E C O N O M I C S   B U L L E T I N

The Segerstrom Model: Stability, Speed of Convergence and Policy Implications

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Abstract

This short paper presents a simple analytical stability proof for the well−known Segerstrom (1998) model of endogenous growth. Moreover, a calibrated version of the model is employed to assess the speed of convergence. The result shows that transitional dynamics are important and, hence, the policy ineffectiveness proposition of non−scale endogenous growth models must be qualified.

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1. Introduction

The Segerstrom (1998) approach has the clear merit of modelling endogenous growth in an empirically plausible manner. Moreover, it establishes itself as one of the workhorses of endogenous growth theory. For example, Dinopoulos and Segerstrom (1999) employ the model to analyse the effects of trade liberalisation between developed countries and Arnold (2002) uses the model to investigate the consequences of education policies on human capital accumulation and economic growth.

The paper at hand consists of parts: First, an analytical proof of stability is presented. Since the model is fairly simple, standard methods can be directly applied. It turns out that the balanced growth path is saddle-point stable. Indeterminacy and cyclical movements can be ruled out. Second, the Segerstrom model, like other non-scale endogenous growth models (e.g., Jones, 1995), bears far-reaching policy implications. Specifically, the model yields a policy ineffectiveness proposition according to which economic policy cannot control the long-run growth rate. Economic policy can nonetheless affect the growth rate along the transition path and, hence, it can influence the level of the balanced growth path. We should, therefore, explore the relative importance of transitional dynamics vis-à-vis balanced growth dynamics. The answer to this relative-importance question mainly hinges on the speed of convergence implied by the model under study.[1] A calibrated version of the model is employed to determine the speed of convergence numerically.

2. Sketch of the model

The Segerstrom (1998) model belongs to the class of R&D-based endogenous growth models of the increasing quality type. It is basically a Grossman-Helpman (1991) model modified to incorporate the idea of an increasing R&D difficulty.[2] Consumers value quality of goods positively. In each industry there is one quality leader who owns the patent to produce the state-of-the-art product and acts as a monopolist. Firms conduct R&D to discover new ideas. In the case of research success they are able to produce higher quality products and take a temporary monopoly position. The underlying R&D technology exhibits an increasing R&D difficulty. This allows Segerstrom to remove the empirically implausible scale effect

[1] The second determinant is the frequency and severity of macroeconomic shocks which push the economy from its balanced growth path (or move it even farer away). This is, of course, primarily an empirical issue.

[2] Segerstrom (1998, p. 1297) describes the basic notion by the words "*The most obvious ideas are discovered first, making it harder to find new ideas subsequently*."

according to which the long run growth rate increases with the scale (size) of the economy. Hence, the model belongs to the second generation of R&D-based endogenous growth models, which are characterised by non-scale growth. On the aggregate level, growth results from a continuous upgrade in the quality of consumer goods. The balanced growth rate of utility (which equals the balanced growth rate of real wages) is independent of the scale of the economy and turns out to read $g = (n/\mu)\log\lambda$, where n denotes the population growth rate, μ an R&D technology parameter indicating the pace at which R&D difficulty increases and λ the size of quality improvements, respectively. For details the reader is referred to Segerstrom (1998, Section II).

3. A simple analytical proof of stability

The dynamic evolution of the market economy is described by the following differential equations together with the initial conditions $c(0) > 0$ and $x(0) > 0$ (Segerstrom, 1998, p. 1300):

$$\dot{c}(t) = c(t)\left[\frac{(\lambda-1)A}{\lambda(1-s_R)}\frac{c(t)}{x(t)} + \frac{(\mu-1)A}{x(t)}\left(1-\frac{c(t)}{\lambda}\right) - \rho\right] \tag{1}$$

$$\dot{x}(t) = \mu A\left(1-\frac{c(t)}{\lambda}\right) - n\,x(t), \tag{2}$$

where $c(t)$ denotes per capita consumption expenditure and $x(t)$ represents relative R&D difficulty. As usual, $\dot{c}(t) := d\,c(t)/dt$ and $\dot{x}(t) := d\,x(t)/dt$. Moreover, $n > 0$ is the population growth rate, $\rho > 0$ the time preference rate, $0 \le s_R \le 1$ the fraction of R&D expenditures paid by the government, $\mu > 0$ the R&D difficulty parameter, $\lambda > 1$ indicates the size of quality improvements in the case of research success and $A > 0$ signifies R&D efficiency. For the analysis of stability, we need only one further restriction concerning the relative size of n and ρ, namely $\rho > n$.[3] The unique stationary solution of (1) and (2) is obtained by solving $\dot{c}(t) = 0$ and $\dot{x}(t) = 0$ for c and x. This solution, denoted as $\{\tilde{c}, \tilde{x}\}$, corresponds to the unique balanced growth path of this economy. The Jacobian matrix of (1) and (2) evaluated at $\{\tilde{c}, \tilde{x}\}$ is given by

[3] This restriction guarantees convergence of the utility integral.

$$\tilde{J} = \begin{pmatrix} \dfrac{[\mu - \lambda - s_R(\mu - 1)][n(\mu - 1) - \mu\rho]}{(\lambda - 1)\mu} & \dfrac{\lambda\rho(s_R - 1)[n - n\mu + \mu\rho]}{A(\lambda - 1)\mu} \\ -\dfrac{A\mu}{\lambda} & -n \end{pmatrix}. \tag{3}$$

The determinant of this Jacobian matrix reads as follows

$$Det(\tilde{J}) = \frac{[n(\mu - 1) - \mu\rho][n(\lambda + s_R(\mu - 1) - \mu) - (s_R - 1)\mu\rho]}{(\lambda - 1)\mu}. \tag{4}$$

Since $\lambda > 1$ and $\mu > 0$ the denominator of (4) is positive, i.e. $(\lambda - 1)\mu > 0$. Next we determine the sign of the numerator. At first, since $\rho > n > 0$ the first term is negative, i.e. $n(\mu - 1) - \mu\rho = \mu(n - \rho) - n < 0$. It remains to check the sign of the second term of the numerator. In order to accomplish this task, let us define the following function $f(s_R) := n(\lambda + s_R(\mu - 1) - \mu) - (s_R - 1)\mu\rho$. Setting s_R equal to zero yields $f(0) = n(\lambda - \mu) + \mu\rho = n\lambda + \mu(\rho - n) > 0$. On the other hand, setting s_R equal to one gives $f(1) = n(\lambda - 1) > 0$. Since $f(s_R)$ is a monotonic function in s_R we conclude that $f(s_R) > 0$ for all $s_R \in [0,1]$. From $(\lambda - 1)\mu > 0$, $n(\mu - 1) - \mu\rho < 0$ and $f(s_R) > 0$ it follows that $Det(\tilde{J}) < 0$. Moreover, let the eigenvalues of the Jacobian matrix be denoted as b_1 and b_2. By remembering that $Det(\tilde{J}) = b_1 b_2$ we immediately obtain $b_1 < 0 < b_2$.[4] Hence, the balanced growth path is saddle-point stable. The equilibrium growth path is unique and indeterminacy cannot arise. Furthermore, since $b_1, b_2 = 0.5\left\{Tr(\tilde{J}) \pm \left[Tr(\tilde{J})^2 - 4Det(\tilde{J})\right]^{0.5}\right\}$ the eigenvalues are real and cyclical movements can be ruled out.

4. Speed of convergence and policy implications

At this stage it is clearly interesting to determine the speed of convergence for two reasons. First, the speed of convergence represents a quantitative empirical implication of the model under study. Second, it allows us to assess the relative importance of transitional dynamics as opposed to balanced growth dynamics.

The speed at which some variable $q(t)$ converges to its balanced growth path $\tilde{q}(t)$ is measured by the (instantaneous) rate of convergence $\psi_q(t) := -\frac{\dot{q}(t) - \dot{\tilde{q}}(t)}{q(t) - \tilde{q}(t)}$. In the limit, the rate of convergence is constant and equals the (negative of the) stable eigenvalue of the Jacobian matrix, i.e. $\lim_{t\to\infty} \psi(t) = \tilde{\psi} = -b_1 > 0$. For the model under study this eigenvalue can be determined analytically. The resulting expression is, however, too unwieldy to yield much insights. The stable eigenvalue can nonetheless be calculated by employing a calibrated model. Fortunately, the Segerstrom (1998) model contains only a small number of parameters, which can largely be determined empirically. The following baseline set of parameters is employed: $n = 0.01$, $\lambda = 1.3$, $\rho = 0.04$, $s_R = 0.1$, $\mu = 0.25$, $A = 1$.[5] In this case, the model implies a local rate of convergence amounting to $\tilde{\psi} \cong 0.019$ or a half-life of about 38 years. This value is compatible with the majority of cross-country studies on the speed of convergence (e.g., Barro and Sala-i-Martin, 1992). Notice that the neoclassical growth model (plausibly calibrated) yields rates of convergence of about $\tilde{\psi} \cong 0.12$ corresponding to a half-life of about 5.5 years (Barro and Sala-i-Martin, 1992, p. 226). On the other hand, Jones (1995) shows that convergence is very slow (a half-life of per capita income of 62 years or even more) in his horizontal innovation model. This result is, however, derived by holding basic allocation variables fixed.

The finding of a half-life of about 38 years indicates that, according to the model under study, transitional dynamics are important and should not be neglected. The importance of this result follows from the fact that non-scale growth models are usually marked by policy ineffectiveness. More specifically, these models imply that public policy cannot control the long-run growth rate. Public policy can, however, influence the growth rate along the transition path and thereby control the level of the balanced-growth path (e.g., Jones, 1995). It

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[4] For quadratic matrices it holds in general that the determinant of the matrix equals the product of its eigenvalues (e.g. Simon and Blume, 1994, Theorem 23.9).

[5] Following Segerstrom (1998) n is interpreted as the world population growth rate and hence $n = 0.01$. Estimates on the average percentage mark-up over marginal cost yield values in the interval [0.1, 0.4] (e.g., Basu, 1996). We choose 0.3 as the benchmark value. Since this percentage mark-up can be considered as a proxy for $\lambda - 1$, it follows that $\lambda = 1.3$. The OECD average share of business enterprise R&D expenditure financed by government amounts to $s_R = 0.1$ (OECD, 2000, p. 31). Moreover, the model implies a steady state growth rate of real wages equal to $g = (n/\mu)\log\lambda$. Setting the average growth rate of real wages (approximated by the average growth rate of real per capita GDP) to 0.01 yields $\mu = 0.25$.

should be clear that different levels of the balanced growth path can be associated with dramatic differences in welfare.

Of course, the rate of convergence calculated above is valid for one point in the parameter space only. In order to assess the robustness of the preceding proposition on the rate of convergence, the stable eigenvalue (b_1) is successively calculated for varying parameters holding the remaining parameters fixed. Figure 1 summarises the result of this experiment. It can be recognised that the stable eigenvalue does not vary substantially with the respective parameters. The strongest impact comes from the population growth rate [plot (a)] and the time preference rate [plot (b)]. Moreover, Figure 1 shows how the rate of convergence is qualitatively affected by changes in the respective parameters.[6]

FIGURE 1

5. Conclusion

The paper at hand presents a simple analytical proof of stability for the Segerstrom (1998) model. The restrictions on the single parameters and one restriction on the relative size of two parameters (namely $\rho > n$) suffice to show that the balanced growth path is saddle-point stable. Indeterminacy and cyclical movements can be ruled out.

A calibrated version of the model is used to assess the speed of convergence. Based on an empirically plausible baseline set of parameters the local rate of convergence amounts to nearly 2 % being in line with the majority of cross-country studies on the speed of convergence. Moreover, the result is largely robust with respect to parameter changes. The findings indicate that transitional dynamics are important and, hence, the policy ineffectiveness proposition of non-scale endogenous growth models must be qualified.

[6] The fact that the rate of convergence is independent of A is due to the general non-scale character of the model. The results should be compared to similar results on the rate of convergence in endogenous growth models (e.g., Ortigueira and Santos, 1997).

6. References

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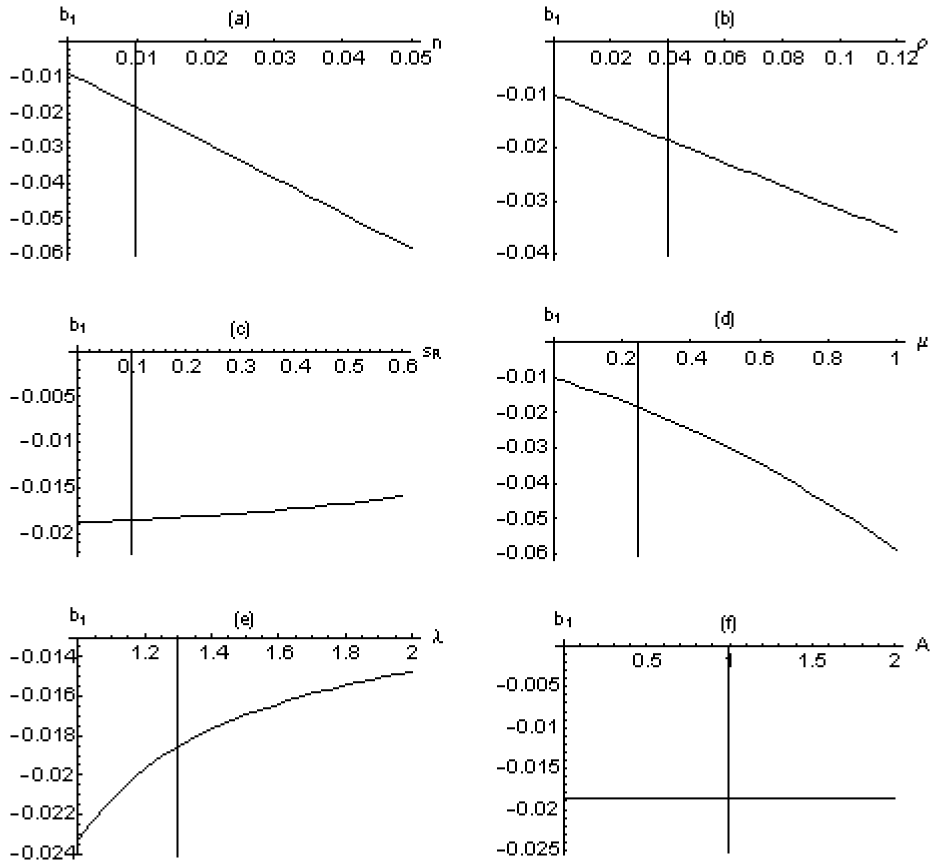

Figure 1: Stable eigenvalue (b_1) in response to parameter variations (vertical lines indicate the value of the respective parameter within the baseline set of parameters).