E C O N O M I C S   B U L L E T I N

On the Existence of Spatial Monopolies Under Free Entry

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Abstract

This paper studies the question of entry in the circular city model when the pre−entry market structure involves local monopolies. In contrast with Salop (1979), we show that the unit profit rate of incumbent monopolists is strictly positive and bounded above. The upper bound decreases with the size of gaps and with the number of incumbents.

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1 Introduction

In horizontal differentiation models, under the assumption of costless relocation of firms upon entry, the free entry outcome is usually characterized by a zero-profit condition (Salop, 1979; McLeod, Norman and Thisse, 1988; Economides, 1989; Norman and Thisse, 1996). However, the existence of long run spatial monopolists has not been studied seriously in the literature, since most of the papers are restricted to market structures where incumbents effectively compete for their marginal consumers (covered market structures).

In this paper, we study the question of entry in the circular city model when the pre-entry market structure involves local monopolies. As we show, the difficulty in analyzing local monopoly market structures under free entry stems from the fact that gross monopoly profits do not depend on the total number of active firms. Hence, the number of local monopolists that deter entry cannot be simply derived by applying a zero-profit condition and a detailed analysis of entry must be carried out. To this aim, we define the Symmetric Local Monopolist Equilibrium (SLME) and we compare it with the monopoly Symmetric Zero-Profit Equilibrium (SZPE) of Salop (1979).

Our results are the following. We first establish that a monopoly market structure candidate for SLME exists for every value of the structural parameters of the model. In contrast both with the situation of restricted entry and with the monopoly SZPE, the unit profit rate of incumbent monopolists is positive and bounded above in equilibrium. The upper bound we find decreases with the size of unserved market areas (gaps) and with the number of incumbent monopolists.

2 Model and definitions

Consumers are differentiated with respect to their location on the circumference of a circle Λ of unit length, with uniform density, normalized to 1. Transportation cost along the circle is linear in distance, at rate t per unit of good. The reservation price of consumers in the differentiated industry is constant and equal to $v > 0$. When n firms are active in the differentiated product market, firm i's ($i = 1, ..., n$) location is denoted by $x_i \in \Lambda$.

For any list of prices p_i's ($i = 1, ..., n$) quoted by the differentiated firms, the consumer located at point $x \in \Lambda$ determines his preferred brand and buys one unit of this brand only if he obtains a surplus greater than the outside option. Formally, consumer x solves:

$$\max \left[0, \max_{i=1,...,n} v - p_i - t\left|x - x_i\right| \right] \tag{1}$$

Production entails a fixed cost F and a variable cost proportional to the quantity produced, which we normalize, without loss of generality, to zero. Though the number of all candidates for incumbency is potentially large, economies of scale ensure that only a bounded number of firms can break even on the market.

Monopoly pricing in the horizontal differentiation model is defined as follows. When a profit-maximizing firm alone on a 'large' differentiated market sets a price p, its market radius is equal to $x = (v - p)/t$ and profits are $\Pi = 2(v - p)p/t$. The monopoly price is therefore $p^m = v/2$. It leads to a *monopoly market area* of $D^m = v/t$ and to *monopoly (gross) profits* of $\Pi^m = v^2/2t$.

With a fixed number of firms, non-cooperative pricing can lead to non covered market structure in the circular city (monopoly market structures with gaps). For, when it leads to market shares that do not overlap, the monopoly price is clearly an equilibrium of the price game.[1] With n equally spaced firms on a circular market of unit size, a monopoly market structure with gaps obtains when $D^m = v/t \leq 1/n$. In the rest of the analysis, we denote by $d = 1/n - v/t \geq 0$ the size of the gaps.

Can a monopoly market structure with gaps maintain when entry is free? The SZPE concept is ill-suited to answer this question. The reason is that, contrary to competitive (or kinked) profits, *monopoly gross profits do not depend on the number of other active firms around the circle.* To circumvent this difficulty, we define the SLME concept as follows.

Definition 1 *A Symmetric Local Monopolists' Equilibrium (SLME) is any monopoly market structure which satisfies the two conditions: (i) incumbents make non negative profits and (ii) no further entrant perceives the possibility of at least breaking even on the market.*

When checking for (ii), potential entrants' anticipations on the post-entry market equilibrium is crucial. We assume that capital is perfectly mobile so that incumbents always relocate upon entry to form a symmetric structure with the entrant. Concerning the price equilibrium, we assume that prices adjust immediately in case of entry.[2]

3 Non covered markets under free entry

3.1 Existence of a candidate SLME

Consider a profitable monopoly market structure with n firms. With costless relocation, entry is blockaded if the profit per firm in the market structure with $n + 1$ equally spaced firms is below the fixed cost. A necessary condition for this is that the post-entry structure does not consist of local monopolists itself. After relocation of incumbents, the entrant has 'free access' to the total mass nd of previously unserved consumers. For entry to be blockaded, it must

[1] Monopoly pricing is then a dominant strategy for firms, at least locally (see the discussion in Novshek, 1980).

[2] This implies that prices in the pre-entry structure have no entry deterring effect.

thus be the case that $n.d < D^m$. Combining this with the *ex ante* monopoly condition $d \geq 0$, we can therefore restrict the analysis to:

$$\frac{t}{v} - 1 \leq n \leq \frac{t}{v} \tag{2}$$

Let us assume, for the rest of the paper, that $t/v \geq 1$, so that the total market is large enough for at least one spatial monopolist. For fixed v and t, there exists exactly one integer n that satisfies equation (2). When v and t vary, n varies from one (the spatial single monopolist) to infinity. Since incumbents cannot make binding location decisions, it is not surprising to find only one monopoly market structure candidate for SLME for every value of the structural parameters of the model.[3] The important result is that, independently of F, we have at least one candidate SLME for every set of parameters values.

We analyze separately the cases of a single spatial monopolist and of monopoly market structures with at least two firms. The reason for this distinction is that it is only in the latter case that relocation of incumbents upon entry plays a role.

3.2 The SLME with at least two firms

We assume that $t/v \geq 2$, so that the integer n which satisfies equation (2) is at least equal to 2. We conduct the analysis with the function:

$$\alpha(n) = \frac{t}{nv} \tag{3}$$

Condition (2) then rewrites

$$\frac{t}{t+v} \leq \alpha(n+1) \leq 1 \tag{4}$$

When $\alpha(n+1) = t/(t+v)$, the size of the uncovered market d is zero, whereas when $\alpha(n+1) = 1$, the uncovered market area d reaches its maximum value compatible with relocation of incumbents, D^m/n.

The question that arises now is the nature of the post-entry price equilibrium: it can be either kinked or competitive. Recall from Salop that the price equilibrium in a covered market structure with n firms is kinked when

$$\frac{t}{v} \leq n \leq \frac{3t}{2v} \tag{5}$$

whereas when

$$\frac{3t}{2v} \leq n \tag{6}$$

it is competitive.

[3] The structural parameters are v, t and the total market size, which is normalized to one.

Lemma 1 *The post-entry price equilibrium is never competitive when the pre-entry monopoly market structure has at least two firms.*

Proof. By equation (6), the price equilibrium with $n+1$ firms is competitive when $n \geq \frac{3t}{2v} - 1$. Since $n \leq \frac{t}{v}$ by (2), a competitive post entry price equilibrium can only occur when $\frac{3t}{2v} - 1 < \frac{t}{v}$, or equivalently, $\frac{t}{v} < 2$, a case we ruled out. ■

The driving force behind this result is the aggregation of all unserved consumers due to the relocation of firms in the post-entry structure. The costless relocation assumption thus softens post-entry price competition, and leads to the kinked equilibrium. As we shall see, this relocation effect does not appear in the single spatial monopolist case.

By definition, the kink equilibrium price is such that the marginal consumer of the representative firm is indifferent between buying the brand of this firm, buying the closest neighboring brand and not buying in the differentiated industry. The post entry price is thus equal to:

$$p^k = v - \frac{t}{2(n+1)} \tag{7}$$

and gross profits per firm are given by:

$$\Pi^k = \frac{v}{n+1} - \frac{t}{2(n+1)^2} \tag{8}$$

After some calculations, the condition for blockaded entry, $\Pi^k \leq F$, writes:

$$\frac{F}{\Pi^m} \geq \alpha(n+1)(2 - \alpha(n+1)) \tag{9}$$

Equation (9) defines, for any value of $\alpha(n+1)$ between $t/(t+v)$ and 1, a lower bound on the ratio F/Π^m. This lower bound is a quadratic function of $\alpha(n+1)$, which is equal to $t(t-2v)/(t+v)^2$ at $\alpha(n+1) = t/(t+v)$ and monotonically increases up to its maximum 1 attained at $\alpha(n+1) = 1$.

In the rest of the paper, we denote by n^M the integer that satisfies equation (2), i.e., the integer part of t/v. We present our results using the *Unit Profit Rate* (UPR) for spatial monopolists, represented by the ratio $(\Pi^m - F)/F$. The following proposition summarizes our results.

Proposition 1 *When $t \geq 2v$, there exists exactly one market structure candidate for SLME, with a number of firms equal to $n^M \geq 2$. The monopoly market structure with n^M firms is a SLME if and only if the UPR of local monopolies is non negative and bounded above by:*

$$\frac{\Pi^m - F}{F} \leq \frac{\left[1 - \alpha(n^M+1)\right]^2}{\alpha(n^M+1)\left[2 - \alpha(n^M+1)\right]} \tag{10}$$

where $\alpha(.)$ is given by (3).

Equation (10) is simply obtained by rearranging equation (9) and by setting $n = n^M$. Finally, a SLME exists under two conditions: (i) gaps must be positive and below the size of the monopoly market areas and (ii) profitability of spatial monopolies cannot be to high in order to deter entry. Using Eq. (10), the upper bound on the *UPR* in a SLME with n^M firms can also be expressed as[4]

$$\frac{\Pi^m - F}{F} \leq \frac{\left(n^M v - t\right)^2}{t(2n^M v - t)} \tag{11}$$

Observe that the profitability of spatial monopolists is not limited when entry is restricted: for given v and t, $(\Pi^m - F)/F$ goes to infinity as F goes to zero.

3.3 The case of the single spatial monopolist

Let us assume now that the only integer that satisfies equation (2) is $n^M = 1$. This is the case when v and t are such that:

$$\frac{1}{2} < D^m = \frac{v}{t} \leq 1 \tag{12}$$

The post-entry structure then consists of two equally spaced firms, each having a market size of 1/2. By applying equation (5) and equation (6) with $n = 2$, we can show the following result.

Lemma 2 *The post-entry price equilibrium is competitive when* $\frac{3}{4} \leq \frac{v}{t} \leq 1$ *(small gap) and is kinked when* $\frac{1}{2} \leq \frac{v}{t} \leq \frac{3}{4}$ *(large gap).*

The intuition behind Lemma 2 is clear. When the total size of the market is close to the monopoly market area, competition is fierce in the post entry structure, whereas when the size of the gap is close to the monopoly market area, post-entry competition is less intense and leads to a kinked equilibrium.

When the post-entry structure is competitive, the price equilibrium is $p^c = t/2$ and the profit per firm is $\Pi^c = t/4$ (see Salop, 1979). The condition for blockaded entry, $\Pi^c < F$, defines then a minimum value for the fixed cost equal to $F^{min} = t/4$. The maximum value for the *UPR* of the single spatial monopolist with a small gap is then $(\Pi^m - F^{min})/F^{min}$, which, after some computations, writes:

$$\frac{\Pi^m - F}{F} \leq 2\frac{v^2}{t^2} - 1 \tag{13}$$

The case when the post entry equilibrium is kinked can be analyzed as in section 3.2. When the gap is large, we thus have a bound on the *UPR* which is given by equation (11) with $n^M = 1$, i.e.,

$$\frac{\Pi^m - F}{F} \leq \frac{2v - t}{t(4v - t)} \tag{14}$$

[4] Observe that this expression depends only on v and t.

Observe that the change from the cases of competitive and kinked post-entry market structures is continuous: at $v/t = 3/4$, equation (13) and equation (14) both give a maximum *UPR* equal to *1/8*. The following proposition summarizes our result.

Proposition 2 *(the case of the single spatial monopolist) Let v and t satisfy equation (12), so that the only candidate SLME has $n^M = 1$. A SLME with exactly one firm exists if and only if the UPR of the incumbent monopolist is bounded above by equation (13) when $\frac{3}{4} \leq \frac{v}{t} \leq 1$ and by equation (14) when $\frac{1}{2} \leq \frac{v}{t} \leq \frac{3}{4}$.*

4 Discussion of the results

The SLME is very different from the monopoly SZPE of Salop. First, a SLME exists for every value of the parameters v and t, whereas the monopoly SZPE exists only when $v = \sqrt{2tF}$. As we have already mentioned, this deficiency of the monopoly SZPE concept comes from the fact that gross *and* net profits of spatial monopolists do not depend on the total number of firms in Salop's analysis. By carefully examining the post entry market structure, we are able to solve this problem and to relate the number of spatial monopolists that can operate under free entry to their *net* profitability.

The SLME has also more economic content than the monopoly SZPE. First, for a given number of firms n^M, the profitability of incumbent monopolists is explicitly linked with the size of the uncovered market areas. From equations (10), (13) and (14), we can derive the following property (see Figure 1):

Corollary 1 *For fixed n^M, the larger the gaps are, the smaller is the maximum UPR for incumbent monopolists. When gaps reach their maximum value ($d = D^m/n^M$), local monopolists must operate with zero profits to deter entry.*

$[Insert\ Figure\ 1\ here]$

When we let n^M vary, a second relation between product proliferation and the maximum *UPR* of local monopolists obtains. Consider for this the upper bound on the *UPR* as a function of the number of firms at the SLME with no gaps, which writes (let $t/v = n^M$ in equation (11)):

$$\frac{\Pi^m - F}{F} \leq \frac{1}{n^M(n^M + 1)} \tag{15}$$

The right hand side of equation (15) is decreasing in n^M and goes to zero as n^M goes to infinity. Since, from Corollary 1, the upper bound on the *UPR* at the SLME with no gaps is the maximum profitability in a SLME with n^M firms, we can derive the following property.

Corollary 2 *(the happy few spatial monopolists) The maximum UPR for incumbent monopolists decreases as n^M increases. At the limit, when n^M goes to infinity, monopolists operate with zero profits.*

We thus have a second link between the number of incumbents and their maximum profitability.[5] The intuition behind this result is that relocation of incumbents upon entry gives the entrant access to a total mass of previously unserved consumers which is increasing in n^M. Thus the larger n^M, the more likely it is that entry is profitable. For very large n^M, even for very small gaps, this relocation effect dominates and net profits must go to zero in order to deter entry.

Table 1 illustrates the result of the 'happy few spatial monopolists'. The relatively important decrease in profitability when n^M changes from 1 to 2 again comes from the fact that relocation plays a role only when $n^M \geq 2$.

[*Insert Table* 1 *here*]

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[5] Observe that allowing n^M to increase does not imply that gross monopoly profits must decrease or even go to zero as n^M goes to infinity. Indeed, although increasing n^M certainly imposes decreasing v/t, it may well be the case that $\Pi^m = v^2/2t$ increases and goes to infinity as n^M goes to infinity (take for instance $v = kt^{2/3}$, k constant, and let t go to infinity).

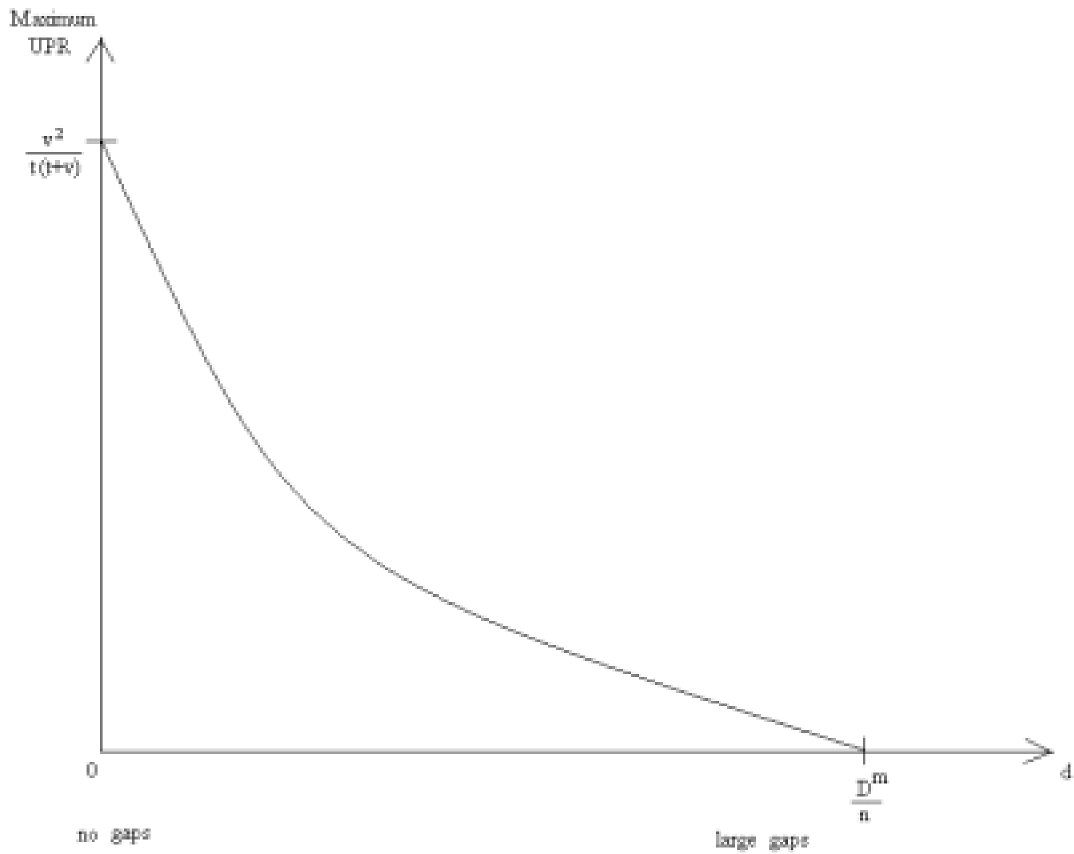

Figure 1: The Unit Profit Rate (UPR) of local monopolies ($n^M \geq 2$)

Number of firms n^M	Maximum value of the UPR
1	100%
2	12.5%
3	6.6%
4	4.2%
5	2.9%
6	2.1%
10	0.8%
100	0.01%

Table 1: Maximum Unit profit Rate (UPR) and number of firms at the SLME