E C O N O M I C S B U L L E T I N

An efficient monte carlo study of two−step generalized least squares estimators for random−effects panel data models

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Abstract

Using efficient Monte Carlo methods, the performance of two−step Generalized Least Squares (GLS) estimators for the one−way error components models in small samples is analyzed. In our approach, we focus on the two−step GLS estimators provided by the programs LIMDEP, RATS and TSP, which mainly differ in the solution of negative variance components problem. Our main result is that the use of non negative first−step estimators, as RATS, produces a considerably efficiency loss. We greatly improve the efficiency of simulations using a control variate that can be implemented with no virtually computational cost.

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1 INTRODUCTION.

The simple static random-effects panel regression model has been one of the main workhorses in the analysis of longitudinal data. However, many problems arise in the application of the two-step GLS estimator, since it requires a prior estimation of the variance of the two random components (an individual specific effect and a remainder disturbance). Firstly, there exists a bewildering variety of estimators and criteria to obtain first-step estimators, which are asymptotically equivalent but have a different performance in finite samples. As a result, an infinite number of two-step GLS estimators with different finite sample efficiency are available. The problem is that the finite properties of competing estimators are unclear, since they are highly sensitive to the sample size together with the true values of the parameters of the variance components (see Maddala and Mount (1973) and Taylor (1980)). Another important problem is that the variance of the individual specific effects is usually obtained residually, and it may take negative values. There exist several practical procedures to solve this problem, but their effects in the efficiency of slope regression parameters surprinsingly have been overlooked in the literature[1].

The lack of evidence about the best choice of first-step estimator provokes that the most used econometric packages, among then LIMDEP, RATS and TSP, rely on different procedures to estimate the variance components. Furthermore, they also differ in practical solutions to the negative variance component problem. TSP and LIMDEP propose alternative first-step estimators, and, as final option, they provide estimators that are always non negative, although they are not the optimal either. In contrast, RATS directly uses estimators that never yield negative values, although they are biased. Several authors have recently considered these problems. Blanchard (1993) detected the problem, but his study did not provide numerical applications and Maudós and Uriel (1996) obtained that LIMDEP offered better estimations for a restricted data set.

In this note we analyze how much efficiency gain can be obtained in two-step GLS estimator by using alternative estimators of variance components, in the context of random individuals effects uncorrelated with explanatory variables. Moreover, we are interested in evaluating practical solutions to negative variance components problem. For these reasons, we focus on the two-step GLS estimators provided by LIMDEP (7.0), RATS for Windows and TSP (4.5). The work most closely to ours is Bruno and De Bonis (2000), that also compare these econometric packages[2]. However, we restrict our attention to the most problematic cases, that is, where some estimators of variance of individuals effects can take negative values and panels in which the cross-section dimension, N, and the time dimension, T, are small[3]. Monte Carlo experiments were performed for these purposes. Moreover, we use variance reduction techniques in order to mitigate their inherently imprecision, specifically, control variates (see Hendry (1984), and more recently Davidson and Mackinnon (1993)).

We show that the choice of first-step estimators affects substantially the efficiency of two-step GLS estimators. More specifically, we observe that the use of non negative estimators of variance components, as RATS, yields a considerable efficiency loss. Moreover, the performance of RATS

[1] An exception could be Baltagi (1981) and Maddala and Mount (1973), that analyze the effects of some procedures used to solve practically the former problem by means of Monte Carlo experiments.

[2] They analyze STATA instead of RATS.

[3] The one way random-effects model can also be estimated by maximum-likelihood estimation (MLE), see Hsiao (1986). Although MLE estimator is asymtotically efficient, the finite sample efficiency is also unclear. Moreover, sometimes the problem of negative value of variance of individuals components arises. We could also analyze its finite sample performance, but the goal of this work is to provided a practical guide for users and we consider that is more interesting to analyze estimators offered by econometric softwares.

is closer to within-groups estimator than LIMDEP and TSP. In addition to this, we observe that LIMDEP offers a more efficient estimator if N grows whereas TSP estimator has less variance if the sample size is smaller. Finally, we greatly improve the efficiency of simulations using a control variate that can be implemented with no virtually computational cost.

The estimation procedures for the random-effects model are reviewed in section 2. In section 3, we explain control variates. In section 4, Monte Carlo results are provided.

2 ESTIMATION OF RANDOM-EFFECTS MODEL.

Consider the simple static random-effects model [4]

$$y_{it} = \mu + \boldsymbol{\beta}'\mathbf{x}_{it} + u_i + \varepsilon_{it}, \;\; i = 1, ..., N, \;\; t = 1, ..., T, \tag{1}$$

where y_{it} is the model variable, $\mathbf{x}_{it}$ is a $(K \times 1)$ vector of explanatory variables and u_i is the random variable of individual effects that is independent of $\mathbf{x}_{it}$. We assume that u_i and ε_{it} are normally random distributed with mean 0 and variance σ_u^2 and σ_ε^2 . In this model, the covariance matrix of the combined unobservables is

$$\Omega = \sigma_\varepsilon^2 \; I_{NT} + \sigma_u^2 \; [I_N \otimes \mathbf{e}_T \mathbf{e}_T'],$$

where $\mathbf{e}_T$ denotes a $T \times 1$ vector of ones. Under these assumptions, the Gauss-Markov estimator of the slope coefficients (Maddala (1971)) is the GLS estimator, that can be expressed as a matrix weighted average of between-groups and within-groups estimators

$$\begin{aligned} \widehat{\boldsymbol{\beta}}^{GLS} &= \Delta \widehat{\boldsymbol{\beta}}^{B} + (I_K - \Delta)\widehat{\boldsymbol{\beta}}^{W}, \\ \Delta &= \gamma T \left[\sum_{i=1}^{N}\sum_{t=1}^{T} (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)' + \gamma T \sum_{i=1}^{N} (\overline{\mathbf{x}}_i - \overline{\mathbf{x}})(\overline{\mathbf{x}}_i - \overline{\mathbf{x}})' \right]^{-1} \left[\sum_{i=1}^{N} (\overline{\mathbf{x}}_i - \overline{\mathbf{x}})(\overline{\mathbf{x}}_i - \overline{\mathbf{x}})' \right] \end{aligned} \tag{2}$$

where $\widehat{\boldsymbol{\beta}}^{W}$ is the within-groups estimator, $\widehat{\boldsymbol{\beta}}^{B}$ is the between-groups estimator and $\gamma = \dfrac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_u^2}$. $\widehat{\boldsymbol{\beta}}^{B}$ is the estimator obtained pre-multiplying the initial model (1) by the matrix operator $B = I_N \otimes \dfrac{1}{T}\mathbf{e}_T\mathbf{e}_T'$ and then estimating it by OLS.

In order to be operative the former estimator, it is required to estimate the variance components (giving rise to the two-step GLS estimator). As there exist many estimators and criteria[5], that have different small-sample properties, it can be obtained a bewildering variety of two-step GLS estimators. For this reason, the most used econometric packages propose alternative estimators of variance components.

TSP[6] proposes two procedures to estimate the variance of error components. Firstly, it uses small sample estimators

$$\widehat{\sigma}_\varepsilon^2 = \frac{\widehat{\mathbf{v}}_w' \widehat{\mathbf{v}}_w}{N(T-1) - K} \quad \text{and} \quad \widehat{\sigma}_u^2 = \frac{\widehat{\mathbf{v}}_{Tot}' \widehat{\mathbf{v}}_{Tot}}{NT - (K+1)} - \widehat{\sigma}_\varepsilon^2, \tag{3}$$

where $\widehat{\mathbf{v}}_w$ denotes the within-groups residuals vector and $\widehat{\mathbf{v}}_{Tot}$ the total residuals vector obtained by estimating the initial model (1) directly by OLS. If negative values are computed using the

[4] See Hsiao (1986).
[5] See Swamy and Arora (1972) and Maddala and Mount (1973).
[6] See Hall (1997).

former estimator of the variance of individuals effects, the program switches over to large sample formulas. The differences between them are just the correction of the degree of freedom in order to obtain positive values for all the possible cases. For this reason, they are consistent, though biased. These can be written as

$$\widehat{\widehat{\sigma}}_{\varepsilon}^{2}=\frac{\widehat{\mathbf{v}}_{w}^{\prime}\widehat{\mathbf{v}}_{w}}{NT} \qquad \text{and} \qquad \widehat{\widehat{\sigma}}_{u}^{2}=\frac{\widehat{\mathbf{v}}_{Tot}^{\prime}\widehat{\mathbf{v}}_{Tot}-\widehat{\mathbf{v}}_{w}^{\prime}\widehat{\mathbf{v}}_{w}}{NT}. \tag{4}$$

On the other hand, LIMDEP proposes three alternative procedures to estimate the variance of the individual effects. Firstly, in contrast with TSP, it estimates σ_u^2 using between-model and the estimator $\widehat{\sigma}_{\epsilon}^{2*}=\frac{\widehat{\mathbf{v}}_{w}^{\prime}\widehat{\mathbf{v}}_{w}}{N(T-1)-(K+1)}$. This estimator takes the form

$$\widehat{\sigma}_{u}^{2*}=\frac{\widehat{\mathbf{v}}_{B}^{\prime}\widehat{\mathbf{v}}_{B}}{N-(K+1)}-\frac{\widehat{\sigma}_{\varepsilon}^{2*}}{T}, \tag{5}$$

where $\widehat{\mathbf{v}}_B$ is the between-groups residuals vector. In case of expression (5) yields a negative result, LIMDEP applies a second criteria consisting of replacing the sum of the squares of the model between-groups by

$$\widehat{\mathbf{v}}_{BT}^{\prime}\widehat{\mathbf{v}}_{BT}=\sum_{i=1}^{N}[\overline{y}_i-(\widehat{\mu}^{Tot}+\widehat{\boldsymbol{\beta}}^{Tot}\overline{\mathbf{x}}_i)]^2, \tag{6}$$

where $\widehat{\mu}^{Tot}$ and $\widehat{\boldsymbol{\beta}}^{Tot}$ are the estimators obtained by regressing the total model. Unfortunately, this estimator may not be positive. When this occurs, LIMDEP uses as a third procedure the estimator proposed by Nerlove (1971), which consists of estimating σ_u^2 directly by means of the sample variance of the fixed effects (obtained in the within-groups model), α_i, i.e.

$$\widehat{\sigma}_{u}^{2***}=\frac{\sum_{i=1}^{N}(\widehat{\alpha}_i-\overline{\widehat{\alpha}})^2}{N} \tag{7}$$

where $\overline{\widehat{\alpha}}=\frac{\sum_{i=1}^{N}\widehat{\alpha}_i}{N}$. It becomes evident that, by construction, this third estimator can never be negative.

Finally, the RATS[7] program uses the decomposition of analysis of variance of total residuals in order to obtain estimators of σ_ε^2 and σ_u^2. Consequently, the estimations of σ_ε^2 and σ_u^2 are given by

$$\widehat{\sigma}_{\varepsilon}^{2R}=\frac{SSR_w}{N(T-1)} \qquad \text{and} \qquad \widehat{\sigma}_{u}^{2R}=\frac{SSR_b}{N-1} \tag{8}$$

where SSR_b is the sum of the squares of the between-groups residuals and SSR_w is the sum of the squares of the within-groups residuals. But the main difference with LIMDEP and TSP is that RATS does not apply the correct degrees of freedom. This fact provokes that these estimators are biased for small samples, although it always obtains non-negative estimations of σ_u^2.

[7] See Doan (1996).

3 Variance reduction method: control variates.

The principle of control variates is to find an auxiliary statistic, θ^*, called control variate, with some known distribution properties, specifically the population mean, and which is also highly correlated with the estimator or statistical test being studied, $\widehat{\theta}$. The divergence between the population mean and the sample mean of the control variates is precisely used to improve the estimation obtained in the small sampling experiments. It should be pointed out that control variates require knowledge of statistics that can only be observed in a Monte Carlo setting. Such Monte Carlo estimators for the bias and the variance can be constructed as follows.

In each Monte Carlo experiment are obtained $\widetilde{\theta}_j$, $j = 1, ...M$, one realization of the estimator the interest, where M is the number of replications, and θ_j^*, the control variate. Using control variates, the optimal control variates estimator of $E(\widehat{\theta})$, are given by

$$\ddot{\theta}\ (\lambda) = \overline{\overline{\theta}} - \lambda(\overline{\theta}^* - E(\theta^*)), \tag{9}$$

where $\overline{\overline{\theta}}$=$\frac{\sum_{j=1}^{M} \widetilde{\theta}_j}{M}$, $\overline{\theta}^*$ is the sample means of $\theta_j^{*\prime}s$, and λ is a parameter that has to be determined. $\overline{\overline{\theta}}$ is the commonly called naive estimator and, although this estimator is not the most efficient, it has become the conventional method for evaluating $E(\widehat{\theta})$. The natural choice of λ is the value that minimizes the variance of (9), $V(\ddot{\theta}\ (\lambda)) = V(\overline{\overline{\theta}}) + \lambda^2 V(\overline{\theta}^*) - 2\lambda Cov(\overline{\theta}^*, \overline{\overline{\theta}})$, that is,

$$\lambda^* = \frac{Cov(\overline{\overline{\theta}}, \overline{\theta}^*)}{Var(\overline{\theta}^*)}. \tag{10}$$

By using a control variate, we achieve a efficiency gain which we clearly observe from (9) and (10). The neat proceeding to obtain efficiency gain by using control variates will be

$$EG = \frac{V(\overline{\overline{\theta}})}{V(\ddot{\theta}\ (\lambda))} = \frac{1}{(1-\rho^2)}, \tag{11}$$

where ρ is the coefficient of correlation between $\widetilde{\boldsymbol{\theta}}$, a M-vector with typical element $\widetilde{\theta}_j$, and $\boldsymbol{\theta}^*$, a M-vector with typical element θ_j^*. In (11), it is easily observed that if ρ is nonzero, then, the use of control variates produces some gains with respect to the naive estimator, $\overline{\overline{\theta}}$. When the number of observations, n, grows, the correlation between the control variates and the quantity of interest must increase, as then the finite-sample of the latter distribution approaches the asymptotic distribution. Consequently, (9) is increasingly efficient with increasing n, offsetting the rising cost of experimentation (see Hendry (1984)).

In much of the literature on control variates (e.g., Henry (1984)), λ is set equal to 1. It is reasonable if θ_j^* and $\widetilde{\theta}_j$ are highly correlated and have similar variances, but it is not the best choice in general. Therefore, we alternatively proceed to estimate[8] λ^*. The easiest way to estimate this value is by means of the OLS estimator, $\widehat{\lambda}$, obtained by regressing $\widetilde{\boldsymbol{\theta}}$ on $\mathbf{e}$, an M-vector of ones, and $\boldsymbol{\theta}^*$.

We can also use control variates to estimate the variance of $\widehat{\theta}$. The optimal control variates estimator of variance (see Hendry (1984) and Campos (1986)) is

$$\frac{1}{M-1}\sum_{j=1}^{M}(\widetilde{\theta}_j - \overline{\overline{\theta}})^2 - \frac{1}{M-1}\sum_{j=1}^{M}(\theta_j^* - \overline{\theta}^*)^2 + Var(\theta^*). \tag{12}$$

[8] See Davidson and Mackinnon (1993) for a detailed analysis of this question.

To estimate the bias and the variance of two-step GLS by means of Monte Carlo experiments [9], it is natural to use the following expression as control variate [10]

$$\beta^* = \Delta\widehat{\beta}^B + (I_K - \Delta)\widehat{\beta}^W, \tag{13}$$

$$\Delta = \gamma_0 T\left[\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it}-\overline{x}_i)^2 + \gamma_0 T\sum_{i=1}^{N}(\overline{x}_i-\overline{x})^2\right]^{-1}\left[\sum_{i=1}^{N}(\overline{x}_i-\overline{x})^2\right],$$

where $\gamma_0 = \dfrac{\sigma^2_{0\varepsilon}}{\sigma^2_{0\varepsilon}+T\sigma^2_{0u}}$, $\sigma^2_{0\varepsilon}$ and σ^2_{0u} are the true values of the variance components. This control variate has known mean that is equal to the true value of parameter of interest and variance $Var(\theta^*) = \sigma^2_{0\varepsilon}(\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it}-\overline{x}_i)^2 + \gamma_0 T\sum_{i=1}^{N}(\overline{x}_i-\overline{x})^2)^{-1}$. In addition, it is highly correlated with the estimator of interest (notice that it is impossible to compute β^* from a real data set).

4 Monte Carlo Experiments.

We consider the following generating data process

$$y_{it} = \mu + \beta x_{it} + \varepsilon_{it} + u_i, \qquad t = 1, ..., T,\ i = 1, ..., N, \tag{14}$$

where $\mu = 4$, $\beta = 1$, $\varepsilon_{it} \sim N(0, \sigma^2_\varepsilon)$ and $u_i \sim N(0, \sigma^2_u)$. We have kept the observations on x_{it} fixed over replications. Furthermore, x_{it} are uncorrelated with the individuals effects.

In these simulations, we study the performance of the two-step GLS estimators and we also compare it with within-groups. Furthermore, we compute the number of times it is used each of the procedures proposed by LIMDEP and TSP for the estimation of σ^2_u. The experiments were carried out for various panel size. Specifically, we consider that N takes values of 20, 25, 30 and 40 and T is assigned values of 5 and 10. For each sample size, we restrict our attention for the three different ratios of variance components: $\dfrac{\sigma^2_\varepsilon}{\sigma^2_u} = 1, 20, 40$, since we are interested in the cases where the estimators of σ^2_u yield negative values. We also set $R^2 = 0.9$ [11]. In addition, the control variate (13) is used to obtain Monte Carlo estimators.

Table 1A of the appendix shows the percentage of times LIMDEP and TSP programs applied the estimation procedures of σ^2_u for each sample size and ratio $\sigma^2_\varepsilon/\sigma^2_u$. The two methods used by TSP are called respectively T1 and T2 and the three estimators used by LIMDEP are called L1, L2 and L3.

The data show that, depending on the ratio $\sigma^2_\varepsilon/\sigma^2_u$ the first estimators of σ^2_u yield negative values, either in LIMDEP and TSP (the maximal negative frequency arises for $T = 5$ and $N = 25$). Furthermore, the problem of negative values is really important, because it appears in the 46 per cent of times. Specifically, the negative values arise when this ratio is greater than or equal to 20, or in other terms, when the heterogeneity effects are less important. For a lower proportion no negative results are observed in any of the cases studied. It is important to point out, that the second procedure proposed by LIMDEP, (L2), is applied to a small number of cases. When N and T are increased, the percentage of cases in which T1 and L1 offer negative results decreases. Moreover, this effect is reinforced for $T = 10$.

[9] In our experiments we only have a regressor.

[10] The formulae can be found in Hendry and Harrison (1974).

[11] Results for other values of R^2 are qualitatively similar to those. These can be reported upon requested.

Table 1 summarizes the performance of two-step GLS and within-groups estimators. Bias and variance of each estimator are estimated by using the optimal control variates estimators (9) and (12), respectively. The tabulated results show that LIMDEP and TSP provide estimators with better properties in all performed experiments. Their estimated variances present no significant differences (under 1 %). RATS, which used biased estimators to calculate variance of error components, always presents worse results, and, surprisingly, it has a performance very close to within-groups estimator. It is also important to note that, all estimators perform worst as the ratio $\sigma_\varepsilon^2/\sigma_u^2$ increases. An interesting result is that TSP offers the most efficient estimator for small samples (T=5 and N=20). In contrast, LIMDEP estimator becomes the most efficient when N grows.

RATS presents up to 25 (%) higher variance than LIMDEP and TSP (the maximum differences arise for T=5, N=40 and $\sigma_\varepsilon^2/\sigma_u^2$= 20). However, notice that differences depend strongly on the ratio of the two variance parameters. When σ_ε^2 equal σ_u^2, all estimators perform in the same way (even the within-groups estimator). In contrast, if $\sigma_\varepsilon^2/\sigma_u^2$ increases, there exists a substantial efficiency gain in practice by using LIMDEP and TSP as opposed to RATS. Furthermore, the differences between the estimated variance of estimators change with sample sizes. When N grows, the performance of LIMDEP and TSP considerably improves as opposed to RATS. In contrast, if T becomes large, all estimators behave similar.

It is important to note that the performance of RATS is closer to within-groups estimator than LIMDEP and TSP. Whereas LIMDEP has a estimated variance up to 35 % smaller than within-groups, RATS presents a maximum difference of 10 % in efficiency with respects to within-groups.

To conclude, we found that the first-step estimators affect substantially the efficiency of two-step GLS estimators. The use of biased estimators as unique procedure to avoid the problem of negative values of σ_u^2 (as RATS) produces a considerably efficiency loss in two-step GLS estimator (around 25%). In addition, finite-sample differences depends on either sample size and the true values of the ratio $\sigma_\varepsilon^2/\sigma_u^2$.

Table 2A of the appendix summarizes the empirical efficiency gains using the optimal control variates estimator of the bias (9). The empirical efficiency gains are obtained by the ratio $1/(1-\widetilde{\rho}^2)$, where $\widetilde{\rho}$ is the empirical coefficient of correlation between the naive estimator and the control variate. The gains are apparently very large (being 71.68 its maximum value). To get a efficiency gain of this magnitude it will be necessary to perform a highly number of replications. In particular, the efficiency gains are greater for LIMDEP and TSP estimators. However, the gain strongly depends on the ratio of true values of variance components (see Davidson and Mackinnon (1993)): the lower is the ratio $\sigma_\varepsilon^2/\sigma_u^2$, the greater are the efficiency gains. Finally, gains grow with sample size, as the theory predicts. Table 3A of the appendix shows the empirical correlations between the naive estimator and the control variate.

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Table 1.
Results of Monte Carlo experiments. Bias and variance of two-step GLS and within-groups estimators.

	T	*N*	$\frac{\sigma_e^2}{\sigma_u^2}$		TSP	LIMDEP	RATS	W-G
1	*5*	**20**	**1**	**Bias**	-0,000046	-0,000091	0,000196	0,000260
				Variance	*0,000767*	*0,000771*	*0,000792*	*0,000804*
			20	**Bias**	0,000377	0,000271	0,000650	0,000695
				Variance	*0,001410*	*0,001420*	*0,001550*	*0,001640*
			40	**Bias**	0,000082	-0,000104	-0,000399	-0,000383
				Variance	*0,001428*	*0,001429*	*0,001550*	*0,001650*
2		**25**	**1**	**Bias**	-0,000215	-0,000298	0,000590	0,000805
				Variance	*0,000628*	*0,000630*	*0,000646*	*0,000657*
			20	**Bias**	-0,000794	-0,000702	-0,002330	-0,003010
				Variance	*0,001090*	*0,001080*	*0,001190*	*0,001280*
			40	**Bias**	-0,000124	-0,000056	-0,000177	-0,000256
				Variance	*0,001090*	*0,001090*	*0,001170*	*0,001260*
3		**30**	**1**	**Bias**	0,000005	0,000012	0,000285	0,000357
				Variance	*0,000533*	*0,000533*	*0,000528*	*0,000532*
			20	**Bias**	-0,000062	-0,000069	-0,000100	-0,000085
				Variance	*0,000953*	*0,000957*	*0,001060*	*0,001140*
			40	**Bias**	0,000048	0,000069	-0,000548	-0,000784
				Variance	*0,000930*	*0,000930*	*0,001050*	*0,001130*
4		**40**	**1**	**Bias**	0,000134	0,000074	-0,000015	-0,000046
				Variance	*0,000387*	*0,000386*	*0,000411*	*0,000421*
			20	**Bias**	0,000398	0,000271	0,002250	0,002780
				Variance	*0,000697*	*0,000697*	*0,000866*	*0,000932*
			40	**Bias**	-0,000032	0,000019	0,000175	0,000104
				Variance	*0,000701*	*0,000694*	*0,000758*	*0,000823*
5	**10**	**20**	**1**	**Bias**	0,000136	0,000251	-0,000220	-0,000261
				Variance	*0,000349*	*0,000347*	*0,000350*	*0,000351*
			20	**Bias**	-0,000054	-0,000071	-0,001020	-0,001130
				Variance	*0,000684*	*0,000680*	*0,000709*	*0,000718*
			40	**Bias**	0,000166	0,000181	0,000113	0,000149
				Variance	*0,000670*	*0,000672*	*0,000644*	*0,000644*

(i) The number of replications computed are 200, 150, 130 and 100 for the experiments 1, 2, 3, 4 respectively and 100 for 5.

APPENDIX

Table 1A
Estimation procedures of s_u^2 (%) in TSP and LIMDEP.

	T=5 *N*=20			*T*=5 *N*=25			*T*=5 *N*=30			*T*=5 *N*=40			*T*=10 *N*=20		
$\frac{s_e^2}{s_u^2}$	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**
T1	100,0	66,5	56,0	100,0	75,3	54,0	100,0	77,6	64,0	100,0	77,0	71,0	100,0	87,0	73,0
T2	0,0	33,5	44,0	0,0	24,6	46,0	0,0	22,4	36,0	0,0	23,0	29,0	0,0	13,0	27,0
L1	100,0	66,5	59,0	100,0	74,0	52,0	100,0	79,2	62,4	100,0	78,0	73,0	100,0	87,0	71,0
L2	0,0	4,0	3,0	0,0	2,0	4,0	0,0	1,6	3,2	0,0	1,0	1,0	0,0	3,0	3,0
L 3	0,0	29,5	38,0	0,0	24,0	44,0	0,0	19,2	34,4	0,0	21,0	26,0	0,0	10,0	26,0

Tabla 2A
Empirical efficiency gains by using optimal control variates estimator (9) in Monte Carlo experiments.

	T=5 *N*=20			*T*=5 *N*=25			*T*=5 *N*=30			*T*=5 *N*=40			*T*=10 *N*=20		
$\frac{s_e^2}{s_u^2}$	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**
TSP	55,81	50,25	25,25	62,75	55,81	26,57	62,75	55,81	26,57	71,68	62,75	28,03	71,68	71,68	50,25
LIMDEP	50,25	50,25	33,59	62,75	55,81	28,03	62,75	55,81	26,57	71,68	55,81	28,03	62,75	71,68	55,81
RATS	33,59	9,19	10,67	35,97	7,95	7,40	33,59	6,93	6,84	38,71	9,87	8,19	50,25	18,11	18,11
W-G	29,66	6,08	7,11	28,03	5,42	4,94	21,09	4,81	4,65	31,50	6,93	5,47	45,71	14,96	14,54

Table 3A.
Empirical correlations between control variate and naive estimator.

	T=5 *N*=20			*T*=5 *N*=25			*T*=5 *N*=30			*T*=5 *N*=40			*T*=10 *N*=20		
$\frac{s_e^2}{s_u^2}$	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**	**1**	**20**	**40**
TSP	0,991	0,990	0,980	0,992	0,991	0,981	0,992	0,991	0,981	0,993	0,992	0,982	0,993	0,993	0,990
LIMDEP	0,990	0,990	0,985	0,992	0,991	0,982	0,992	0,991	0,981	0,993	0,991	0,982	0,992	0,993	0,991
RATS	0,985	0,944	0,952	0,986	0,935	0,930	0,985	0,925	0,924	0,987	0,948	0,937	0,990	0,972	0,972
W-G	0,983	0,914	0,927	0,982	0,903	0,893	0,976	0,890	0,886	0,984	0,925	0,904	0,989	0,966	0,965