E C O N O M I C S   B U L L E T I N

Non−separable utility, wealth effects, and economic growth in a monetary economy

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Abstract

This paper investigates the effects of wealth−enhanced social status using an optimizing monetary growth model with non−separable utility function between consumption and wealth. Within this framework, we first arrive a conclusion that, in the case of no wealth effects, an increase in the rate of money growth does not stimulate the steady−state growth rate. Moreover, in the case of existing wealth effects, we show that an increase in the rate of money growth has a negative effect on the long−run growth rate of the economy. This result is in sharp contrast with the typical conclusion of the relevant field.

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1. Introduction

This paper investigates the effects of wealth-enhanced social status in an endogenous growth economy. Using an optimizing monetary growth model with non-separable utility function and cash-in-advance constraint (hereinafter the CIA constraint) on consumption goods. The role of wealth-enhanced social status in an optimal growth model has been investigated, for example Wirl (1994), Zou (1994), Corneo and Jeanne (1997, 2001), and Futagami and Shibata (1998). The remarkable feature of these models is to define wealth-enhanced social status and wealth directly or indirectly in the agent's utility function.[1] This implies that individuals accumulate wealth not only for consumption, but also for its own sake. Consequently, the agent's preference should depend on one's wealth (capital) holdings as well as one's consumption. Such a linkage is usually called "wealth effects".[2] These studies, however, stand on the viewpoints of *real* rather than *monetary* aspects.

On the other hand, the effects of money growth on economic growth have long been a central issue in the macroeconomic literatures (e.g. Tobin, 1965; Sidrauski, 1967; Stockman, 1981). In particular, Marquis and Reffett (1991) are relevant to our studies. They incorporate money into the two-sector Uzawa-Lucas structure through the CIA constraint, and conclude that changes in the rate of money growth do not affect the long-run growth rate when only consumption or physical capital is liquidity constrained. Very few efforts, however, have been made to study the linkages among wealth accumulation through capital holdings, agent's preference, and monetary economy. In actuality, the capital stock contributes as an input to production but also reflects cultural assets that provide direct utility. Therefore, our important task in this paper is to integrate the above noted two research directions under more standard environment.

Our starting point is the model of Chang *et al*. (2000). They showed that an increase in the rate of money growth raises the long-run growth rate of the economy as well as an increase in the degree of wealth-enhanced social status. In the optimizing monetary growth models including Chang *et al*. (2000), however, the relation between preference structure and wealth-enhanced social status has not been fully investigated. This is because most of the models concerning growth and wealth effects assume simple utility functions that satisfy additive separability between consumption and wealth. The principal difference between these models and our present model is that we employ the non-separable utility function concerning consumption and wealth. Within this framework, we mainly investigate that the economic impact of an increase in the rate of

[1] This means that the accumulated wealth through capital holdings induces social status.
[2] See for example Kurz (1968).

money growth under existing wealth-enhanced social status. By this analysis, we first arrive a conclusion that if the effects of wealth-enhanced social status are disregarded, an increase in the rate of money growth does not affect the steady-state growth rate. In addition, if we consider wealth effects, changes in the rate of money growth decrease the rate of long-run growth. From the former result, it is found that the basic implications of Marquis and Reffett (1991) and Chang *et al*. (2000) are preserved even when employing the non-separable utility function. On the other hand, the latter result differs from conventional wisdom in the field of growth and wealth effects (e.g. Chang *et al*. (2000)). This is our significant contribution due to employ the broader class of utility function.

The rest of the paper is organized as follows. Section 2 presents the optimizing model of monetary growth with the AK type production technology. Section 3 summarizes some typical conclusions.

2. The model

In this section, we present an extended version of Chang *et al*. (2000) model. The economy consists of a representative agent and a government, with a perfect-foresight model of monetary growth in which the CIA constraint is imposed only on consumption. Labor force is inelastically supplied and normalized to one. The representative agent has the following additively non-separable utility function:

$$U = \int_0^\infty u(c,k)e^{-\rho t}dt, \qquad \rho > 0.$$

Following Jones and Manuelli (1997, p.78), the instantaneous utility function is given by the modified form of their model:

$$u(c,k) = \begin{cases} \dfrac{c^{1-\sigma}}{1-\sigma}k^{\beta}, & \sigma \geq 0 \\ \ln c \times k^{\beta}, & for\ \sigma = 1 \end{cases},$$

where $0 \leq \beta < 1$.[3] As in Barro (1990) and Rebelo (1991), we assume that the production function takes the familiar AK form,

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[3] In this class of instantaneous utility, another general form is $u(c,k) = \left\{(c^{1-\sigma} - 1)/(1-\sigma)\right\}k^{\beta}, \sigma \geq 0.$ Given this modified form of instantaneous utility, of course, Eq.(9) of the first-order conditions would be changed. As a result, the steady-state relationships in Eq.(18) would be also changed. However, this modification does not bring about us easily new expressions replaced with Eq.(18). More concretely, as we shall note later in the footnote 5, the three-dimensional system in λ_1, m, and k is not fully transformed by the new two-dimensional system in x and z. For this technical difficulty, which is due to non-separability of utility, we will not employ the above form of instantaneous utility in this paper. This technical problem needs further consideration.

$$y = f(k) = Ak, \qquad A > 0. \tag{1}$$

Therefore, the representative agent's optimization problem is

$$\max \; U = \int_0^\infty \frac{c^{1-\sigma}}{1-\sigma} k^{\beta} e^{-\rho t} dt, \tag{2}$$

subject to

$$\dot{m} = Ak + \tau - c - \pi m - i, \tag{3}$$

$$\dot{k} = i, \tag{4}$$

$$m \geq c, \tag{5}$$

where c, $m \; (= M / P)$, M, P, k, τ, $\pi \; (\equiv \dot{P} / P)$, i, $1/\sigma$ and ρ denote the real consumption, real cash balances, nominal money holdings, price level, physical capital stock, real lump-sum transfers from the government, the rate of inflation, investment, the intertemporal elasticity of substitution, and a rate of time preference, respectively. Moreover, a dot mark denotes the time derivative.

The utility function in Eq.(2) contains the remarkable feature of direct benefits derived from the capital stock holdings (i.e. wealth effects). Eqs.(3) and (4) are the budget constraint with M and k given at their initial values $M(0)$ and $k(0)$. Eq.(5) is the CIA constraint. This constraint implies that the agent's real cash balances must finance purchases of consumer goods.

To solve the corresponding dynamic optimization problem, we set up the current-value Hamiltonian. It is specified as

$$\mathcal{H}(c, k, m, i, \lambda_1, \lambda_2, \lambda_3, t) \equiv \frac{c^{1-\sigma}}{1-\sigma} k^{\beta} + \lambda_1 \left[Ak + \tau - c - \pi m - i \right] + \lambda_2 i + \lambda_3 (m - c),$$

where λ_1, λ_2, and λ_3 denote the co-state variables and the multiplier of the current-value Hamiltonian associated with Eqs.(3)-(5), and c and i are the control (choice) variables. The necessary conditions for an optimum are the following:

$$c: \qquad c^{-\sigma} k^{\beta} = \lambda_1 + \lambda_3, \tag{6}$$

$$i: \qquad \lambda_1 = \lambda_2, \tag{7}$$

$$m: \qquad -\lambda_1 \pi + \lambda_3 = -\dot{\lambda}_1 + \lambda_1 \rho, \tag{8}$$

$$k: \qquad \frac{c^{1-\sigma}}{1-\sigma} \beta k^{\beta-1} + \lambda_1 A = -\dot{\lambda}_2 + \lambda_2 \rho, \tag{9}$$

plus the usual two transversality conditions,

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)m(t) = 0, \quad \lim_{t\to\infty} e^{-\rho t}\lambda_2(t)k(t) = 0.$$

In many literatures on monetary growth, including Stockman (1981), Marquis and Reffett (1991), and Chang *et al*. (2000), the government maintains the rate of money growth at a constant rate, μ, and distributes seigniorage to the agent as a transfer payment in a lump-sum fashion. This means

$$\tau = \mu m. \tag{10}$$

On the other hand, by definition, real cash balances change according to

$$\frac{\dot{m}}{m} = \mu - \pi. \tag{11}$$

Combining Eqs.(3), (4), (10), and (11) leads to the goods market equilibrium condition:

$$\dot{k} = Ak - c. \tag{12}$$

As mentioned before, we impose the following CIA constraint:[4]

$$m = c. \tag{13}$$

Eqs.(6)-(13) characterize the equilibrium of the model. We first ask for the equation of rate of inflation. From Eqs.(6)-(9), the rate of inflation is endogenously determined by

$$\pi = -(1+A) - \frac{\beta}{\lambda_1}\frac{c^{1-\sigma}}{1-\sigma}k^{\beta-1} + \frac{k^{\beta}}{\lambda_1 c^{\sigma}}. \tag{14}$$

Substituting Eqs.(7) and (13) into Eq.(9) leads to the evolution of shadow price λ_1,

$$-\frac{\dot{\lambda}_1}{\lambda_1} = A - \rho + \frac{\beta}{\lambda_1}\frac{m^{1-\sigma}}{1-\sigma}k^{\beta-1}. \tag{15}$$

When we substitute Eq.(14) into (11) and use Eq.(13), the evolution of real cash balances is rewritten as

$$\frac{\dot{m}}{m} = \mu + (1+A) + \frac{\beta}{\lambda_1}\frac{m^{1-\sigma}}{1-\sigma}k^{\beta-1} - \frac{k^{\beta}}{\lambda_1 m^{\sigma}}. \tag{16}$$

Finally, substitution of Eq.(13) into (12) leads to the following dynamic equation:

$$\frac{\dot{k}}{k} = A - \frac{m}{k}. \tag{17}$$

The dynamical system of the model is fully described by Eqs.(15)-(17).

Let us now consider the steady-state situation. In the steady-state, the following relationships for the growth rates of shadow price, real cash balances, and capital should

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[4] At the equilibrium, the CIA constraint must be binding.

be satisfied:

$$\left(\frac{1}{\beta-\sigma}\right)\frac{\dot{\lambda}_1}{\lambda_1}=\frac{\dot{m}}{m}=\frac{\dot{k}}{k}=R^*, \tag{18}$$

where R^* is the rate of economic growth at the steady-state equilibrium.[5] Combining Eqs.(15)-(18), we can obtain a quadratic equation with respect to R^*:

$$\beta(1+\beta-\sigma)(A-R^*)R^* = \beta(1+\mu+\rho)(A-R^*)-(1-\sigma)\left\{(\sigma-\beta)R^*-(A-\rho)\right\}. \tag{19}$$

As for the steady-state properties, we first investigate for the case of *no* wealth effects ($\beta=0$). Substituting $\beta=0$ into Eq.(19) we obtain

$$R^*=\frac{A-\rho}{\sigma}. \tag{20}$$

We can confirm that Eq.(20) is identical with a standard result in the AK model.[6] Applying a regular assumption $A>\rho$, R^* has a positive sign.

Proposition 1: *In the case of no wealth effects, changes in the rate of money growth do not affect the rate of long-run growth.*

This is consistent with the former findings of Marquis and Reffett (1991) and Chang *et al*. (2000). However, this paper and Chang *et al*. (2000) employ a modified Barro-Rebelo model, while Marquis and Reffett (1991) adopt a generalized version of Uzawa-Lucas model. All of these analyses proved the property that money is *superneutral*, in each specific circumstance.

Second, let us examine more standard case as $\beta\neq 0$. However, in this case, it is difficult for us to calculate explicit (closed form) solutions about Eq.(19). Allowing for this fact, we limit the present analysis to the special case of $\sigma=1+\beta>1$. This case corresponds to the relatively small elasticity case compared with Chang *et al.* (2000). Applying this assumption to the quadratic form of Eq.(19) leads to

$$R^*=\frac{A\beta(1+\mu+\rho)+(1-\sigma)(A-\rho)}{\beta(1+\mu+\rho)+(1-\sigma)(\sigma-\beta)}=A+\frac{\rho}{\mu+\rho}. \tag{21}$$

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[5] To derive the relationships in Eq.(18), we define new stationary variables: $x=m/k$ and $z=k^{\beta-\sigma}/\lambda_1$. Using these variables, Eqs.(15)-(17) can be rewritten as follows:

$$-\dot{\lambda}_1/\lambda_1 = A-\rho+\left\{\beta/(1-\sigma)\right\}x^{1-\sigma}z,$$
$$\dot{m}/m=\mu+(1+A)+\left\{\beta/(1-\sigma)\right\}x^{1-\sigma}z-x^{-\sigma}z,$$
$$\dot{k}/k=A-x.$$

Taking the new stationary variables as given, we can derive dynamic equations in terms of x and z:

$$\dot{x}/x=(\dot{m}/m)-(\dot{k}/k)=\mu+1+\left\{\beta/(1-\sigma)\right\}x^{1-\sigma}z-x^{-\sigma}z+x,$$
$$\dot{z}/z=(\beta-\sigma)(\dot{k}/k)-(\dot{\lambda}_1/\lambda_1)=(\beta-\sigma)(A-x)+A-\rho+\left\{\beta/(1-\sigma)\right\}x^{1-\sigma}z.$$

From these equations, we can obtain the expressions of Eq.(18) applying the conditions $\dot{x}=\dot{z}=0$.

[6] See Barro (1990) and Rebelo (1991).

Eq.(21) represents the steady-state growth rate at $\sigma = 1 + \beta > 1$. In this case, we can obtain the following result:

Proposition 2: *Let* $\sigma > 1$ *and* $0 < \beta < 1$ *(where we assume* $\sigma = 1 + \beta > 1$*); then the steady-state growth rate is negatively affected by an increase in the rate of money growth. From Eq.(21)*

$$\frac{\partial R^*}{\partial \mu} = \frac{-\rho}{(\mu + \rho)^2} < 0, \quad \text{as} \quad \beta > 0.$$

Proposition 2 contradicts the result of Marquis and Reffett (1991). As mentioned above, in their model, the long-run growth rate was independent to changes in the rate of money growth. Moreover, in the same situation ($\beta > 0$), Chang *et al*. (2000) have derived the result opposite to us. This means that they obtained $\partial R^* / \partial \mu > 0$. Probably, the difference between Chang *et al*. (2000) and our model stems from the size of intertemporal elasticity of substitution because they studied the relatively large elasticity case ($\sigma = 1$). The case they examined corresponds to the situation that the agent's preference is relatively "non-myopic" compared with present case. The growth enhancing mechanism in Chang *et al*. (2000) was explained intuitively as follows. An increase in the rate of growth of money supply (μ) depresses real cash balances, which also oppresses consumption demand through the channel of CIA constraint. Moreover, a higher rate of inflation caused by an increase in μ enhances the CIA cost, while the rate of return on capital stays at a constant level (because of the "AK" model). At the same time, the capital (wealth) accumulation leads to the higher social status, which brings further incentive to the agent's capital accumulating behavior. As a consequence, the higher rate of inflation shifts the agent's demand from consumption to capital holdings. Since the production function exhibits constant returns to scale with respect to physical capital, the stimulation of capital accumulation enhances the growth rate of the economy. In contrast with Chang *et al*. (2000), the more "myopic" agent in the present model values current consumption to future consumption. Therefore, changes in the rate of growth of money supply, which oppress consumption demand, depress the long-run growth rate of the economy. For the other general case, as conjecture, the case $0 < \sigma < 1$ ($\sigma > 1$ and $\sigma \neq 1 + \beta$) will lead to the conclusion that an increase in the rate of money growth positively (negatively) affects the steady-state growth rate.

In addition, from the result of Eq.(21), we can confirm that wealth effects do not affect the long-run growth rate. This phenomenon is derived from our specific assumption of $\sigma = 1 + \beta > 1$. However, if we consider more general case as

$0 < \sigma < 1$ or $\sigma > 1$ (where $\sigma \neq 1+\beta$), wealth-enhanced social status will play a role determining the rate of long-run growth.

On the results of Eqs.(20) and (21), we can present another explanation like in Chang *et al*. (2000). The steady-state property $\{1/(\beta-\sigma)\}\dot{\lambda}_1/\lambda_1 = \dot{k}/k$ as given, we obtain the following relationship using Eqs.(15) and (17),

$$\left(\frac{1}{\beta-\sigma}\right)\left(\rho - A - \frac{\beta}{\lambda_1}\frac{m^{1-\sigma}}{1-\sigma}k^{\beta-1}\right) = A - \frac{m}{k}. \qquad (22)$$

Ignoring the effects of social status ($\beta = 0$), Eq.(22) indicates that the real money balances-capital ratio has a rigid relationship. This means that the ratio is completely determined by the constant parameters of A, σ, and ρ:

$$\frac{m}{k} = A + \frac{1}{\sigma}(\rho - A).$$

Together with Eq.(17), we confirm that the steady-state growth rate of capital stock is independent of the rate of money growth; i.e. $R^* = \dot{k}/k = (A-\rho)/\sigma$. This result is identical with Eq.(20). On the other hand, the existence of wealth effects ($\beta > 0$ and $\sigma = 1+\beta$) would break the above rigid relationship:

$$\frac{m}{k} = \rho + \frac{1}{\lambda_1 k}\left(\frac{k}{m}\right)^{\beta}.$$

Consequently, wealth-enhanced social status allows an additional degree of freedom to affect the real money balances-capital ratio, and this leads to changes in the long-run growth rate of capital stock.

3. Concluding remarks

Our first contribution is to confirm that, in the case of no wealth effects, the equilibrium properties of Marquis and Reffett (1991) and Chang *et al*. (2000) are preserved even under employing the non-separable utility function. Second is to show that, in the case of existing wealth effects, the negative correlation between the long-run growth rate of the economy and a rise in the money growth rate. This result is in sharp contrast with the typical conclusion of the relevant field including Chang *et al*. (2000). It is likely that the reason for such a conclusion stems from the broader class of utility function we employed.

Through the whole analysis, we have used the *absolute* level of wealth as the proxy of wealth-enhanced social status for technical reason. Social status is essentially a *relative* phenomenon, however. In view of this point, further analysis may dramatically change with a concern for relative wealth.

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