

## Shipping the good apples out: a note on contributions of time and money

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### *Abstract*

This note applies insights from a famous problem in price theory, "Shipping the Good Apples Out," to expand the comparative static predictions of the model of monetary donations and volunteering developed by Andreoni, Gale and Scotz (1996). We show that the Hicksian supply of volunteering may be an increasing function of the wage, if volunteering is strongly complementary to either donations or consumption. It is also possible for government charity to crowd-in private contributions.

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## 1 Introduction

People make charitable contributions of both time and money. Most empirical studies of charitable giving have focused on either donations of money or donations of time, but not both.<sup>1</sup> Over the last decade, as data on volunteer labor supply have become more accessible, several empirical papers have estimated donations and volunteering simultaneously [Menchik and Weisbrod (1987), Brown and Lankford (1992), Callen (1994)]. However, only recently have theoretical models of giving and volunteering been developed. Duncan (1999) provides a public goods model where the presence of labor supply allows imperfect crowding-out of donations. Andreoni, Gale, and Scholz (1996, henceforth AGS) develop and estimate what we take to be the canonical model of warm-glow contributions of time and money.<sup>2</sup>

This note extends and elaborates upon the comparative statics of the AGS model. AGS consider a consumer who derives utility from an aggregate consumption good, donations, volunteering, and leisure. However, they assume in their theoretical model that the cross-partial derivatives of the utility function are all zero (AGS footnote 6, p. 6).<sup>3</sup> Although this allows sharp predictions, it obscures some interesting possibilities. When there are more than two goods in the utility function, the demand for one good will be affected not only by its degree of substitutability or complementarity with each of the other goods, but also by the degree of substitutability or complementarity of the other goods with each other. A full accounting of the comparative statics of the model must capture the “indirect,” as well as the “direct,” interactions across goods. This is not to say, “anything can happen” without further restrictions on preferences. “Indirect” feedback between goods lies at the heart of a famous problem in price theory, first posed by Alchian and Allen (1969), “Shipping the Good Apples Out.” We exploit an unexpected isomorphism between the mathematics of this problem [Gould and Seagall (1969), Borchering and Silberberg (1978)] and the AGS model. The model yields sufficient conditions to sign comparative static derivatives that may suggest cross-equation restrictions that may be of use in simultaneous estimation of donations and volunteering.

The outline of the paper is as follows. Section I lays out the model. In order to abstract from income effects, Section II characterizes the Hicksian supplies of donations and volunteering and donations. In Section II.A we establish a surprising proposition: an increase in the wage may actually *increase* the Hicksian supply of volunteering, provided that volunteering is sufficiently complementary with either donations *or* the consumption good. In Section II.B we also show that government provision of charity need not crowd out private volunteering or donations. It is possible for an increase in government charity to *increase* the Hicksian supply of donations. Section IV offers some concluding thoughts.

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<sup>1</sup> Empirical studies of monetary contributions include Reece (1979), Reece and Zieschang (1985), Feenberg (1987), Kingma (1989), Randolph (1995). Empirical studies of volunteer labor supply include Dye (1980), Segal (1992), Segal, Mauser, and Weisbrod (1992), Menchik and Weisbrod (1987).

<sup>2</sup> There is now a large literature on warm-glow giving. See Cornes and Sandler (1984,1986), Schiff (1985), Posnett and Sandler (1986), Sandler and Posnett (1991), Andreoni (1989,1990), Menchik and Weisbrod (1987), Kingma (1989), Lucas and Stark (1985), Steinberg (1986,1987) and Harbaugh (1998).

<sup>3</sup> In their empirical model, AGS assume that preferences are quadratic.

## 2 The Model

Imagine a person with the utility function

$$U = U(x, m, h, \ell; g). \quad (1)$$

$x$  is a consumption good, a composite commodity of goods and services purchased on the market.  $m$  is the quantity of charitable goods financed by the consumer's donations.  $h$  is the number of hours she spends in volunteering activities.  $\ell$  is the number of hours she spends in household activities, which we will call "home leisure."  $g$  denotes government provision of charity. The consumer derives a "warm-glow" from giving both money  $m$  and time  $h$ . For the moment, we merely assume that  $U(\bullet)$  is twice-differentiable in all of its arguments, and increasing and quasi-concave in  $x, m, h$ , and  $\ell$ . We will need to make more restrictive assumptions later, in order to arrive at specific comparative static results. Our focus in this paper is solely on the choice problem of an individual consumer, so we abstract from strategic interactions.<sup>4</sup>

Suppose that the price of the aggregate consumption good is  $p_x$ , the price of donations is  $p_m$  (equal to 1 minus the marginal tax rate on charitable deductions), and  $w$  is the wage (all measured in dollars). Non-wage income is  $y$ . If the consumer has a total of  $H$  available hours, her budget constraint is<sup>5</sup>

$$wH + y = p_x x + p_m m + w(h + \ell) \quad (2)$$

Notice that volunteering  $h$  and home leisure  $\ell$  have the same nominal opportunity cost,  $w$ . In other words, when the consumer chooses not to work an additional hour, she sacrifices the same amount regardless of whether she volunteers or enjoys home leisure. This has the important implication that the sum of volunteering and home leisure can be thought of as a composite commodity. We will call this composite commodity "total leisure" (in the sense that it is the total time spent on non-market activities),  $L = h + \ell$ . This fact will prove useful in developing the predictions of the model mathematically, since a change in  $w$  changes *two* nominal prices.

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<sup>4</sup> Our formulation of preferences differs from that of AGS in two respects. First, they assume that utility depends upon the value of volunteering to the charity, rather than the number of hours volunteered. Second, they do not incorporate government charity into their model. Menchik and Weisbrod (1987) wrote down a version of this model. However, they only derived the first-order conditions, and did not develop the comparative statics of the model. Schiff (1990) discusses donations and volunteering as different aspects of charitable behavior, but develops a formal model only of donations (Chapter 2).

<sup>5</sup> Given the monotonicity of the utility function, the budget constraint will be binding at the optimum, so we write it as an equality.

### 3 Hicksian Demand and Supply Functions

Since the burden of our argument will rest upon *substitution* between volunteering, donations, and consumption, it will be useful to abstract from income effects entirely. To this end, consider the expenditure minimization problem

$$\min_{x, m, h, \ell} \quad p_x x + p_m m + w(h + \ell) - wH - y \quad \text{s.t.} \quad \bar{U} = U(x, m, h, \ell; g) \quad (3)$$

If  $e(p_x, p_m, w)$  is the expenditure function associated with this problem, then Shephard's Lemma leads to the Hicksian demand for consumption and the Hicksian supplies for donations and labor

$$m(p_x, p_m, w) = \frac{\partial e}{\partial p_m}, \quad (4)$$

$$L(p_x, p_m, w) - H = h(p_x, p_m, w) + \ell(p_x, p_m, w) - H = \frac{\partial e}{\partial w} \quad (5)$$

Notice that Shephard's Lemma allows us to recover the Hicksian demand for total leisure  $L(p_x, p_m, w)$ , but not its constituent parts, volunteering  $h(p_x, p_m, w)$  and home leisure  $\ell(p_x, p_m, w)$ .

#### 3.a Donations, Volunteering, and Their Prices

Ignoring the components of the composite commodity, our model is identical to the standard textbook model of consumer behavior. This means that the expenditure function has all of the usual properties [see Varian (1995, p. 72-73), for example]. In particular, it is strictly concave in  $p_x$ ,  $p_m$ , and  $w$ , so we may invoke the standard result:

**Proposition 1.** *The own-price effects are non-negative. That is,*

$$\frac{\partial m}{\partial p_m} = \frac{\partial^2 e}{\partial p_m^2} \leq 0, \quad (6)$$

$$\frac{\partial L}{\partial w} = \frac{\partial^2 e}{\partial w^2} \leq 0. \quad (7)$$

AGS [Equation (3), p. 6] derive the Marshallian version of (6), showing that the uncompensated supply of donations is decreasing in its price. Notice, that (7) does *not* imply that either the Hicksian supply of volunteering  $h(p_x, p_m, w)$  or the Hicksian demand for home leisure  $\ell(p_x, p_m, w)$  will not increase when the wage increases; it only asserts that the *sum* of these non-market activities cannot increase. We will have to resort to other arguments to uncover the effects of a change in the wage on volunteering and home leisure.

How does an increase in the wage affect the Hicksian supply of volunteering? Let us answer this question by posing another, seemingly unrelated question: Suppose that two types of apples are produced in one locality and exported to another locality. They differ in quality, so that the better apples fetch a higher price to the producer. There is a common, constant cost of transporting both types of apples to the market. Question: if transportation costs increase, what happens to the relative demands for the two types of apples at the market? Is it possible for the relative demand for the good apples to increase, so that more of the good apples get exported from the producing locality (to the chagrin of apple lovers in the producing locale). This is a classic problem in price theory - "Shipping the Good Apples Out" - variants of which have appeared in textbooks for decades [Alchian and Allen (1969), Stigler (1966), Hirshleifer (1976), Silberberg (1990)].

The textbook answer, offered by Alchian and Allen (1969), is that the demand for good apples must increase, since their price (including transportation costs) relative to the bad apples has decreased. We will call this the "direct" substitution effect.

Gould and Seagall (1969) challenged this argument by introducing a third, composite commodity in the budget set. In this case, an increase in transportation costs changes the relative prices of both types of apples with the third good. Demand for the good apples might increase or decrease, depending upon the extent to which consumers regard good apples as substitutes or complements for the third good.<sup>6</sup> We call this the "indirect" substitution effect.

*The problem of "shipping the good apples out" is isomorphic to our problem, except that the two "apples" (volunteering and home leisure) have the same dollar price (the wage), and that there are two other goods (consumption and donations). The rise in the wage has the same effect as an increase in transportation costs. Since the prices of the "apples" are the same, there is no "direct" substitution effect at all, but only an "indirect" substitution effect. The effects of the change in transportation costs (wages) thus depend upon the whether consumers view the two types of apples as substitutes or complements for the other goods in the budget set. Our problem might be described as "shipping the good times out." Is it possible that an increase in the wage will induce people to spend less time at home, and more time volunteering?*

Adapting the proof in Gould and Seagall (1969), we prove the following proposition in the Appendix:

***Proposition 2.*** *The Hicksian supply of volunteering may actually be an **increasing** function of the wage, provided that volunteering is a sufficiently strong Hicksian complement to either consumption or donations.*<sup>7</sup>

To provide some insight into when this paradoxical outcome is likely to occur, we adapt an argument employed by Borchering and Silberberg (1978) and Silberberg

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<sup>6</sup> Borchering and Silberberg (1978) defended the empirical relevance of the "Alchian and Allen Theorem" against the theoretical argument of Gould and Seagall (1969). Umbeck (1980) later criticized Borchering and Silberberg (1978).

<sup>7</sup> AGS (p. 6) note that the sign of the comparative static effect of the wage on the Marshallian supply of volunteering is ambiguous. They attribute this ambiguity to the usual offsetting income and substitution effects. In fact, even the slope of the Hicksian supply of volunteering is of ambiguous sign.

(1990, pp. 388). Since the Hicksian supply of volunteering is homogeneous of degree zero with respect to price of donations and the wage, Euler's theorem leads to the familiar elasticity restriction:

$$\varepsilon_{hw} + \varepsilon_{hp_m} + \varepsilon_{hp_x} = 0, \quad (8)$$

where  $\varepsilon_{hw}$ ,  $\varepsilon_{hp_m}$ , and  $\varepsilon_{hp_x}$  are the elasticities of  $h(p_x, p_m, w)$  with respect to  $w$ ,  $p_m$  and  $p_x$ , respectively. This means that if both consumption and donations are Hicksian substitutes for volunteering ( $\varepsilon_{hp_x} > 0, \varepsilon_{hp_m} > 0$ ) then the Hicksian supply of volunteering will unambiguously decrease when the wage increases ( $\varepsilon_{hw} < 0$ ). However if either consumption or donations is a Hicksian complement to volunteering ( $\varepsilon_{hp_x} < 0$  or  $\varepsilon_{hp_m} < 0$ ), then the Hicksian supply of volunteering will **increase** when the wage increases ( $\varepsilon_{hw} > 0$ ).

Using the same kind of reasoning, it is easy to establish that an analogous result holds for home leisure: if either consumption or donations is a Hicksian complement to home leisure, then the Hicksian demand for home leisure will **increase** when the wage increases.

Notice that these paradoxical possibilities -- that volunteering and home leisure may actually increase with the wage -- rest upon strong complementarities between volunteering and home leisure, on the one hand, and donations, on the other. These complementarities cannot be "too" strong, however, since -- according to Proposition 1 -- the Hicksian demand for total leisure (volunteering plus home leisure) must be a non-increasing function of the wage. In other words, either volunteering or home leisure may be an increasing function of the wage, but not both.

What are we to make of the counter-intuitive prediction that volunteering may be an increasing function of the wage? To develop some intuition for this result, consider a simpler model where there is no consumption, only donations, volunteering, and leisure. We have seen that if the wage increases, then total leisure  $L(p_x, p_m, w)$  will decrease as the consumer substitutes toward donations. This is where the analysis would stop in the standard model of leisure-consumption choice. Here, however, the increased donations may feedback to alter the marginal utilities of volunteering and home leisure in different ways. Suppose, on the one hand, that the increase in donations were to decrease the marginal utility of both volunteering and home leisure. In this case, the demand for both volunteering and home leisure would unambiguously decrease. Now suppose, on the other hand, that the increase in donations were to raise the marginal utility of volunteering, but lower the marginal utility of home leisure. Given the complementarity between volunteering and donations, the supply of volunteering actually *increases*. The same kind of intuition applies in our more complicated setting, except that the response of volunteering to the increase in wage depends upon the interaction of volunteering and home leisure with consumption, as well as donations.

### 3.b Private Philanthropy and Public Giving

We now analyze the impact of changes in public giving ( $g$ ) on the Hicksian provision of private philanthropy  $h(p_x, p_m, w)$  and  $m(p_x, p_m, w)$ . In general, it is hard to say much about the comparative static effects of changes in  $g$ , since, in principle, a change

in  $g$  may alter the marginal utilities of all four of the goods, as well as confer utility itself. We impose the plausible simplifying assumption that changes in  $g$  do not affect the marginal utilities of consumption or home leisure ( $U_{xg} = U_{lg} = 0$ ). However, we will allow  $g$  to affect the marginal utilities of the two philanthropic activities available to the consumer, donations and volunteering. Finally, we assume that the direct effect of  $g$  on utility ( $U_g$ ) is fairly small. In keeping with the partial-equilibrium flavor of the model, we ignore the question of how the government finances these expenditures. In particular, we assume that government expenditures are not funded by taxes on the consumer.

Consider two cases. First, imagine that an increase in  $g$  decreases the marginal utility of donations, but has no impact on the marginal utility of volunteering. In the Appendix we prove

**Proposition 3.** *Suppose that  $U_g > 0$  is small, that  $U_{xg} = U_{hg} = U_{lg} = 0$  but that  $U_{mg} < 0$ . An increase in government charity will unambiguously decrease the Hicksian supply of private donations,  $\partial m / \partial g < 0$ . It will increase the supply of volunteering,  $\partial h / \partial g > 0$ .*

The intuition for this result is straightforward. An increase in public giving reduces the marginal rate of substitution of giving money relative to volunteering, causing the consumer to substitute away from donations toward volunteering as an outlet for her philanthropic tastes.<sup>8</sup>

A more complicated example allows the marginal utilities of both donations and volunteering to be affected by the change in  $g$ . In the Appendix we prove

**Proposition 4.** *Suppose that volunteering and donations are Hicksian substitutes, that  $U_g > 0$  is small, that  $U_{xg} = U_{lg} = 0$ , but that  $U_{mg} < 0$  and  $U_{hg} < 0$ . It is possible for an increase in government charity to increase the Hicksian supply of private donations, so  $\partial m / \partial g > 0$ . It may increase or decrease the supply of volunteering  $\partial h / \partial g ? 0$ .*

## 5 Conclusion and Discussion

This note fleshes out the comparative statics of the AGS model by applying insights from a classic problem in price theory, “Shipping the Good Apples Out.” We establish two paradoxical theoretical possibilities.

First, the Hicksian supply of volunteering may theoretically be positively sloped. It is of course possible for the Marshallian supply of volunteering to increase with the wage if volunteering is a normal good. Our argument is that complementarity between volunteering and consumption may make the Marshallian supply of volunteering become positively sloped, even if volunteering is an inferior good. At first glance, this may seem

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<sup>8</sup> Actually, there is a bit more to the story. An increase in  $g$  causes a direct increase in utility ( $U_g > 0$ ).

This amounts to an “in-kind” transfer of utility that shifts the consumer to a higher indifference contour (analogous to an increase in output in a cost minimization problem). This generally tends to increase the demand for all goods, even if the marginal rates of substitution between them do not change (assuming this is true is analogous to assuming that factors of production in a cost minimization problem are not inferior).

to be more than a theoretical curiosity, since Menchik and Weisbrod (1987), Callen (1994), and Brown and Lankford (1992) find giving and volunteering to be Marshallian complements. AGS also find contributions of time and money to be Marshallian complements. However, their estimates also suggest that (1) giving and volunteering are (weak) Hicksian substitutes, and (2) volunteer labor supply is in fact a decreasing function of the wage.

Second, it is also possible for government provision of charity to cause private donations to increase. That is, there may actually be crowding-in. There has been ample evidence of partial crowding-out in monetary donations [Abrams and Schitz (1978), Roberts (1984), Khanna, Posnett, and Sandler (1995), Kingma (1989), Steinberg (1985), Reece (1979), and Schiff (1985)]. However, Khanna and Sandler (2000) have recently demonstrated evidence of crowding-in of government grants in the UK.

While neither of these paradoxical outcomes has much empirical support, they highlight the complex web of interactions that is hidden beneath the surface of such a simple model. Our theoretical results may suggest cross-equation restrictions -- such as the elasticity condition in equation (8) -- in empirical estimation of contributions of time and money.



**Appendix**  
**Derivation of Propositions 1, 2, and 3**

Our derivation follows that of Gould and Seagall (1969) closely.

In order to highlight the “direct” and “indirect” effects of a change in the wage, suppose initially that the opportunity costs of volunteering and home leisure are not equal. For example, volunteering may offer some small compensation ( $s$ ) per hour to cover transportation costs, so that if the market wage is  $w$ , then  $w = w_\ell$  and  $w_h = w_\ell - s$ . It will also save space to define  $w_\ell = p_\ell$  and  $w_h = p_h$ . The budget constraint then becomes

$$p_\ell H + y = p_x x + p_m m + p_h h + p_\ell \ell. \quad (\text{A.1})$$

The Lagrangian for the minimization problem in Equation (6) of the text becomes

$$L = p_x x + p_m m + p_h h + p_\ell \ell - p_\ell H - y + \lambda [\bar{U} - U(x, m, h, \ell; g)]. \quad (\text{A.2})$$

The first-order conditions are

$$P_i = \lambda U_i, \quad i = x, m, h, \ell \quad (\text{A.3})$$

$$\bar{U} = U(x, m, h, \ell; g). \quad (\text{A.4})$$

The bordered Hessian of the Lagrangian is:

$$\begin{bmatrix} -\lambda U_{xx} & -\lambda U_{xm} & -\lambda U_{xh} & -\lambda U_{x\ell} & -U_x \\ -\lambda U_{mx} & -\lambda U_{mm} & -\lambda U_{mh} & -\lambda U_{m\ell} & -U_m \\ -\lambda U_{hx} & -\lambda U_{hm} & -\lambda U_{hh} & -\lambda U_{h\ell} & -U_h \\ -\lambda U_{\ell x} & -\lambda U_{\ell m} & -\lambda U_{\ell h} & -\lambda U_{\ell\ell} & -U_\ell \\ -U_x & -U_m & -U_h & -U_\ell & 0 \end{bmatrix}, \quad (\text{A.5})$$

where  $\lambda > 0$  is the Lagrange multiplier. The sufficient second-order conditions are that the naturally ordered principal minors of this matrix be negative [Varian (1995, pp. 499-500)].

Let  $M < 0$  be the determinant of the bordered Hessian, and let  $M_{ij}$  be the cofactor for the element in the  $i$ th row and  $j$ th column in the bordered Hessian,  $i, j = x, m, h, \ell, \lambda$ . It will also save space to write the vector of endogenous variables as  $[x, m, h, \ell] = [X_x, X_m, X_h, X_\ell]$ . Using the notation of Gould and Seagall (1969), the substitution effect of a change in price  $j$  on good  $i$  is then

$$\frac{\partial X_j}{\partial P_i} = -\frac{M_{ij}}{m} = K_{ji}. \quad (\text{A.6})$$

Now suppose that the prices of both volunteering and home leisure increase by the same amount  $dp_h = dp_\ell = dw > 0$  (the market wage increases). The resulting change in the Hicksian supply of volunteering is

$$dX_h = (K_{hh} + K_{h\ell})dw. \quad (\text{A.7})$$

$M < 0$  by the second-order conditions, and  $M_{hh} < 0$  since it is a border-preserving principle minor. Therefore  $K_{hh} = -M_{hh} / M < 0$ . However,  $K_{h\ell}$  cannot be signed.

Intuitively,  $K_{hh}$  captures the standard “own-price” substitution effect of a wage increase on volunteering: it is unambiguously negative. Unlike the standard, two-good model, however, an increase in the wage increases raises the prices of *two* goods, volunteering and home leisure.  $K_{h\ell}$  captures the complex interactions between volunteering, home leisure, donations, and consumption caused by this second price change; it need not be negative.

To see when  $dX_h$  may be positive, we follow Gould and Seagall (1969) in invoking Hicks’ third law [Silberberg (1990, pp. 342-343)]:

$$p_x K_{xh} + p_m K_{mh} + p_h K_{hh} + p_\ell K_{\ell h} = 0. \quad (\text{A.8})$$

Since  $K_{h\ell} = K_{\ell h}$ , we can use (A.8) to write (A.7) as

$$dX_h = \left[ K_{hh} \left( 1 - \frac{p_h}{p_\ell} \right) - \frac{p_x}{p_\ell} K_{xh} - \frac{p_m}{p_\ell} K_{mh} \right] dw. \quad (\text{A.9})$$

This corresponds to Equation (7a) in Gould and Seagall (1969, p. 134), except that they consider the case of only three goods.

In our model, the opportunity costs of volunteering and donations are the same, so that  $p_h = p_\ell = w$ . This causes the first term in (A.9) – the “direct” substitution effect – to disappear, so that we are left with

$$dX_h = - \left[ \frac{p_c}{p_\ell} K_{xh} + \frac{p_m}{p_\ell} K_{mh} \right] dw. \quad (\text{A.10})$$

If both consumption and donations are substitutes for volunteering ( $K_{xh} > 0$  and  $K_{mh} > 0$ ), then volunteering must decrease with the wage. However, if either consumption or donations is a sufficiently strong complement with volunteering ( $K_{xh} \ll 0$  or  $K_{mh} \ll 0$ ), then it is possible for an increase in the wage to increase volunteering. This proves Proposition 1.

Now consider the effects of an increase in public charity,  $g$ . Suppose that this decreases the marginal utilities of donations and volunteering ( $U_{mg} < 0, U_{hg} < 0$ ) but has no impact on the marginal utilities of consumption or home leisure ( $U_{xg} = U_{\ell g} = 0$ ). An increase in  $g$  does confer utility itself ( $U_g > 0$ ). The comparative static effects of the change in  $g$  on the Hicksian supplies of donations and volunteering are then:

$$dX_m = -\left\{\lambda\left[U_{mg}K_{mm} + U_{hg}K_{mh}\right] + U_gK_{\lambda m}\right\}dg, \quad (\text{A.11})$$

$$dX_h = -\left\{\lambda\left[U_{mg}K_{hm} + U_{hg}K_{hh}\right] + U_gK_{\lambda h}\right\}dg. \quad (\text{A.12})$$

The own-price substitution effects are negative ( $K_{mm} < 0, K_{hh} < 0$ ). We will assume that volunteering and donations are substitutes, so that the cross-price substitution effects are positive ( $K_{hm} = K_{mh} > 0$ ). We also assume that  $K_{\lambda m} < 0$  and  $K_{\lambda h} < 0$  (Since  $K_{\lambda i} = -M_{i\lambda} / M$ , this requires  $M_{i\lambda} > 0$ , which is analogous to assuming that the factors of production in a cost minimization problem are not inferior,).

Consider the two cases discussed in the text. First, suppose that an increase in  $g$  increases the marginal utility of donations, but has no effect on the marginal utility of volunteering ( $U_{mg} < 0, U_{hg} = 0$ ). In this case

$$dX_m = -\left[\lambda U_{xg}K_{mm} + U_gK_{\lambda m}\right], \quad (\text{A.13})$$

$$dX_h = -\left[\lambda U_{xg}K_{hm} + U_gK_{\lambda h}\right]. \quad (\text{A.14})$$

Assuming that the direct effect of  $G$  on utility ( $U_g$ ) is small, public charity crowds-out private donations, but stimulates volunteering (Proposition 3).

Second, consider the more complicated case where an increase in  $g$  decreases the marginal utilities of both donations and volunteering ( $U_{xg} < 0, U_{hg} < 0$ ). From (A.11) and (A.12), it is clear that if  $U_{hg}$  is large enough in absolute value, it is possible that  $dX_m > 0$  and  $dX_h < 0$  (Proposition 4).

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