ECONOMICS BULLETIN

Investment and the tragedy of the commons

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Abstract

This note studies the exploitation of a common proerty resource when firms choose a technology before exploiting the resource. The sequential choice leads to overinvestment. Moreover, exploitation levels are higher than they would be if technology and labour input were chosen simultaneously. The tragedy of the commons is aggravated.

I would like to thank Pio Baake for helpful comments.
Citation: Borck, Rainald, (2001) "Investment and the tragedy of the commons." *Economics Bulletin,* Vol. 4, No. 2 pp. 1−6
Submitted: February 1, 2001. **Accepted:** March 19, 2001.
URL: http://www.economicsbulletin.com/2001/volume4/EB−01D60001A.pdf

1. Introduction

Common property resources are subject to the well known "tragedy of the commons" (Hardin, 1968). If I fish on the open sea, I diminish your catch, and the neglect of this externality leads to overfishing.

In this note, we show that the problem may get worse when firms using common pool resources first choose an investment which determines the productivity of their labour input. For instance, farmers have to make substantial investments in tractors, milking machines, and other equipment before grazing their cows on the commons; fishermen must buy vessels before going out to fish. To a large extent, these investments are irreversible.

The literature has implicitly assumed that the choice of technology is made simultaneously with the choice of factor input. However, when firms commit to a technology before choosing inputs, there is an additional externality: When my labour force gets more productive, I will exploit the resource more and thereby reduce the profits of all other firms. Hence, the level of investment is too high from a social point of view.

The idea that commitment changes the incentives of players is of course familiar in economics. Applications range from bargaining (Schelling, 1960) to industrial organisation (e.g., Fershtman and Judd, 1987), strategic trade policy (Brander and Spencer, 1985), and monetary policy (Rogoff, 1985). Konrad (1994) shows that individuals may strategically choose to be poor in order to free ride on others' contributions in a game of private provision of public goods. Similar reasoning is applied to the common pool problem in this note.

2. The Model

Suppose for simplicity there are two firms which use a common property resource, say, a fishing ground. The total catch, X, is a function of aggregate labour employed, $L := L_1 + L_2$, where L_i is labour input of firm i: $X = F(L), F' > 0, F'' < 0$. Firm i's profit depends on labour supplied and on its productivity parameter, θ_i. Let $c(\theta_i)$ be the cost of productivity increases, with $c(0) = 0, c', c'' > 0$. Firm i's profit function is:

$$\pi_i = \theta_i L_i \frac{F(L)}{L} - wL_i - c(\theta_i), \tag{1}$$

where w is the constant wage rate. The assumption that firms' catch rates are proportional to their input levels is made for convenience and is common in the literature.

Suppose first that the choices of labour and technology are made simultaneously. The first order conditions are:

$$\frac{d\pi_i}{dL_i} = \theta_i \left(\frac{L_i}{L} F' + \frac{L_j}{L} \frac{F}{L} \right) - w = 0, \tag{2}$$

$$\frac{d\pi_i}{d\theta_i} = L_i \frac{F}{L} - c' = 0. \tag{3}$$

These equations characterize the Nash equilibrium of the simultaneous move game. The resulting equilibrium levels of L and θ will be denoted by (L^N, θ^N), where $\theta := \theta_1 + \theta_2$ is the aggregate productivity parameter. We will concentrate throughout the paper on symmetric equilibria.

3. Sequential Choice

Suppose now that firms commit to a technology before deciding on the level of exploitation of the common pool resource. We now have a two stage game: At the first stage, the irreversible technology choice is made, and at the second stage, each firm chooses how much labour to employ for a given technology. Assuming complete information, we solve backwards for a subgame perfect equilibrium.

The first order condition at the second stage, for given technology choices, is completely analogous to (2). Total differentiation shows that the reaction functions, $R_i := L_i(L_j, \theta_i, w)$, are downward sloping. Let $L_1(\theta_1, \theta_2, w)$ and $L_2(\theta_1, \theta_2, w)$ be the resulting equilibrium levels of labour for given technology choices. Substitute into the profit function to get the "indirect profit function" $\Pi_i(\theta_1, \theta_2, w)$. We assume that the equilibrium is locally stable. This implies $dL_i/d\theta_i > 0, dL_j/d\theta_i < 0$.

Firm i's technology choice then satisfies the first order condition

$$\frac{d\Pi_i}{d\theta_i} = \theta_i \frac{L_i}{L} \left(F' - \frac{F}{L} \right) \frac{dL_j}{d\theta_i} + L_i \frac{F}{L} - c' = 0. \tag{4}$$

Denote the outcome of the sequential game by (L^S, θ^S).

Proposition 1. *At the subgame perfect equilibrium of the sequential game, technology choices and labour inputs are larger than at the Nash equilibrium of the simultaneous move game.*

Proof. Since $F' < F/L$ and $dL_j/d\theta_i < 0$, at the sequential equilibrium, we have

$$L_i \frac{F}{L} - c' < 0. \tag{5}$$

The LHS of this inequality corresponds to $d\pi_i/d\theta_i$ with simultaneous technology and input choices. Since $d^2\pi_i/(d\theta_i dL_i) > 0$, the result follows. ∎

The welfare consequences are readily deduced. Let $\pi := \pi_1 + \pi_2$ be aggregate profit. Suppose $L_1 = L_2$, and $\theta_1 = \theta_2$. We have

$$\frac{d\pi}{dL} = \theta F' - w, \tag{6}$$

$$\frac{d\pi}{d\theta} = F - 2c'. \tag{7}$$

Let L^* and θ^* be the choice of labour and technology where (6) and (7) are zero.

Note that, conditional on the suboptimal labour choice L^N, θ^N is optimal from the social point of view. This is because in the simultaneous move game, there is no technological spillover between the firms. In the sequential game, evaluate (4) at θ^*, using symmetry:

$$\left.\frac{d\Pi_i}{d\theta_i}\right|_{\theta^*} = \frac{\theta_i}{2}\left(F' - \frac{F}{L}\right)\frac{dL_j}{d\theta_i} > 0. \tag{8}$$

Thus, firms have an incentive to overinvest in technology.

Proposition 2. *Aggregate profits are higher at the Nash equilibrium of the simultaneous move game than at the subgame perfect equilibrium of the sequential game.*

Proof. The assumptions on F and c imply that both (6) and (7) are negative iff $L > L^*$ and $\theta > \theta^*$. Since $L^S > L^N > L^*, \theta^S > \theta^N > \theta*$, the result follows. ∎

4. Policy

An obvious implication is that the sequential move game requires different policies to internalize the externalities. The standard prescription is a unit tax on labour. The firm's optimisation problem is then

$$\max \pi_i = \theta_i L_i \frac{F(L)}{L} - (w + t_i) L_i - c(\theta_i). \tag{9}$$

Let us assume firms are symmetric, and the equilibrium is symmetric too, i.e., $L_1 = L_2, \theta_1 = \theta_2$. Then we have the following.

Proposition 3. *In the simultaneous move game, the optimal tax rate just equals (minus) the marginal externality (i.e., loss of profits) imposed on the other firm:*

$$t_i^* = -\frac{d\pi_j}{dL_i} = -\theta_j \frac{L_j}{L} \left(F' - \frac{F}{L} \right) > 0. \tag{10}$$

Proof. The firm's first order conditions are

$$\begin{aligned} \frac{d\pi_i}{dL_i} &= \theta_i \left(\frac{L_i}{L} F' + \frac{L_j}{L} \frac{F}{L} \right) - (w + t) = 0, && (11) \\ \frac{d\pi_i}{d\theta_i} &= L_i \frac{F}{L} - c' = 0. && (12) \end{aligned}$$

Inserting (10) into (11), using symmetry, gives the result. ■

In the sequential move game, the formula remains valid, *conditional* on the choice of technology. If there is no corrective policy to ensure optimal investment, however, the taxation of labour according to this rule will not achieve the first best. In fact, it needs to be supplemented by a tax on investment. Let τ be the unit tax rate on investment. The firm's problem now is

$$\max \pi_i = \theta_i L_i \frac{F(L)}{L} - (w + t_i) L_i - c(\theta_i) - \tau_i \theta_i. \tag{13}$$

Proposition 4. *The optimal tax system in the sequential move game consists of tax rates t_i^*, τ_i^*, where*

$$\tau_i^* = \frac{d\Pi_j}{d\theta_i} \frac{dL_j / d\theta_i}{dL_i / d\theta_i} = \theta_i \frac{L_i}{L} \left(F' - \frac{F}{L} \right) \frac{dL_j}{d\theta_i} > 0. \tag{14}$$

Proof. The first order condition for the first stage is now

$$\frac{d\Pi_i}{d\theta_i} = \theta_i \frac{L_i}{L} \left(F' - \frac{F}{L} \right) \frac{dL_j}{d\theta_i} + L_i \frac{F}{L} - c' - \tau_i = 0. \quad (15)$$

Substitute (14) into (15), which shows that with this tax rate, (7) is zero, and hence, the individual technology choice is socially optimal. Differentiating Π_j gives

$$\frac{d\Pi_j}{d\theta_i} = \theta_j \frac{L_j}{L} \left(F' - \frac{F}{L} \right) \frac{dL_i}{d\theta_i} < 0, \quad (16)$$

which, using symmetry, gives the result. ■

Note that, since labour input is a function of θ, one really needs a tax on labour and technology: While one might set a tax rate, say, $\hat{\tau}$, which would result in $L(\theta(\hat{\tau})) = L^*$, the corresponding technology level would not equal θ^*. Likewise, a tax on labour alone would not suffice.

5. Conclusion

This note has shown that there is an extended tragedy of the commons when firms make irreversible investments before choosing their input levels. An important assumption was that technology is a private good. Many technological improvements, however, have clear public good properties. For instance, the benefits of research and development activities are often available to all firms in an industry (Kamien, Muller and Zang, 1992, study the incentives of firms to internalize these externalities in cartels or joint ventures). If this is the case, there is a tendency to underinvest in technology, which would counteract the tendency of overexploitation. On the other hand, there may also be negative externalities in R & D patent races (see Reinganum, 1989, for an overview): If I am the first to get a patent on an invention, you can't get it. This externality is akin to the common pool problem. This would lead to overinvestment and therefore make the tragedy even more tragic.

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