncertainty." In Essays on Economic Behaviour under Uncertainty, Edited by Balch, M., McFadden, D. and Wu, S. North-Holland: Amsterdam.

Quiggin, J. and Chambers, R. G. (1998), 'A state-contingent production approach to principal-agent problems with an application to point-source pollution control', Journal of Public Economics 70, 441–72.