E C O N O M I C S   B U L L E T I N

Predicting the Cyclical Phases of the Post−War U.S. Leading and Coincident Indicators

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Abstract

A bifactor model of the unobserved common leading and coincident indicators with Markov switching, introduced via the common factor intercept term, is examined. The model has four regimes and the lag between the leading and coincident factors is reflected in transition probabilities matrix. Three hypotheses concerning the relationship between the two factors are evaluated: (1) cyclical dynamics of the two factors are independent; (2) cyclical dynamics are common for both factors; (3) dynamics are interrelated, with coincident factor lagging behind the leading factor. The models are estimated using US monthly macroeconomic time series. The estimated recession probabilities reveal close correspondence to NBER business cycle dating. Moreover, model 3 shows that the leading factor is entering the recession 5 months and the expansions 9 months earlier than the coincident one. This permits timely forecasting of the future evolution of the coincident economic indicator.

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1 Introduction

The objective of the paper is the simultaneous construction of both coincident indicator and leading indicator of the macroeconomic activity, which would allow tracing of evolution of the business cycle in order to be able to predict the recessions of the coincident indicator using those of the leading indicator. While the coincident indicator can be used to characterize the current state of the "business climate", the leading indicator may serve to forecast the changes in this state which would take place within the next several periods.

For the construction of the indicators in question we are using the dynamic factor analysis methodology. One of the most influential dynamic single-factor models of the coincident economic indicator with linear dynamics is that by Stock and Watson (1988), while the single-factor models with Markov-switching dynamics were proposed first by Chauvet (1998), Kim and Yoo (1995). Until now, to our best knowledge, no one has come with a multifactor model (incorporating both leading and coincident common factors) with regime switching.

This paper introduces a two-factor model with regime-switching dynamics where one of the latent factors is postulated as a common leading indicator, while the second factor is taken to be the common coincident indicator. The common leading and coincident factors are estimated from a set of observed time series which is split into a subset of leading and a subset of coincident variables. The assumed leading relationship is reflected in the transition probabilities matrix governing the shifts in the regimes of the two common factors. Here we base our analysis on that of Phillips (1991) who showed, using a bivariate regime switching model, how the time lags between two observed variables with Markov-switching dynamics can be expressed in terms of the transition probabilities.

The remainder of paper is organized as follows. Section 2 formulates three bifactor models with regime-switching dynamics differing in the way the cyclical evolution of both factors is defined. In section 3 these three Markov-switching models with leading and coincident common factors are estimated using the U.S. Post-War monthly macroeconomic data. Section 4 summarizes the results of the paper. All the tables and graphs are put into the Appendix following the list of references.

2 Model

It was observed by many authors, among them by Diebold and Rudebusch (1996) that the model of the business cycle would be incomplete if it would not take into account both the comovement of various macroeconomic variables and the asymmetries between the phases of the cycle. The linear leading-coincident factors model would incorporate only the phenomenon of the simultaneous changes in the levels of different individual time series. However, it lacks a mechanism which would reflect the qualitatively different behavior of these series during recessions and expansions. One of the ways to introduce this mechanism in our model is to add to it the regime-switching dynamics.

We consider two sets of the observed time series: leading and coincident. The common dynamics of the time series belonging to each of these groups are explained by a single common factor: leading factor for the first group and coincident factor for the second group. The idiosyncratic dynamics of each time series in particular are captured by one specific factor per each observed time series. Therefore the model can be written as:

$$\Delta y_t = \Gamma \Delta f_t + u_t \tag{1}$$

where $\Delta y_t = (\Delta y_{Lt} \mid \Delta y_{Ct})'$ is the $n \times 1$ vector of the observed time series in the first differences; $\Delta f_t = (\Delta f_{Lt} \mid \Delta f_{Ct})'$ is the 2×1 vector of the latent common factors in the first differences; $u_t = (u_{Lt} \mid u_{Ct})'$ is the $n \times 1$ vector of the latent specific factors; Γ is the $n \times 2$ factor loadings matrix linking the observed series with the common factors.

The dynamics of the latent common factors can be described in terms of a nonlinear VAR model:

$$\Delta f_t = \mu(s_t) + \Phi(L)\Delta f_{t-1} + \varepsilon_t \tag{2}$$

where $\mu(s_t) = \{\mu_L(s_t), \mu_C(s_t)\}$ is the 2×1 vector of the state-dependent intercepts of the common leading and coincident factors, correspondingly, which take different values depending on the regime; $\Phi(L)$ is the sequence of p ($p = \max\{p_L, p_C\}$, where p_L is the order of the AR polynomial of the leading factor, and p_C is the order of the AR polynomial of the coincident factor) 2×2 lag polynomial matrices; ε_t is the 2×1 vector of the serially and mutually uncorrelated common factor disturbances with possibly state-dependent variance:

$$\varepsilon_t \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_L^2(s_t) & 0 \\ 0 & \sigma_C^2(s_t) \end{pmatrix} \right)$$

where s_t is the unobserved regime variable. In the two-regime (expansion-recession, or high-low growth rate) case it takes two values: 0 or 1. Depending on the regime, the common factor's intercept assumes different values: low in contractions and high in expansions. Thus, the common factors grow faster during the upswings and slower (or even have negative growth rate) during the downswings of the economy.

The changes in the regimes are governed by the first-order Markov chain process, which is summarized by the transition probabilities matrix, whose characteristic element is $p_{ij} = prob(s_t = j | s_{t-1} = i)$.

Since we have two common factors each of which passes through its own low and high regimes, the whole process is to be cast in a four regimes framework as it is done in Phillips (1991). Namely:

$s_t = 0$	$s_t = 1$	$s_t = 2$	$s_t = 3$
$s_t^L = 0$	$s_t^L = 1$	$s_t^L = 0$	$s_t^L = 1$
$s_t^C = 0$	$s_t^C = 0$	$s_t^C = 1$	$s_t^C = 1$

where s_t^L and s_t^C are the unobserved state variables for leading factor and coincident factor, respectively.

The way the unobserved regimes of leading indicator and coincident indicator are interrelated affects the form of the 4×4 transition probabilities matrix. First, we may suppose that the state variables s_t^L and s_t^C are mutually independent. In that case the transition matrix, π, looks like:

$$\begin{pmatrix} p_{11}^L p_{11}^C & (1-p_{11}^L)p_{11}^C & p_{11}^L(1-p_{11}^C) & (1-p_{11}^L)(1-p_{11}^C) \\ (1-p_{22}^L)p_{11}^C & p_{22}^L p_{11}^C & (1-p_{22}^L)(1-p_{11}^C) & p_{22}^L(1-p_{11}^C) \\ p_{11}^L(1-p_{22}^C) & (1-p_{11}^L)(1-p_{22}^C) & p_{11}^L p_{22}^C & (1-p_{11}^L)p_{22}^C \\ (1-p_{22}^L)(1-p_{22}^C) & p_{22}^L(1-p_{22}^C) & (1-p_{22}^L)p_{22}^C & p_{22}^L p_{22}^C \end{pmatrix}$$

In fact, $\pi = \pi^C \otimes \pi^L$, where π^L and π^C are the transition probabilities matrices for state variables s_t^L and s_t^C.

Second hypothesis may be that there is no two different state variables, but only one representing a single process and that both common factors enter into each regime simultaneously, without any lags among the two factors. In other words, the recessions (expansions) of the leading factor are the recessions (expansions) of the coincident factor. In this case there is no sense to talk about leading factor, because both factors are coincident. This case may be represented with an ordinary two-regime transition probabilities matrix:

$$\pi = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}$$

Under the third hypothesis the two unobserved processes are interrelated, with leading factor entering the recessions (expansions) several periods earlier than the coincident indicator. As Phillips (1991) remarks, the model with an integer lag exceeding one period would require a Markov process with the order higher than 1. However, the real-valued (positive) lag can be modeled with a first-order Markov process by constructing the following transition probabilities matrix:

$$\pi = \begin{pmatrix} p_{11} & 1-p_{11} & 0 & 0 \\ 0 & 1-\frac{1}{A} & 0 & \frac{1}{A} \\ \frac{1}{B} & 0 & 1-\frac{1}{B} & 0 \\ 0 & 0 & 1-p_{22} & p_{22} \end{pmatrix}$$

where A and B are the expected leads in the recession and expansion, correspondingly. The expected duration of this regime ($s_t = 1$) is:

$$A = 1 + p(s_t = 1|s_{t-1} = 1) + p(s_t = 1|s_{t-1} = 1)^2 + ...$$

that is

$$A = \frac{1}{1-p(s_t=1|s_{t-1}=1)}$$

Thus, we can analyze the three above stated cases - independent cyclical evolution of leading and coincident factors (let us call it model 1), identical cyclical evolution of both factors (model 2), and similar cyclical evolution with coincident factor lagging behind the leading indicator (model 3) - and compare the resulting three hypotheses to check whether the leading common factor is really leading and if so, how far it is advancing the coincident common factor.

We assume for simplicity that the two factors are related only through the transition probabilities, no correlation and no Granger-causality among these factors being assumed. The last premise is, however, feasible and it was considered in our previous paper - Kholodilin (2001). This assumption means that the matrices Φ_i $(i = 1, ..., p)$ are diagonal or lower diagonal for all i:

$$\Phi_i = \begin{pmatrix} \phi_{L,i} & 0 \\ 0 & \phi_{C,i} \end{pmatrix}$$

The idiosyncratic factors are by definition mutually independent and are modelled as the AR processes:

$$u_t = \Psi(L)u_{t-1} + \eta_t \tag{3}$$

where $\Psi(L)$ is the sequence of q ($q = \max\{q_{1,...}, q_n\}$, where q_i is the order of the AR polynomial of the $i-th$ idiosyncratic factor) $n \times n$ diagonal lag polynomial matrices and η_t is the $n \times 1$ vector of the mutually and serially uncorrelated Gaussian shocks:

$$\eta_t \sim \left(\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{pmatrix} \right)$$

To estimate this model we express it in a state-space form:

$$\Delta y_t = A\beta_t \tag{4}$$

$$\beta_t = \alpha(s_t) + C\beta_{t-1} + v_t \tag{5}$$

where $\beta_t = (f_t|u_t)'$ is the state vector containing stacked on top of each other vector of common factors and the vector of specific factors; v_t is the vector of the common and idiosyncratic factors' disturbances with mean zero and variance-covariance matrix Q; $\alpha(s_t) = (\mu_L(s_t), \mu_C(s_t), ..., 0)'$ is the state-dependent vector of intercepts. The structure of the system matrix A is defined as in Kholodilin (2001), while matrix C has somewhat different structure given the fact that the assumption of Granger causality between the common factors has been removed from this model:

$$C = \begin{pmatrix} \Phi^L & O & & & 0 \\ O & \Phi^C & & & \\ & & \Psi^1 & & \\ & & & \ddots & \\ 0 & & & & \Psi^n \end{pmatrix}$$

where matrices Φ^L, Φ^C, and Ψ^i $(i = 1, ..., n)$ are formulated exactly the same way as in Kholodilin (2001).

There are different ways of estimating the unknown parameters and the latent factors (maximum likelihood, EM, MCMC techniques - see Kim and Nelson (1999) for more details). Here we applied the maximum likelihood method with log-likelihood function obtained using Kalman filter recursions. To save space we will not present them here, referring the reader, for instance, to Hamilton (1994) who gives very clear and systematic explanation of the Kalman filter methodology.

3 Real example

The linear two-factor model was estimated using the US monthly data from January 1959 to December 1998. To construct the leading common factor the data from Watson (2000) were used, namely one real and three financial time series: authorized housing - total new housing units in thousands (HSBR), spread between the US Treasury bills 3-month interest rate and federal funds effective annualized rate (SFYGM3), spread between the US Treasury bills with constant maturity 1-year interest rate and federal funds effective annualized rate (SFYGT1), and NYSE common stock price index (FSNCOM). The common coincident factor was estimated based on the four real time series borrowed from Mariano and Murasawa (2000): employees on nonagricultural payrolls; personal income less transfer payments; index of industrial production; and manufacturing and trade series.

The leading time series were selected by comparing them individually to a coincident factor computed as if it were not dependent of a hypothetical leading common factor. Figure 1 shows that the correlation between these series (SFYGM3, SFYGT1 in levels and the first differences of the log of HSBR and FSNCOM), on one hand, and the growth rate of the common coincident indicator, on the other hand, is relatively high at leads 4-5. It is also very important that the series are sufficiently highly correlated among each other, thus permitting to postulate existence of a latent common factor standing behind their common evolution.

All the three models were estimated under the identifying assumption of the first factor loading for each common factor being equal 1. The parameter estimates, together with their standard errors and corresponding p-values, of the three models can be found in Tables 1-3 of Appendix. In all the models both the common and specific factors are taken to follow autoregressive processes of order 0. Otherwise the number of parameters to estimate would be too high.

Both the model with independent leading and coincident indicators and the model with the leading and coincident indicators having the same cyclical dynamics with coincident factor lagging behind the leading one seem to bring significant increase in the maximum likelihood compared to the model with two common factors having common cyclical dynamics. The independent factors model turns out to outperform slightly the model 2. However, model 2 delivers some additional information of interest, namely the leads in low and high states. According to Table 2 the leading factor enters the recessions on average approximately 5 months earlier than the coincident factor and goes into the expansions roughly 9 months earlier than the common coincident indicator does.

Figure 2 represents the three estimates of the common leading and coincident factors corresponding to the three models: independent leading and coincident factors, two coincident factors with common dynamics, and leading and coincident factors with common dynamics. The common factors were reconstructed as the partial sums of their growth rates obtained as an output of the estimation procedure. Therefore they follow random walks. Nevertheless, their profiles are quite similar regardless of the model. The coincident factors in all three cases seem to lag almost always the leading indicators.

On Figure 3 the conditional (smoothed) probabilities of the regimes 1, 2, and 3 are depicted estimated for the case of supposedly independent leading and coincident indicators. Remind that regime 1 corresponds to the leading indicator signalling the low state and coincident indicator still being in the high state, while in the regime 2 the leading factor is already in high state and coincident factor staying in the low state. Not surprisingly the conditional probabilities of state 1 are leading those of state 2. Only one recession fails to be captured by the regime 1 probabilities - that of the end of 1950s. However, it is easy to explain - the leading factor recession must have started well before the beginning of our sample. Regime 3 means that both common factors are in the low state. This regime is superimposed on the National Bureau of Economic Research (NBER) business cycle chronology for the U.S. economy. The conditional probabilities of both factors being simultaneously in the recession coincide almost perfectly with the NBER dating.

Figure 4 reflects the recession probabilities for the model 2. In this case, since both factors have common cyclical dynamics, there are only two states: low and high. Hence we display only the (filtered and smoothed) recession probabilities against the NBER dates. These probabilities are slightly leading the NBER cycle, the leading factor playing more prominent role in the determination of the conditional probabilities.

Figure 5 is pretty similar to Figure 3. On the upper panel one can see the regime 1 and regime 2 conditional probabilities, the former leading the latter. While on the lower panel the regime 3 probabilities are plotted versus the NBER cycle. Again, there is quite close correspondence between our dating and that of the NBER.

4 Summary

The paper introduces a multifactor model with two common factors (leading and coincident) having regime-switching dynamics. The lead-lag relationship between the common leading and coincident factors is reflected in the transition probabilities matrix.

The model was applied to the US Post-World War II monthly macroeconomic data. Its estimation results imply that the common coincident factor is lagging behind the common leading factor 5 months when entering into contractions and around 9 months when going into expansions. Moreover, there exists a close correspondence between our estimated recession chronologies and those provided by the NBER.

The overall conclusion is that it is feasible to use this bifactor model for forecasting the evolution of the US Post-War coincident economic indicator and its turning points up to the horizon of nine months.

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5 Appendix

Table 1. Estimated parameters of model 1
Log-likelihood: -4633.5

Parameter	Estimated	St. error	p-value
$p_{L.11}$	0.988	0.01	0.0
$p_{L.22}$	0.921	0.04	0.0
$p_{C.11}$	0.980	0.01	0.0
$p_{L.22}$	0.892	0.04	0.0
μ_{L1}	0.084	0.02	0.0
μ_{L2}	-0.527	0.11	0.0
μ_{C1}	0.255	0.04	0.0
μ_{L2}	-1.330	0.132	0.0
γ_{12}	0.745	0.241	0.0
γ_{13}	3.520	0.693	0.0
γ_{14}	3.46	0.672	0.0
γ_{26}	0.786	0.05	0.0
γ_{27}	0.898	0.06	0.0
γ_{28}	0.655	0.06	0.0
σ_1^2	0.936	0.06	0.0
σ_2^2	0.963	0.06	0.0
σ_3^2	0.228	0.03	0.0
σ_4^2	0.256	0.03	0.0
σ_5^2	0.279	0.04	0.0
σ_6^2	0.554	0.04	0.0
σ_7^2	0.419	0.04	0.0
σ_8^2	0.689	0.05	0.0
σ_L^2	0.018	0.007	0.01
σ_C^2	0.380	0.042	0.0

Table 2. Estimated parameters of model 2
Log-likelihood: -4682.6

Parameter	Estimated	St. error	p-value
p_{11}	0.988	0.01	0.0
$1 - p_{22}$	0.066	0.03	0.01
μ_{L1}	0.084	0.02	0.0
μ_{L2}	-0.434	0.09	0.0
μ_{C1}	0.134	0.04	0.0
μ_{L2}	-0.689	0.10	0.0
γ_{12}	0.746	0.25	0.0
γ_{13}	3.980	0.82	0.0
γ_{14}	3.61	0.75	0.0
γ_{26}	0.829	0.06	0.0
γ_{27}	0.998	0.06	0.0
γ_{28}	0.738	0.06	0.0
σ_1^2	0.945	0.06	0.0
σ_2^2	0.969	0.06	0.0
σ_3^2	0.164	0.03	0.0
σ_4^2	0.311	0.03	0.0
σ_5^2	0.358	0.04	0.0
σ_6^2	0.558	0.04	0.0
σ_7^2	0.360	0.04	0.0
σ_8^2	0.649	0.05	0.0
σ_L^2	0.016	0.01	0.01
σ_C^2	0.548	0.06	0.0

Table 3. Estimated parameters of model 3
Log-likelihood: -4641.1

Parameter	Estimated	St. error	p-value
p_{11}	0.983	0.01	0.0
p_{22}	0.873	0.05	0.0
A	4.950	2.18	0.01
B	8.840	3.5	0.01
μ_{L1}	0.090	0.02	0.0
μ_{L2}	-0.467	0.09	0.0
μ_{C1}	0.278	0.04	0.0
μ_{L2}	-1.040	0.10	0.0
γ_{12}	0.760	0.25	0.0
γ_{13}	3.590	0.71	0.0
γ_{14}	3.530	0.69	0.0
γ_{26}	0.791	0.05	0.0
γ_{27}	0.889	0.06	0.0
γ_{28}	0.656	0.06	0.0
σ_1^2	0.938	0.06	0.0
σ_2^2	0.963	0.06	0.0
σ_3^2	0.229	0.03	0.0
σ_4^2	0.254	0.03	0.0
σ_5^2	0.277	0.04	0.0
σ_6^2	0.548	0.04	0.0
σ_7^2	0.429	0.04	0.0
σ_8^2	0.688	0.05	0.0
σ_L^2	0.018	0.01	0.01
σ_C^2	0.433	0.04	0.0

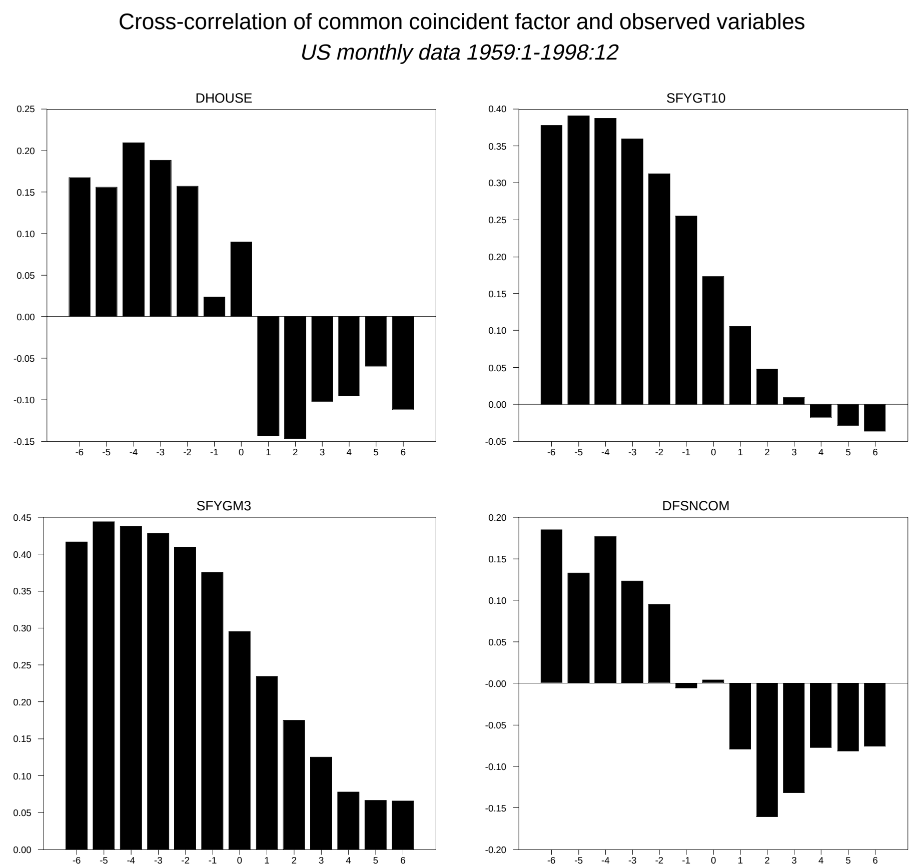

Cross-correlation of common coincident factor and observed variables
US monthly data 1959:1-1998:12
DHOUSE
SFYGT10
SFYGM3
DFSNCOM

Figure 1:

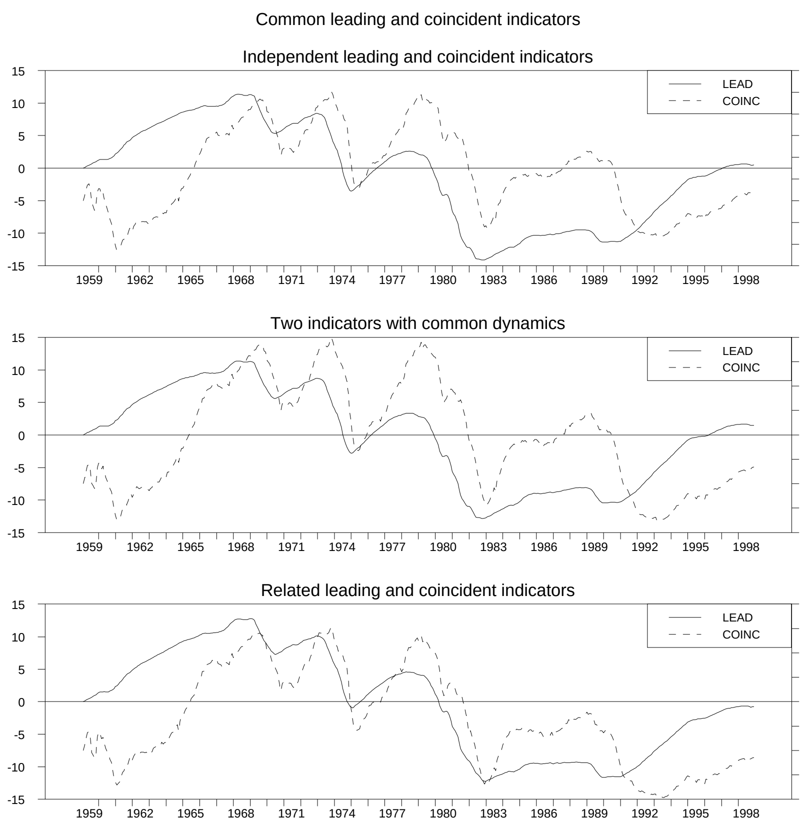

Common leading and coincident indicators
Independent leading and coincident indicators
LEAD
COINC
Two indicators with common dynamics
LEAD
COINC
Related leading and coincident indicators
LEAD
COINC
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 2:

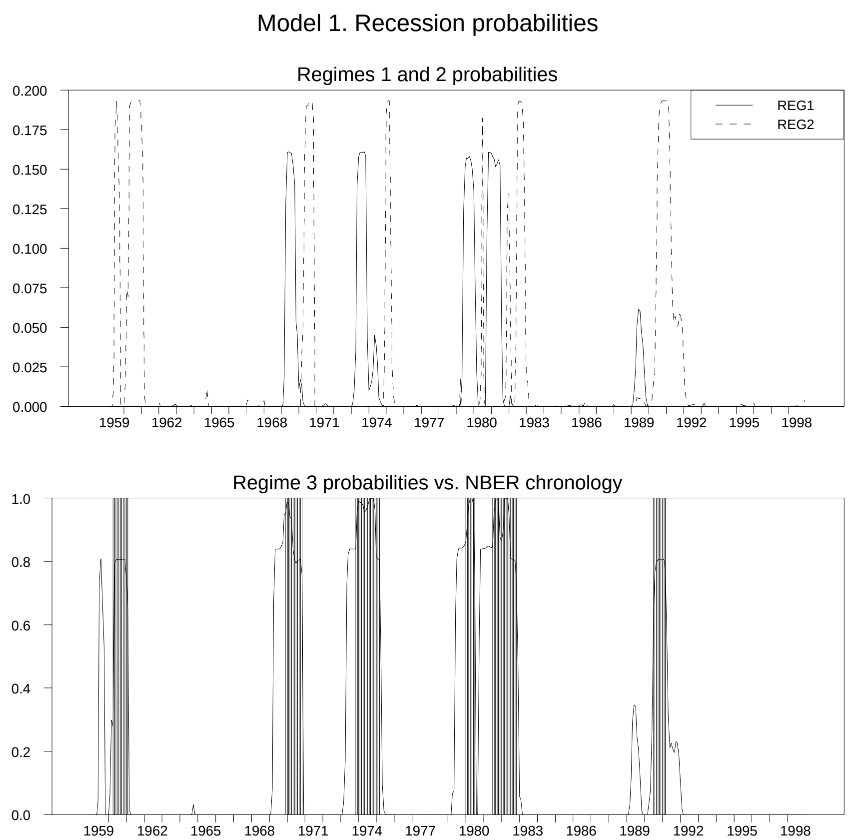

Model 1. Recession probabilities
Regimes 1 and 2 probabilities
REG1
REG2
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Regime 3 probabilities vs. NBER chronology
1.0
0.8
0.6
0.4
0.2
0.0
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 3:

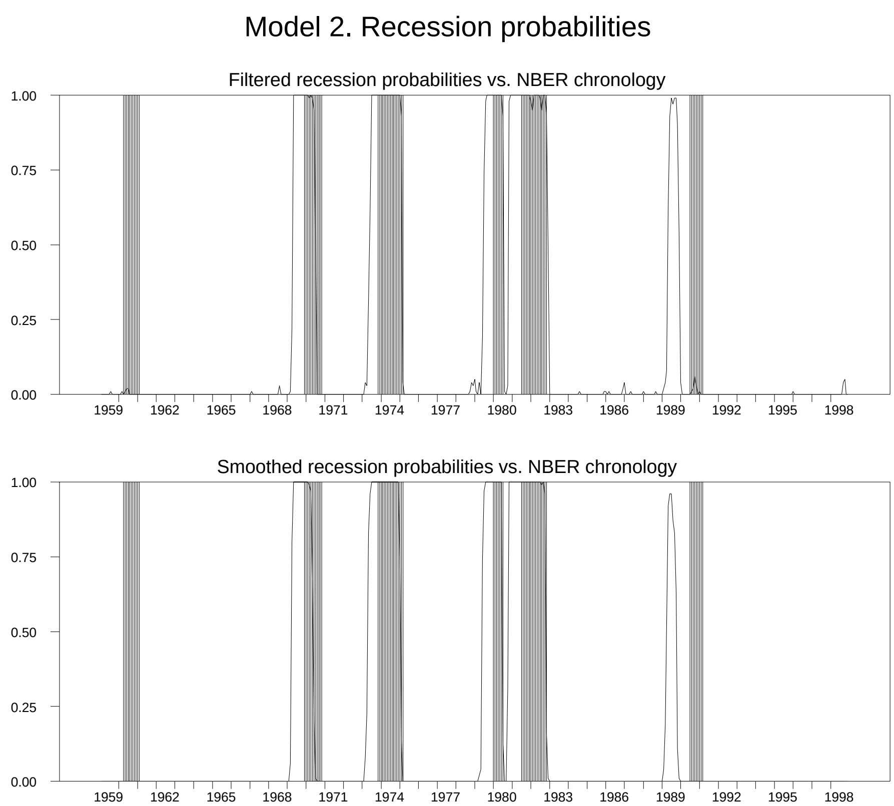
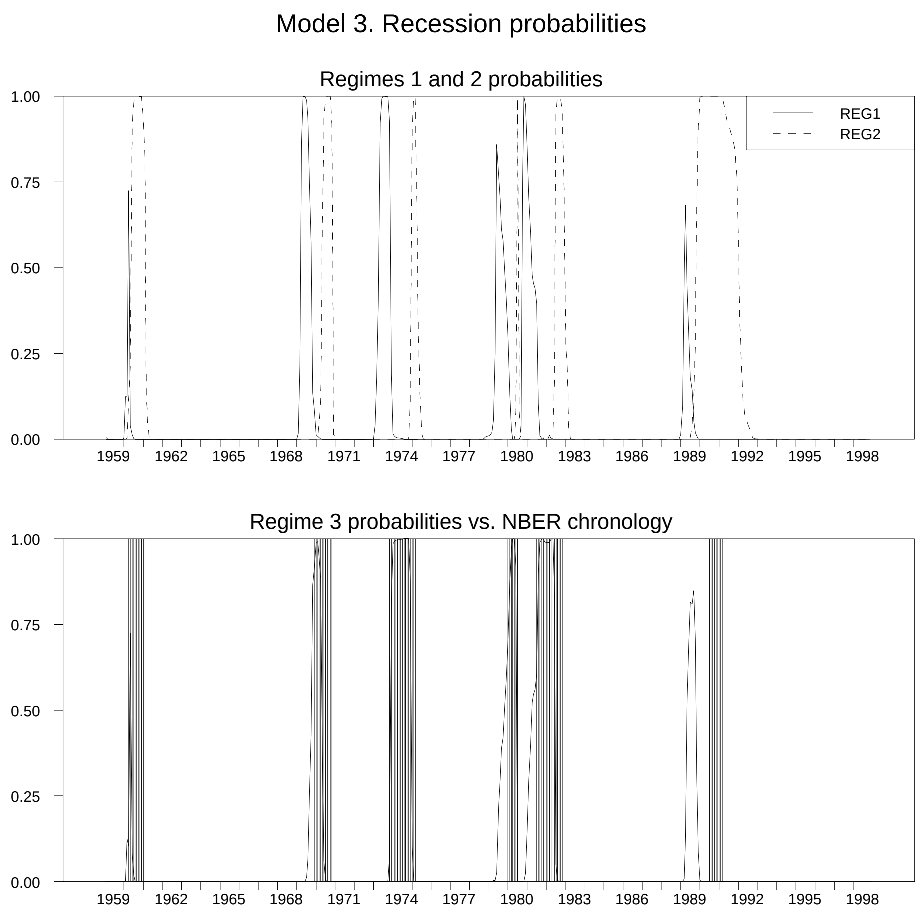

Model 2. Recession probabilities
Filtered recession probabilities vs. NBER chronology
1.00
0.75
0.50
0.25
0.00
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Smoothed recession probabilities vs. NBER chronology
1.00
0.75
0.50
0.25
0.00
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 4:


Model 3. Recession probabilities
Regimes 1 and 2 probabilities
REG1
REG2
1.00
0.75
0.50
0.25
0.00
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Regime 3 probabilities vs. NBER chronology
1.00
0.75
0.50
0.25
0.00
1959
1962
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1995
1998

Figure 5: