E C O N O M I C S   B U L L E T I N

Elasticity and revenue: do we need a reappraisal?

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Abstract

The relationship between price−elasticity of demand, price variations and total revenue changes might be considered as one of the most widely accepted results arising from consumer theory. Recently, however, this relationship has been put under suspicion on the basis of a misinterpretation of what economists have in mind when writing about it. In this paper we try to clarify concepts incorporating new elements into discussion with the aim of reaffirming the validity of this relationship.

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1. Introduction

In a paper recently published in this review (Quesada, 2002) it was held that the well-known relationship between price elasticity of demand, price changes and changes in seller's total revenue is far from being universally valid. This paper uses a definition of price elasticity of demand in which proportional changes are evaluated with respect to initial quantity and price. Here, we show that the results of the above-mentioned paper depend critically on the adopted measure of price elasticity and we prove that a more consistent characterisation of demand elasticity allows to easily overcome this author's criticism of the accepted viewpoint.

In section 2, we enlarge the scope of his study by introducing a definition of price elasticity according to which proportions are calculated with respect to final price and quantity. Then, we go on to prove that unexpected results are not obtained when both elasticities (that based on initial point and that based upon final point) are simultaneously greater (smaller) than one. Section 3 considers both definitions of price elasticity and shows that if both are simultaneously greater (smaller) than one traditional rules regarding price elasticity, price changes and revenue changes remain valid. Section 4 presents some reflections about the use (and abuse) of the concept of elasticity and recovers an aged measure that allows to circumvent any difficulty.

2. Price elasticity of demand and total revenue: Are there real problems?

In order to make comparison easier, we use an identical notation to that in the above-mentioned paper. So, let $a = (p_0, q_0)$ and $b = (p_1, q_1)$ be two different points of a strictly decreasing demand function defined for non-negative prices and quantities. Set $Q = q_1 / q_0$ and $P = p_1 / p_0$. Obviously, $P > 1$ implies $Q < 1$ and vice versa. We initially define price elasticity of demand from a to b as

$$\varepsilon = -[(q_1 - q_0) / q_0] / [(p_1 - p_0) / p_0] = (1 - Q) / (P - 1) \tag{1}$$

The change in total revenue due to the price change is $\Delta R = p_1 q_1 - p_0 q_0$. Thus, we have $\Delta R \gtreqless 0$ depending on whether $PQ \gtreqless 1$.

The relationship between price elasticity ε, price changes and changes in total revenue can be easily represented (see Figure 1) in the space (Q, P). A constant total revenue associates with the locus where $PQ = 1$, which corresponds to a rectangular hyperbola. Given our assumption of a strictly decreasing demand function, points located in the north-east region, where both Q and P are greater than one, and in the south-west region, were both Q and P are smaller than one, are not of interest for the analysis. Moreover, definition (1) implies that $\varepsilon = 1$ where $Q + P = 2$. Note that the curve $PQ = 1$ never lies below the straight line $Q + P = 2$ and both are tangent at (1, 1).

In Figure 1 we can identify six regions relevant to the analysis. Regions **A**, **B** and **C** imply $P > 1$ and $Q < 1$ (i.e. $\Delta p = p_1 - p_0 > 0$ and $\Delta q = q_1 - q_0 < 0$) while **D**, **E** and **F** display $P < 1$ and $Q > 1$ (i.e. $\Delta p < 0$ and $\Delta q > 0$). It can be easily shown that $\varepsilon > 1$ in regions **A**, **E** and **F**, whereas $\varepsilon < 1$ in **B**, **C** and **D**. Also, it is obvious that $\Delta R > 0$ in **C** and **F**, while $\Delta R < 0$ in the others. In **A**, **C**, **D**, and **F** the usual statements about elasticity, price changes and changes in total revenue hold true. However, these seem to be challenged in regions **B** and **E** inasmuch as, disputing the general viewpoint, in **B** we have $\Delta p > 0$ and $\Delta R < 0$ being $\varepsilon < 1$, and in **E** we have $\Delta p < 0$ and $\Delta R < 0$ being $\varepsilon > 1$. It is worth noting that these two latter results are not dependent on the magnitude of price change.

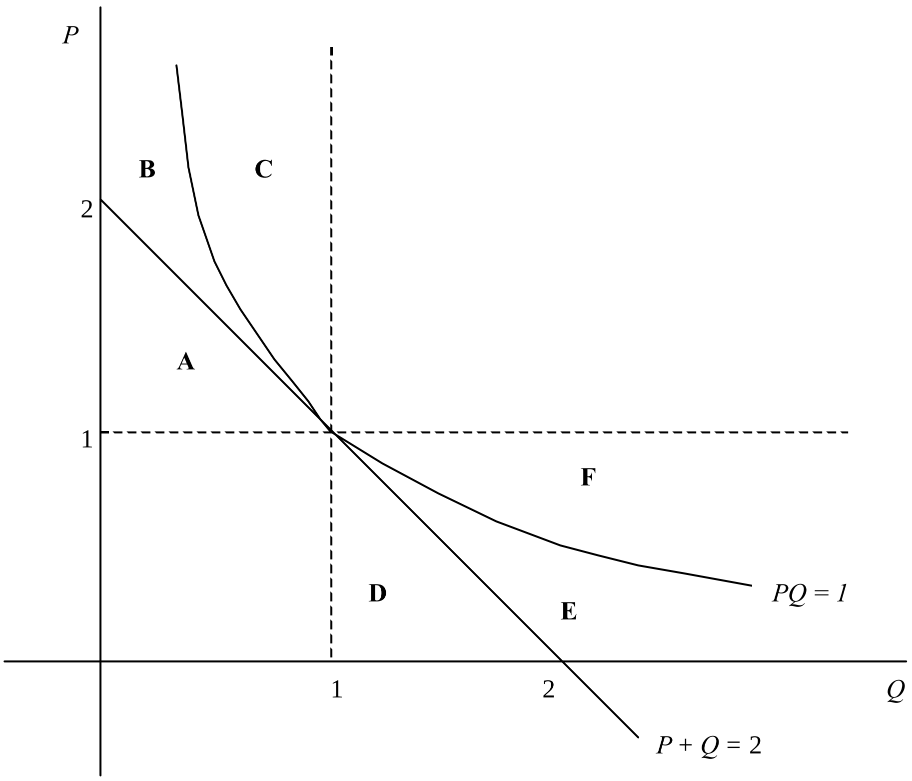

Figure 1. Relationship between ε and total revenue change

Are these results truly surprising? Clearly, they are not. In order to show that, let us define a new price elasticity of demand as

$$\varepsilon' = -\,[(q_1 - q_0)\,/\,q_1]\,/\,[(p_1 - p_0)\,/\,p_1] = [(1 - Q)\,/\,(P - 1)]\,[P\,/\,Q]\,, \qquad (2)$$

which differs from (1) only in the use of final point values to calculate the relevant proportions.

At this stage, it seems convenient to recover the usual practice of economists. For sure when they speak about a demand (or a segment of a demand) as elastic (inelastic) they assume that ε and ε' are both greater (smaller) than one for any pairs (q, p) belonging to that demand. If this were not the case, concepts such as elastic or inelastic demand, which are of maximum interest for the current practice, would be meaningless. In this way, we can summarize the conventional wisdom in the following proposition:

H1: The price change and the change in total revenue move in the same (opposite) direction if and only if demand is inelastic (elastic).

Consider now the situation in regions **B** and **E** if we introduce the requirement $\varepsilon' < 1$ when $\varepsilon < 1$ and, in the same way, $\varepsilon' > 1$ when $\varepsilon > 1$. Focusing on region **B**, where $P > 1$ and $\varepsilon < 1$, associated with previous requirements we also have

$$\varepsilon' < 1 \quad \Leftrightarrow \quad [(1 - Q) / (P - 1)]\, [P / Q] < 1 \quad \Leftrightarrow \quad [(Q + P) / 2] < PQ$$

Moreover in region **B** we have $Q + P > 2$. Then, we have to conclude that $PQ > 1$, which contradicts the fact that region **B** associates with $PQ < 1$.

Turning to region **E**, where $P < 1$ and also $\varepsilon > 1$, under the above mentioned requirements we have

$$\varepsilon' > 1 \quad \Leftrightarrow \quad [(1 - Q) / (P - 1)]\, [P / Q] > 1 \quad \Leftrightarrow \quad [(Q + P) / 2] < PQ$$

Moreover, in region **E** we have $Q + P > 2$. Then, we have to conclude that $PQ > 1$, which contradicts the fact that region **E** associates with $PQ < 1$.

Now, the causes of the results obtained by Quesada are very clear. The two cases that apparently disprove H1 do correspond to movements in which one goes from one point where $\varepsilon > 1$ ($\varepsilon < 1$) to another one where $\varepsilon' < 1$ ($\varepsilon' > 1$). The numerical examples presented by the author provide clear evidence on this point. Adjusting a straight line through the corresponding points it is obvious that we are going from the elastic (inelastic) part of the demand curve to the inelastic (elastic) one. Thus, these numerical examples do not provide evidence against H1 if the concept of elastic (inelastic) demand is treated in the way economists do.

3. Elasticity and revenue: the complete analysis

Figure 2 shows the complete partition of the space (Q, P) when it is considered not only ε but also ε'. In addition to the functions $PQ = 1$ and $Q + P = 2$, which represent the set of pairs (Q, P) associated with $\Delta R = 0$ and $\varepsilon = 1$, respectively, we have a new function corresponding to $\varepsilon' = 1$. This function is $P = Q / (2Q - 1)$ and, for the purposes of our analysis, it has two differentiated parts. If P and Q are both greater than 0.5, the locus where $\varepsilon' = 1$ associates with the strictly convex branch of the rectangular hyperbola that goes through the vertex (1, 1). However, when either P or Q are smaller than 0.5, the locus where $\varepsilon' = 1$ corresponds to the strictly concave branch of the rectangular hyperbola that cross through the vertex (0, 0).

Now, we can characterise the different regions in Figure 2 in terms of the behaviour of ε and ε'. Consider, for example, into region **A** the sub-area where $P > 1$ and $0 < Q < 0.5$. In this zone, P lies above the line corresponding to $\varepsilon' = 1$. Thus,

$$P > Q / (2Q - 1) \quad \Leftrightarrow \quad 2PQ - P < Q \quad \Leftrightarrow \quad P(Q - 1) < Q(1 - P) \quad \Leftrightarrow$$

$$[(1 - Q) / (P - 1)]\, [P / Q] < 1 \quad \Leftrightarrow \quad \varepsilon' > 1$$

Let us turn now over the sub-area where $P > 1$ and $0.5 < Q < 1$. In this zone, P lies below the line corresponding to $\varepsilon' = 1$. Thus,

$P < Q / (2Q - 1) \quad \Leftrightarrow \quad 2PQ - P < Q \quad \Leftrightarrow \; [(1 - Q) / (P - 1)] \, [P / Q] < 1 \quad \Leftrightarrow \; \varepsilon' > 1$

Operating in this way and taking advantage of the information provided by $PQ = 1$ and $P + Q = 2$, we can determine the behaviour of ε, ε', Δp, and ΔR in each region of Figure 2. Table 1 at the end of the section provides detailed information about these points. Using such information the relationship between price elasticity, price changes and changes in revenue can be established in a more precise way. First, we consider the regions where both ε and ε' offer a consistent measure of price elasticity, which implies that both measures are simultaneously greater or smaller than one. In region **A**, where ε > 1 and ε' > 1, Δp > 0 implies ΔR < 0. In **D**, where ε < 1 and ε' < 1, Δp < 0 implies ΔR < 0. In **C1**, where ε < 1 and ε' < 1, Δp > 0 implies ΔR > 0. Finally, in **F1**, where ε > 1 and ε' > 1, Δp < 0 implies ΔR >0. In short, as traditional rules and H1 predicate,

- If ε and ε' are both greater than one, then an increase (decrease) of price causes a decrease (increase) of total revenue.
- If ε and ε' are both smaller than one, then an increase (decrease) of price causes an increase (decrease) of total revenue.

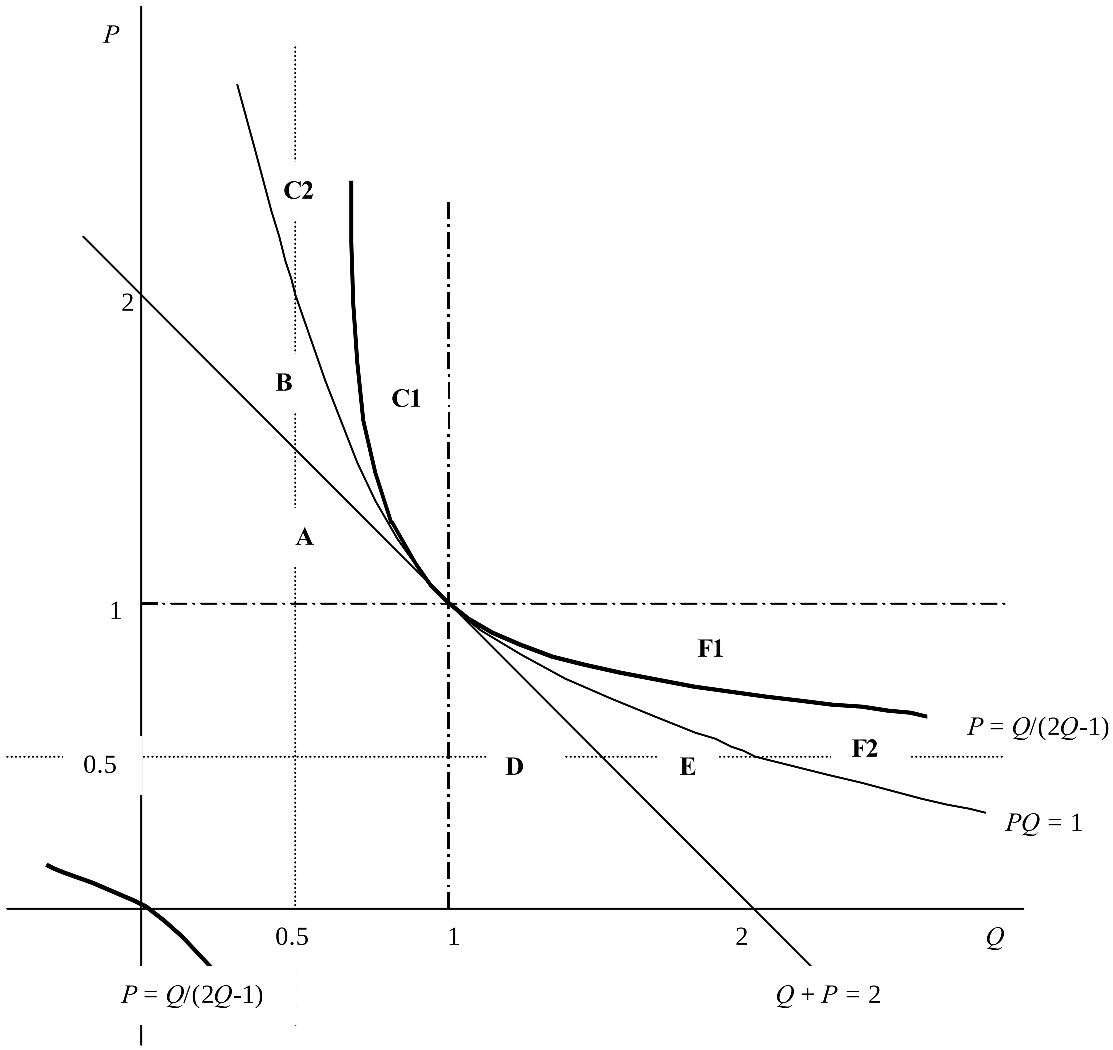

Figure 2. The complete partition of the space (Q, P) in terms of ε and ε'

Now let us turn our attention to regions where ε > 1 while ε' < 1 or vice versa. First, as proved in the previous section, in region **B**, ε < 1 and ε' > 1, while, in **E**, ε > 1 and ε' < 1. In these regions the usual relationship between price elasticity, price changes and changes in total revenue fails if we measure price elasticity through ε, but it holds true if we use ε'.

Turning now to **C2**, where ε < 1 and ε' > 1, we observe that $\Delta p > 0$ implies $\Delta R > 0$. Finally, in **F2**, where ε > 1 and ε' < 1, $\Delta p < 0$ implies $\Delta R > 0$. Thus, in these regions the expected relationship between price elasticity, price changes and revenue changes works if we consider ε, but fails in terms of ε'.

Table I. Price changes and revenue changes according to ε and ε' in Figure 2

Regions	In Figure 1	ε	ε'	Δp	ΔR
A	A	> 1	> 1	> 0	< 0
B	B	< 1	> 1	> 0	< 0
C1	C	< 1	< 1	> 0	> 0
C2	C	< 1	> 1	> 0	> 0
D	D	< 1	< 1	< 0	< 0
E	E	> 1	< 1	< 0	< 0
F1	F	> 1	> 1	< 0	> 0
F2	F	> 1	< 1	< 0	> 0

The previous discussion, summarised in Table I, allows us to conclude the validity of H1.

4. Final remarks

At this stage, any keen reader would think that our results are obvious. In fact, it would be enough to draw a straight demand, as for example $q = 100 - p$, to replicate most of the results obtained. For more than a century, the bulk of economists have been conscious that when measuring elasticity through a ratio of proportions they have to be very careful about the quantities and prices taken as reference. This is the only reason for using the concept of arc-elasticity, for which traditional rules hold as Quesada himself admits.

But arc-elasticity is not the only concept that allows us to overcome these difficulties. We can consider, for example, the following definitions of elasticity[1]:

$$\varepsilon'' = -[(q_1 - q_0) / q_0] / [(p_1 - p_0) / p_1] = (P - PQ) / (P - 1) \quad (3)$$

$$\varepsilon''' = -[(q_1 - q_0) / q_1] / [(p_1 - p_0) / p_0] = (1 - Q) / (PQ - Q) \quad (4)$$

[1] See Robinson and Eatwell (1976). Both authors attribute these definitions to the pioneering work of Marshall (1920) and Lerner (1933).

In this case, $\varepsilon'' = \varepsilon''' = 1$ imply $PQ = 1$. Moreover, it is easy to prove that $\varepsilon'' > 1$ (<1) if and only if $\varepsilon''' > 1$ (<1) and vice versa. All the problems associated with ε and ε' disappear when we use either ε'' or ε''' as a measure of price elasticity, both measures allowing to confirm H1 and to corroborate traditional rules as we can see in the following expression:

$$\Delta R / \Delta P = q_1(1 - \varepsilon''/PQ) = q_1(1 - \varepsilon''') \quad (5)$$

Leaving apart $\Delta R / \Delta P = q_1(1 - \varepsilon''')$, that mirrors the usual relationship between price elasticity, price changes and changes in total revenue as it is formulated in continuous terms, notice that regarding $\Delta R / \Delta P = q_1(1 - \varepsilon''/PQ)$ we have:

$$\varepsilon'' < 1 \iff PQ > 1 \iff \varepsilon''/PQ < 1$$

$$\varepsilon'' > 1 \iff PQ < 1 \iff \varepsilon''/PQ > 1$$

To finish let us pose a very simple question: what interest does price elasticity have to the seller if he disposes of full information about pairs (q, p) and, thus, can directly evaluate revenue change? The problem is usually that the seller does not have such information and price elasticity helps him to surpass this difficulty. Elasticity estimates are usually obtained from log–log demand relations, which make price elasticity independent of the variation in price (or quantity), allowing decision-makers to consider price elasticity as a constant for the estimated value. So, when elasticity estimates are usually around a specific value, decision-makers operate as if they were facing constant elasticity demand function. In this way, different elasticity values, not necessarily referred to demand relations, have come to be parameters almost universally accepted for policy makers and serve as a basis for action in real world[2].

References

Fuchs, V. R., Krueger, A. B. and J. M. Poterba (1998) "Economists' Views about Parameters, Values, and Policies: Survey Results in Labor and Public Economics" *Journal of Economic Literature* **XXXVI**, 1387-1425.

Lerner, A. P. (1933) "The Diagrammatic Representation of the Elasticity of Demand" *Review of Economic Studies* **1**.

Marshall, A. (1920) *Principles of Economics*, 8th edition, MacMillan: London.

Quesada, A. (2002) "Elasticity and Revenue: A Reappraisal" *Economics Bulletin* **4**, 1-5.

Robinson, J. and J. Eatwell (1976) *An Introduction to Modern Economics*, 2nd edition, McGraw-Hill: New York.

[2] Cfr. Fuchs, Krueger and Poterba (1998).