ECONOMICS BULLETIN

The Dynamic Correlation Between Growth and Unemployment

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Abstract

We apply the measure of dynamic correlation developed by Croux, Forni and Reichlin (2001) [A measure of comovement for economic variables: theory and empirics. Review of Economics and Statistics 83, 232−241] to the relation between the quarterly rates of unemployment and labor productivity growth for the post−war United−States economy. The application of the dynamic correlation reveals that these variables are strongly related, but in a different manner according to the frequencies considered: negatively at low frequencies and positively at business cycle frequencies.

I would like to thank Julien Matheron and Tristan−Pierre Maury for their helpful comments. Any remaining errors are mine.
Citation: Tripier, Fabien, (2002) "The Dynamic Correlation Between Growth and Unemployment." *Economics Bulletin,* Vol. 5, No. 4 pp. 1−9
Submitted: December 11, 2001. **Accepted:** April 6, 2002.
URL: http://www.economicsbulletin.com/2002/volume5/EB−01E30001A.pdf

1 Introduction

In modern macroeconomics, distinct methodologies have been developed to study the relation between productivity growth and unemployment depending on whether we are interested in the short-run or the long-run (see the both theoretical and empirical contributions of Christiano and Eichenbaum, 1992, for the short-run and of Bean and Pissarides, 1993, for the long-run). In this paper, we follow an alternative approach and propose to deal with the overall comovements between these two variables in an unified setup by means of spectral analysis. To do so, we resort to the notion of dynamic correlation developed by Croux et al. (2001).

The dynamic correlation is defined in section 2 and applied in section 3 to the relation between the quarterly rates of unemployment and labor productivity growth for the post-war United-States economy. It permits us to describe the sign and the amplitude of their comovements according to different frequencies: from long-run relations to short-run movements. If we look only at the contemporaneous comovements of the raw series, we would conclude that these two variables are weakly or not correlated. The application of the dynamic correlation reveals that these variables are strongly related, but in a different manner according to the frequencies considered. The dynamic correlation is negative at frequency zero, crosses the zero value at the frequency which corresponds to a cycle with a period of eight years, and reaches a peak at the frequency which corresponds to a cycle with a period of four years.

We then conclude that the rates of labor productivity growth and unemployment are negatively related in the long-run and positively in the short-run. This conclusion is discussed in section 4. In particular, we highlight the contribution of our empirical results to the theoretical controversy on the sign of the structural relations between growth and unemployment. Section 5 concludes.

2 Definition

This section takes up the Croux et al. (2001) definition of dynamic correlation in the particular case of a bivariate analysis (for more than two series, the authors introduce the notion of cohesion). Spectral analysis consists in depicting the covariance matrices in the frequency domain instead of the time domain. This approach has become very popular for describing the dynamic properties of univariate series through their spectrum. The dynamic correlation permits us to extend this approach to the comovements between two series in a more convenient way than traditional indeces used in time series analysis.

Consider two zero-mean real stochastic and stationary processes y and x. Let $s(\omega)$ be the spectral density matrix of the covariance matrices at frequency $-\pi \leq \omega \leq \pi$. The spectrum of x and y are denoted $s_x(\omega)$ and $s_y(\omega)$, and the cross spectrum between y and

x is denoted $s_{xy}(\omega)$. Let us introduce the coherency between x and y, denoted $h_{xy}(\omega)$, as a first comovement index. It is defined by

$$h_{xy}(\omega) = \frac{s_{xy}(\omega)}{\sqrt{s_x(\omega)\, s_y(\omega)}} \tag{1}$$

It measures the correlation between the complex representation of x_t and y_t at frequency ω. Unfortunately, since the cross spectrum has an imaginary part this index is not real. To obtain a more convenient measure of comovement, the squared coherency $h^2_{xy}(\omega)$ is then generally preferred in the literature. It is defined by

$$h^2_{xy}(\omega) = \frac{|s_{xy}(\omega)|^2}{s_x(\omega)\, s_y(\omega)} \tag{2}$$

and can be interpreted as the contribution of the frequency ω to the squared correlation coefficient. Nevertheless, this index is invariant with respect to a shift in the time process: $h^2_{xy}(\omega)$ is equal to $h^2_{zy}(\omega)$ where $z_t = x_{t+k}$ and k accounts for a shift in time process[1]. To avoid this limitative feature, Croux et al. (2001) propose to take the real part of the coherency, and call it the dynamic correlation

$$\rho_{xy}(\omega) = \frac{c_{xy}(\omega)}{\sqrt{s_x(\omega)\, s_y(\omega)}} \tag{3}$$

for $0 \leq \omega \leq \pi$. $c_{xy}(\omega) = \text{real}(s_{xy}(\omega))$ is the cospectrum between x and y. Contrary to the coherency it measures the correlation between the real waves of x_t and y_t instead of the complex ones. As emphasized by Croux and al. (2001), the dynamic correlation has the advantage of being real, taking values between -1 and 1, and being dependant upon a shift in the time process[2]. To illustrate the differences between the dynamic correlation and the squared coherency, an example is developed in the appendix .

We can also define the dynamic correlation on a frequency band. Let Ω be a frequency band which satisfies the following definition: $\Omega = \Omega_+ \cup \Omega_-$, where $\Omega_+ = [\omega_1, \omega_2)$ and $\Omega_- = [-\omega_1, -\omega_2)$ and $0 \leq \omega_1 \leq \omega_2 \leq \pi$. The static correlation coefficient between x_t and y_t over the frequency band Ω is defined by

$$\rho_{xy}(\Omega_+) = \frac{\int_{\Omega_+} c_{xy}(\omega)\, d\omega}{\sqrt{\int_{\Omega_+} s_x(\omega)\, d\omega \cdot \int_{\Omega_+} s_y(\omega)\, d\omega}} = \frac{\int_{\Omega_+} \rho_{xy}(\omega) \cdot \sqrt{s_x(\omega)\, s_y(\omega)} d\omega}{\sqrt{\int_{\Omega_+} s_x(\omega)\, d\omega \cdot \int_{\Omega_+} s_y(\omega)\, d\omega}} \tag{4}$$

For the particular band $\omega_1 = 0$ and $\omega_2 = \pi$, $\rho_{xy}(\Omega_+)$ is the static contemporaneous correlation coefficient. In other cases, the simple mean of the dynamic correlation values over a band is not a consistent measure of the static correlation coefficient.

[1] More precisely, the information encompassed in the phase on the delay between the comovements is lost (see Brockwell and Davis, 1991, ch. 11).

[2] In fact, it depends only on the value of such a shift and not on its sign: $\rho_{z(k)y}(\omega)$ is equal to $\rho_{z(-k)y}(\omega)$ where $z_t(k) = x_{t-k}$.

3 Application

Our data set comes from the Bureau of Labor Statistics and covers the period 1948:1-2000:4 at a quarterly periodicity for the United-States. The first data is the first differenced logged labor productivity and the second data is the unemployment rate (also taken in logarithm). They are denoted g and u, respectively. For both series a constant is removed.

To discuss the results, we retain a standard decomposition of the frequency band. Long-run comovements belong to the low frequency band $[0, \pi/16]$ (which corresponds to cycles with a period longer than 8 years). Business cycle comovements belong to the high frequency band $[\pi/16, \pi/3]$ (which corresponds to cycles with a period between 1.5 and 8 years). As traditional in the literature, we neglect movements at shorter horizons, on the frequency band $[\pi/3, \pi]$.

The computation of the dynamic correlation requires to estimate the empirical spectral density matrix of the process $\{g_t, u_t\}$. To do so, the empirical autocovariance function is estimated and smoothed with a Barlett window equal to $\sqrt{T}$, where T is the number of observations. The dynamic correlation defined by (3) is depicted on figure 1 and the static correlation coefficient per frequency band defined by (4) is reported in table 1.

Over the frequency band $\Omega_+ = [0, \pi]$, $\rho_{gu}(\Omega_+)$ is the static correlation coefficient between g_t and u_t. The corresponding value appears to be approximately zero, around 0.06. This result hides strong differences according to the frequency considered. The dynamic correlation takes its smallest value at frequency zero (around -0.375) and reaches a peak (around 0.663) near the frequency $\omega = \pi/7.5$, which corresponds to movements about 15 quarters. Between these two frequencies, the dynamic correlation is monotonically increasing and crosses the zero line around the frequency $\pi/16$. Its values are always negative over the frequency band $\Omega_+ = [0, \pi/16]$ and always positive over the frequency band $\Omega_+ = [\pi/16, \pi/3]$. Notice that for the last frequency band, there is no other peak or dip in the dynamic correlation.

The consequences for the static correlation coefficient are straightforward. Whereas the static correlation coefficient is close to zero for the frequency bands $[0, \pi]$ and $[\pi/16, \pi]$, it exhibits high absolute values for the two others frequency bands considered: $[0, \pi/16]$ and $[\pi/16, \pi/3]$. The static correlation coefficient is negative (-0.26) for long-run movements and positive (0.33) for the business cycle movements.

In conclusion, the rates of unemployment and labor productivity growth do not comove in the same manner on all the frequency bands. They comove negatively at low frequencies and positively at high frequencies. To assess the robustness of these results, we propose to retain alternative methods of estimation of the spectral density matrix (see Wei, 1990, for an exposition).

Table 2 and figure 2 report the results obtained with alternative estimation methods of the spectral density matrix. We first consider two alternative values for the Bartlett window ($\sqrt{T/2}$ and $\sqrt{2 \times T}$), we then apply a Hanning window (with a size of $\sqrt{T}$), and finally we estimate a bivariate VAR and deduce the corresponding spectral density matrix. The pattern of the results does not change with the method of estimation. For these four estimation methods, the dynamic correlation is negative at frequency zero and reaches a peak at business cycle frequencies. Nevertheless, the previously described pattern is minored with the small Bartlett window and reinforced with the three other estimation methods. In particular, the VAR estimation delivers the highest static correlation coefficient per frequency bands. We conclude that our basic results are robust to changes in the estimation of the spectral density matrix.

4 Discussion

Our results on business cycle frequencies recover the puzzling Dunlop-Tarshis observation: contrary to the prediction of standard real business cycle models, the empirical correlation between the cyclical components of labor productivity and employment is not strongly positive, but rather close to zero or negative (see Hansen and Wright, 1992). Our results confirm this finding by stating it the frequency domain. For long-run movements, it is more difficult to link our results to the literature due to the lack of studies on this dimension. A notable exception is Staiger et al. (2001) who confront the univariate trends of productivity growth and unemployment rate and conclude at a "striking and intriguing" negative correlation but make also caution about the relevance of the apparent correlation. Indeed, as already shown by Caballero (1993) such results depend crucially on the filter's choice to define the univariate trends. The results obtained in this paper confirm the finding of Staiger et al. (2001), but since we use spectral analysis our conclusion is not contingent upon a particular decomposition between the cyclical and trend components.

The theoretical implications of our results are straightforward for the short-run. A wide literature has been developed to account for the Dunlop-Tarshis observation (for example, Christiano and Eichenbaum, 1992, suggest to consider nontechnology shocks whereas Gali, 1999, puts forward the role of nominal rigidities). Our results confirm the importance of this issue and could be fruitfully used to assess the relevance of the different theoretical explanations by studying their spectral predictions on the dynamic correlation.

For long-run analysis, our results could contribute to the debate on the structural relation between growth and unemployment. Aghion and Howitt (1994) gave rise to a controversy about the sign of this relation by describing the steady state adverse effects of the long-run growth rate on the equilibrium rate of unemployment. Our results provide evidence against this view and suggest that these variables are more probably negatively related in the long-run. Nevertheless, Aghion and Howitt (1994) and the subsequent literature consider only the steady state properties of deterministic models. Further researches should then be devoted to study stochastic versions of these models in order to confront their spectral properties to the empirical relation described here.

5 Conclusion

In this paper, we applied the dynamic correlation measure developed by Croux et al. (2001) to describe the relation between the quarterly rates of unemployment and productivity growth for the post-war U.S. economy. The approximately zero value of the static correlation coefficient between the raw series hides strong differences between low and high frequencies: the correlation is negative for long-run movements and positive at business cycle frequencies. We then discussed the usefulness of this approach for assessing the relevance of alternative theoretical explanations of the relation between growth and unemployment either in the short-run or in the long-run.

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Appendix

To illustrate the properties of dynamic correlation, let us take an example developed by Brockwell and Davis (1991, ch. 11)

$$x_t = \varepsilon_{1t}$$
$$y_t = \varepsilon_{2t} + bx_{t-\lambda}$$

where ε_{1t} and ε_{2t} are two white noise with a variance σ^2. The spectral density matrix is

$$s(\omega) = \frac{\sigma^2}{2\pi} \begin{bmatrix} 1 & be^{i\lambda\omega} \\ be^{-i\lambda\omega} & 1+b^2 \end{bmatrix}$$

and the squared coherency and dynamic correlation are

$$\begin{aligned} h_{xy}^2(\omega) &= \frac{b^2}{1+b^2} \\ \rho_{xy}(\omega) &= \frac{b}{\sqrt{1+b^2}} \cos(\lambda\omega) \end{aligned}$$

From this simple example, we can see how the dynamic correlation is a more informative index than the squared coherency. The squared coherency depends only on the absolute value of the parameter b, that is the strength of the comovements. The dynamic correlation provides two additional information. The first one is the sign of b: the dynamic correlation gives information both on the strength and the sign of comovement. The second one is the influence of the absolute value of λ: whereas the squared coherency is constant, the dynamic correlation varies with ω for $\omega \in [0, \pi]$. To obtain a measure of comovements that gives information about the sign of λ, we would have to compute the phase between x and y. Finally, notice that over the frequency band $\Omega_+ = [0, \pi]$, we have $\rho_{xy}(\Omega_+) = 0$ which is natural since the static contemporaneous correlation coefficient between x_t and y_t is zero.

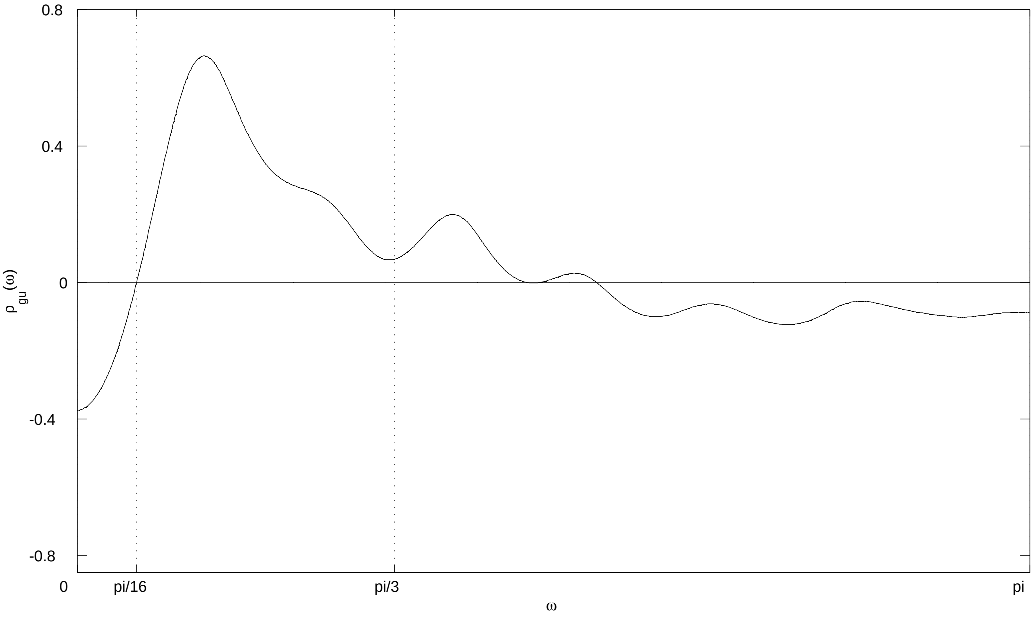

Figure 1: Dynamic correlation between g and u for the frequencies $\omega \in [0, \pi]$.

Table 1: Static correlation coefficient between g and u over the frequency bands Ω_+.

	Frequency bands Ω_+			
	$[0, \pi]$	$[0, \pi/16]$	$[\pi/16, \pi/3]$	$[\pi/3, \pi]$
Values of $\rho_{gu}(\Omega_+)$	0.060	-0.255	0.328	-0.020

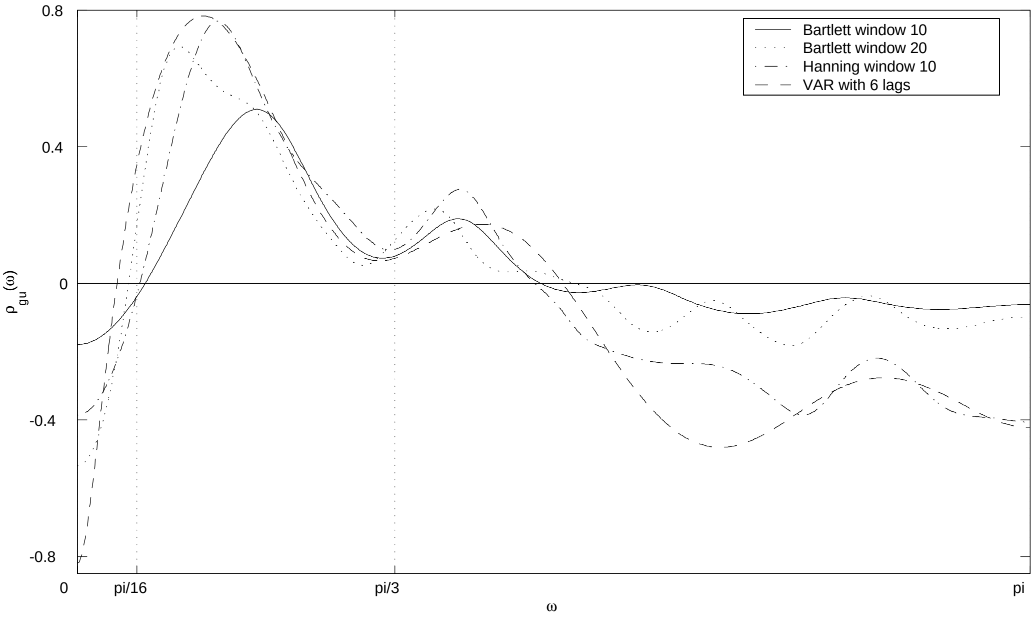

Figure 2: Dynamic correlation between g and u for the frequencies $\omega \in [0, \pi]$ for alternative estimation methods of the spectral density matrix. *Estimation methods:* Bartlett window 10 (solid line), Bartlett window 20 (dotted line), Hanning window 15 (dashdot line), and VAR with six lags (dashed line).

Table 2: Static correlation coefficient between g and u over the frequency bands Ω_+ for alternative estimation methods of the spectral density matrix.

	Frequency bands Ω_+			
	$[0, \pi]$	$[0, \pi/16]$	$[\pi/16, \pi/3]$	$[\pi/3, \pi]$
Values of $\rho_{gu}(\Omega_+)$ using the method:				
Bartlett window 10	0.060	-0.132	0.226	-0.006
Bartlett window 20	0.060	-0.328	0.405	-0.025
Hanning window 15	0.060	-0.274	0.357	-0.101
VAR with six lags	0.035	-0.460	0.514	-0.139