E C O N O M I C S   B U L L E T I N

A new set of critical values for systems cointegration tests with a prior adjustment for deterministic terms

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Abstract

In this note I present a new set of simulated percentiles of asymptotic distributions regarding systems cointegration tests with a prior adjustment for deterministic terms suggested by Saikkonen and Lütkepohl (2000a, 2000b, 2000c) and Saikkonen and Luukkonen (1997). The new percentiles are based on an improved random number generator implemented in GAUSS V3.6 and make critical values available for a larger range of percentage points and higher−dimensional systems.

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1 Introduction

In this note I present a new set of percentiles of asymptotic distributions regarding systems cointegration tests with a prior adjustment for deterministic terms suggested by Saikkonen and Lütkepohl (2000*a*; 2000*b*; 2000*c*) and Saikkonen and Luukkonen (1997). These test versions differ with respect to the deterministic terms they allow for. The procedures of Saikkonen and Lütkepohl (2000*b*) are the most general ones by taking account of shifts in the level of the time series. One test version assumes a linear trend while the other one excludes it. The test by Saikkonen and Lütkepohl (2000*c*) is the corresponding one without level shifts. It originally allows for a linear trend but can be adjusted in order to rule out a trend explicitly. A similar test with a mean term only is due to Saikkonen and Luukkonen (1997). Finally, Saikkonen and Lütkepohl (2000*a*) assume a linear trend that is orthogonal to the cointegration space. Furthermore, seasonal dummy variables can be incorporated into all procedures without changing the asymptotic results. The idea of the tests is to estimate the deterministic terms in a first step and to adjust the original time series by these estimated terms. Then, a likelihood ratio type test like in Johansen (1988) is applied to the adjusted data. The resulting asymptotic distributions are nonstandard and functions of Brownian motions. However, their percentiles can be obtained by simulation.

I recalculate and extend the sets of percentiles stated in Lütkepohl and Saikkonen (2000) and Saikkonen and Lütkepohl (2000*c*) for the tests of Saikkonen and Lütkepohl (2000*a*; 2000*b*; 2000*c*). The new sets are computed using GAUSS V3.6 which incorporates a random number generator that is supposed to be less affected by non-randomness problems than the one in GAUSS V3.2. The latter programm was applied for the calculations of the old percentiles. Moreover, critical values for up to 15-dimensional systems can be obtained from eight different percentiles in contrast to the previous limitation of at most five-dimensional systems and three different percentage points. Since the test version in Saikkonen and Luukkonen (1997) follows the same asymptotic distribution as the cointegration test statistic in Johansen (1988) the recalculated percentiles with respect to this version correspond to the ones in Johansen (1995, Table 15.1). The quantiles in Johansen (1995) were calculated by using the program DisCo written in Turbo Pascal 5 (see Johansen and Nielsen, 1993).

The note is organized as follows. The next section describes the model framework and the test hypotheses and Section 3 presents the test statistics and their limiting distributions. Finally, the simulation and computation details as well as the percentiles are given in Section 4.

2 Model Framework and Test Hypotheses

Let us consider a n-dimensional times series $y_t = (y_{1t}, \ldots, y_{nt})'$ $(t = 1, \ldots, T)$ which is generated by

$$y_t = \mu_t + x_t, \quad t = 1, 2, \ldots, \tag{1}$$

where μ_t contains the deterministic terms depending on the tests' assumptions. The term x_t is an unobservable stochastic error process which is assumed to follow a vector autoregressive process of order p (VAR(p)). The corresponding vector error correction model (VECM) has the form

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \varepsilon_t, \quad t = 1, 2, \ldots, \tag{2}$$

where Π and Γ_j $(j = 1, \ldots, p-1)$ are $(n \times n)$ unknown parameter matrices. The error term ε_t is assumed to be a martingal sequence such that $E(\varepsilon_t | \varepsilon_s, s < t) = 0$, $E(\varepsilon_t \varepsilon_t' | \varepsilon_s,\ s < t) = \Omega$ is a non-stochastic positive definite matrix and the fourth moments are bounded. For the validity of the limiting distributions it suffices that the initial values x_t $(t = -p+1, \ldots, 0)$ have a fixed distribution which does not depend on the sample size. Furthermore, it is assumed that x_t is at most integrated of order one and cointegrated with a rank r implying the same properties for y_t. Moreover, it follows that the matrix Π can be written as $\Pi = \alpha\beta'$, where α and β are $(n \times r)$ matrices of full column rank. When determining the number of cointegration relations one tests for the rank of the matrix Π. I consider the so-called trace test versions, i.e. the pair of hypotheses

$$H_0(r_0) : \mathrm{rk}(\Pi) = r_0 \quad vs. \quad H_1(r_0) : \mathrm{rk}(\Pi) > r_0. \tag{3}$$

is tested.

3 Test Statistics and Asymptotic Distributions

For the proposal of Saikkonen and Lütkepohl (2000b) we have $\mu_t = \delta d_t + \mu_0 + \mu_1 t$ allowing for a linear trend and assuming only one level shift. The shift is modelled by the dummy variable d_t which is one for all $t \geq T_1$ and zero otherwise where T_1 is the break point. The case of several level shifts can be treated in the same way by defining further shift dummies. Obviously, the remaining two quantities μ_0 and $\mu_1 t$ represent the mean and linear trend terms. The unknown $(n \times 1)$ parameter vectors δ, μ_0 and μ_1 are estimated by a GLS procedure. To obtain feasible GLS estimators $\hat{\delta}$, $\hat{\mu}_0$, and $\hat{\mu}_1$ the model (1) is transformed accordingly by using first stage estimators from a reduced rank (RR) regression of

$$\Delta y_t = \nu + \alpha(\beta' y_{t-1} + \phi d_{t-1} + \tau(t-1)) + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \sum_{j=0}^{p-1} \gamma_j \Delta d_{t-j} + \varepsilon_t, \quad t = p+1, p+2, \ldots, \tag{4}$$

with $\nu = -\Pi\mu_0 + (I_n - \sum_{j=1}^{p-1} \Gamma_j)\mu_1$, $\phi = -\beta'\delta$, $\tau = -\beta'\mu_1$, $\gamma_j = \delta$ for $j = 0$, and $\gamma_j = \Gamma_j \delta$ for $j = 1, \ldots, p-1$. This VECM model for y_t is derived from (1) using the aforementioned definition of μ_t and (2). For the RR regression the rank r_0 specified under H_0 is applied since the transformation of (1) considers the structure of x_t under the null hypothesis. Having estimated the deterministic terms one can adjust y_t and compute $\hat{x}_t = y_t - \hat{\delta} d_t - \hat{\mu}_0 - \hat{\mu}_1 t$. Then, a Johansen-type test is performed on $\hat{x}_t$. Since $\hat{x}_t$ is adjusted the test version in Johansen (1988) assuming no deterministic terms is applied. Hence, we have to solve a generalized eigenvalue problem. Using the resulting eigenvalues $\hat{\lambda}_1 \geq \cdots \geq \hat{\lambda}_n$ the test statistic for the pair of hypotheses in (3) is[1]

$$LR^{\delta}_{trend}(r_0) = T \sum_{j=r_0+1}^{n} \log(1 + \hat{\lambda}_j). \tag{5}$$

If no linear trend is present one proceeds in the same way by making the necessary adjustments. In line with Saikkonen and Lütkepohl (2000*b*) μ_1 is set to zero, i.e. $\mu_t = \delta d_t + \mu_0$ in (1), and the intercept term in (4) is restricted to the cointegration relations since $\nu = \Pi\mu_0$ and $\tau = 0$ in this case. The resulting test statistic is denoted by $LR^{\delta}_{mean}(r_0)$.

In case of no level shifts one can set up cointegration tests as before by setting all terms associated with the level shift in (1) and (4) to zero. These tests are due to Saikkonen and Lütkepohl (2000*c*). Interestingly, their test statistics, abbreviated as $LR_{trend}(r_0)$ and $LR_{mean}(r_0)$, have the same limiting distributions as $LR^{\delta}_{trend}(r_0)$ and $LR^{\delta}_{mean}(r_0)$ respectively.

[1]Note that the generalized eigenvalue problem in Saikkonen and Lütkepohl (2000*c*) is formulated in a different way than in Johansen (1988). Therefore, the obtained eigenvalues and the specific form of the test statistic differ. However, the two kinds of eigenvalue problems can be transformed into each other by an appropriate redefinition of the respective eigenvalues. Thus, the test statistics based on the two different sets of eigenvalues are identical apart from minor numerical differences.

To be precise, allowing for level shifts does not affect the asymptotic distribution. For the situation of a mean term only, Saikkonen and Luukkonen (1997) have also proposed to estimate μ_0 by a GLS procedure based on first-stage estimators from a VAR model for y_t imposing no rank restriction. Lütkepohl, Saikkonen and Trenkler (2001), however, have pointed out that using a GLS procedure as suggested by Saikkonen and Lütkepohl (2000*c*) results in better size properties in small samples. Note that the alternative way of estimating μ_0 does not change the asymptotic null distribution of $LR_{mean}(r_0)$.

The test version of Saikkonen and Lütkepohl (2000*a*) is similar to the one in Saikkonen and Lütkepohl (2000*c*) with a linear trend but considers the restriction that the linear trend is orthogonal to the cointegration space. Therefore, the restriction $\tau = \beta'\mu_1 = 0$ is imposed within the RR regression and secondly, the adjustment of the data occurs according to the model

$$\Delta y_t - \mu_1 = \nu + \Pi(y_{t-1} - \mu_0) + \sum_{j=1}^{p-1} \Gamma_j(\Delta y_{t-j} - \mu_1) + \varepsilon_t, \quad t = p+1, p+2, \ldots, \tag{6}$$

which is obtained from (4) by applying $\tau = \beta'\mu_1 = 0$ and $\delta = 0$. Otherwise the test-setup is the same and the corresponding test statistic is denoted as $LR_{ort}(r_0)$.

Let $\mathbf{B}_p(s) = (B_1(s), \ldots, B_p(s))'$ be a p-dimensional standard Brownian motion, then the test statistics have the following asymptotic null distributions: $LR^{\delta}_{trend}(r_0)$, $LR_{trend}(r_0) \xrightarrow{d} \mathrm{tr}(D_{tr})$, $LR_{ort}(r_0) \xrightarrow{d} \mathrm{tr}(D_{ort})$, and $LR^{\delta}_{mean}(r_0)$, $LR_{mean}(r_0) \xrightarrow{d} \mathrm{tr}(D_{mean})$ where

$$\begin{aligned}
D_{tr} &= \left(\int_0^1 \mathbf{B}_* \mathrm{d}\mathbf{B}_*'\right)' \left(\int_0^1 \mathbf{B}_*\mathbf{B}_*' \mathrm{d}s\right)^{-1} \left(\int_0^1 \mathbf{B}_* \mathrm{d}\mathbf{B}_*'\right), \\
D_{ort} &= \left(\int_0^1 \bar{\mathbf{B}}^s \mathrm{d}\mathbf{B}_{(n-r_0)}'\right)' \left(\int_0^1 \mathbf{B}^s\mathbf{B}^{s'} \mathrm{d}s\right)^{-1} \left(\int_0^1 \bar{\mathbf{B}}^s \mathrm{d}\mathbf{B}_{(n-r_0)}'\right), \\
D_{mean} &= \left(\int_0^1 \mathbf{B}_{(n-r_0)} \mathrm{d}\mathbf{B}_{(n-r_0)}'\right)' \left(\int_0^1 \mathbf{B}_{(n-r_0)}\mathbf{B}_{(n-r_0)}' \mathrm{d}s\right)^{-1} \left(\int_0^1 \mathbf{B}_{(n-r_0)} \mathrm{d}\mathbf{B}_{(n-r_0)}'\right),
\end{aligned} \tag{7}$$

$\mathbf{B}_* = \mathbf{B}_{(n-r_0)}(s) - s\mathbf{B}_{(n-r_0)}(1)$ is an $(n-r_0)$-dimensional Brownian bridge, $\mathrm{d}\mathbf{B}_* = \mathrm{d}\mathbf{B}_{(n-r_0)}(s) - \mathrm{d}s\mathbf{B}_{(n-r_0)}$, $\bar{\mathbf{B}}^s = \mathbf{B}^s(s) - \int_0^1 \mathbf{B}^s(u)du$, $\mathbf{B}^s(s) = [\mathbf{B}_{(n-r_0-1)}(s)' : s]'$, $\mathbf{B}_{n-r_0}$ and $\mathrm{d}\mathbf{B}_{n-r_0}$ are short for $\mathbf{B}_{n-r_0}(s)$ and $\mathrm{d}\mathbf{B}_{n-r_0}(s)$, and $\xrightarrow{d}$ signifies convergence in distribution.

Obviously, the null distributions depend on $n - r_0$, not on n and r_0 separately. They are independent of the actual values of μ_0 and, regarding $LR_{trend}(r_0)$, independent of μ_1. Furthermore, $LR_{mean}(r_0)$ has the same limiting distribution as the cointegration test statistic proposed by Johansen (1988) which assumes no deterministic terms ($\mu_0 = \mu_1 = 0$).

Table 1. Percentiles of the Asymptotic Distribution of $LR^{\delta}_{trend}(r_0)$ and $LR_{trend}(r_0)$ Test Versions Allowing for a Linear Trend (μ_1 arbitrary)

$n - r_0$	50%	75%	80%	85%	90%	95%	97.5%	99%
1	2.092	3.544	3.997	4.592	5.423	6.785	8.217	10.042
2	8.318	10.924	11.674	12.556	13.784	15.826	17.700	19.854
3	18.275	22.031	23.063	24.317	25.931	28.455	30.914	33.757
4	32.163	37.065	38.360	39.983	42.083	45.204	48.142	51.601
5	50.052	56.045	57.601	59.480	61.918	65.662	69.227	73.116
6	71.854	79.007	80.874	83.142	86.015	90.346	94.395	98.990
7	97.338	105.755	107.917	110.453	113.711	118.898	123.497	128.801
8	126.994	136.528	138.891	141.764	145.423	150.985	156.028	162.142
9	160.411	171.071	173.735	177.035	181.213	187.242	193.114	199.584
10	197.999	209.650	212.721	216.299	220.921	227.989	234.149	241.795
11	239.120	252.001	255.452	259.257	264.210	271.707	278.265	285.934
12	284.478	298.505	302.105	306.284	311.711	319.827	326.750	334.987
13	333.018	348.575	352.507	357.086	363.028	371.287	378.977	388.476
14	386.071	402.484	406.560	411.438	417.682	427.362	435.864	445.787
15	442.611	460.253	464.781	469.959	476.405	486.527	495.017	504.545

4 Simulated Percentiles

The limiting distributions are simulated numerically by approximating the standard Brownian motions in (7) with 1,000-step random walks of the same dimension. To this end, I generate p-dimensional independent standard normal variates ϵ_t of length $T = 1000$. The random walks are then obtained as $p \times T$ vector $\mathbf{W}_p^T = (W_p^1, W_p^2, \ldots, W_p^T)$ with $W_p^t = \Sigma_{i=1}^t \epsilon_i$. Accordingly, $\mathbf{B}^s$, $\bar{\mathbf{B}}^s$, $\mathbf{B}_{n-r_0}$, and d$\mathbf{B}_{n-r_0}$ are replaced by their corresponding discrete counterparts using $\mathbf{W}^T_{n-r_0-1}$ and $\mathbf{W}^T_{n-r_0}$ for the first two and the last two expressions respectively. The terms $\int_0^1 \mathbf{B}_* d\mathbf{B}_*'$ and $\int_0^1 \mathbf{B}_*\mathbf{B}_*'$ are approximated by $T^{-1}[\mathbf{W}^{T-1}_{n-r_0} - \bar{W}^T_{n-r_0}\mathbf{T}_{-1}][(\mathbf{W}^T_{n-r_0} - \mathbf{W}^{T-1}_{n-r_0}) - \bar{W}^T_{n-r_0}]'$ and $T^{-2}[\mathbf{W}^{T-1}_{n-r_0} - \bar{W}^{T'}_{n-r_0}\mathbf{T}_{-1}][\mathbf{W}^{T-1}_{n-r_0} - \bar{W}^{T'}_{n-r_0}\mathbf{T}_{-1}]'$ respectively where $\mathbf{W}^{T-1}_{n-r_0}$ are the first $T-1$ observations of $\mathbf{W}^T_{n-r_0}$, $\mathbf{T}_{-1}$ is a row vector representing a linear trend running from 1 to $T-1$, and $\bar{W}^T_{n-r_0} = W^T_{n-r_0}/T$. The percentiles in Tables 1-3 are derived

Table 2. Percentiles of the Asymptotic Distribution of $LR_{ort}(r_0)$
Test Version Assuming a Trend Orthogonal to the Cointegration Space ($\mu_1 \neq 0$, $\beta'\mu_1 = 0$)

$n - r_0$	50%	75%	80%	85%	90%	95%	97.5%	99%
2	3.717	5.749	6.376	7.143	8.187	9.890	11.545	13.640
3	11.798	15.059	15.949	17.047	18.473	20.819	22.937	25.687
4	23.749	28.230	29.424	30.858	32.807	35.886	38.607	42.016
5	39.624	45.194	46.707	48.463	50.775	54.280	57.599	61.301
6	59.434	66.290	68.054	70.138	72.878	77.010	80.785	85.587
7	83.083	90.999	93.092	95.555	98.784	103.534	107.851	112.854
8	110.849	119.897	122.228	124.987	128.562	134.058	138.787	144.979
9	142.378	152.637	155.276	158.304	162.434	168.443	173.819	180.246
10	178.077	189.420	192.367	195.793	200.200	207.071	213.145	219.954
11	217.280	229.935	233.126	236.823	241.786	249.157	255.560	263.067
12	260.782	274.636	278.046	281.997	287.198	295.172	301.842	310.375
13	307.517	322.568	326.311	330.650	336.432	344.922	352.118	360.882
14	358.592	374.603	378.667	383.513	389.726	398.848	406.993	416.952
15	413.250	430.590	434.820	440.000	446.327	456.119	464.695	474.272

from 50,000 replications of this simulation experiment by means of a program written in GAUSS V3.6. Independent realizations of ϵ_t are used for each dimension $n - r_0$.

To generate the independent standard normal variates ϵ_t I use the Monster-KISS random number generator which was suggested by Marsaglia (2000). The Monster-KISS algorithm implemented in GAUSS V3.6 passes all of the DIEHARD tests which are used to evaluate the suitability of a random number generator (compare Ford and Ford 2001, and Marsaglia 1996). Therefore, the Monster-KISS algorithm is preferred to the linear congruential random number generator incorporated in earlier versions of GAUSS. This latter algorithm failed a number of the DIEHARD tests indicating non-randomness problems as pointed out by Vinod (2000) and Ford and Ford (2001).

Tables 1 and 2 contain the percentiles of the null limiting distributions of the test statistics with respect to the test versions allowing for a linear trend and an orthogonal trend respectively. They replace and extend the respective Tables in Lütkepohl and Saikkonen (2000)

Table 3. Percentiles of the Asymptotic Distribution of $LR^{\delta}_{mean}(r_0)$ and $LR_{mean}(r_0)$ Test Versions Allowing for a Mean Term Only ($\mu_1 = 0$, μ_0 arbitrary)

$n - r_0$	50%	75%	80%	85%	90%	95%	97.5%	99%
1	0.599	1.546	1.891	2.343	2.996	4.118	5.283	6.888
2	5.482	7.799	8.485	9.331	10.446	12.276	14.172	16.420
3	14.403	18.008	19.011	20.251	21.801	24.282	26.524	29.467
4	27.287	32.027	33.316	34.908	36.903	40.067	42.905	46.305
5	44.153	50.092	51.657	53.522	55.952	59.749	63.107	67.170
6	64.958	72.122	73.983	76.248	79.062	83.364	87.439	92.338
7	89.805	97.944	100.161	102.710	105.841	110.721	115.304	120.902
8	118.350	127.663	130.034	132.985	136.487	142.222	146.971	153.066
9	150.953	161.505	164.234	167.400	171.519	177.801	183.479	190.053
10	187.370	199.307	202.406	205.927	210.461	217.325	223.701	231.072
11	227.782	240.875	244.127	248.032	252.969	260.676	266.873	274.618
12	272.275	286.247	289.871	294.026	299.156	307.161	314.050	323.007
13	319.961	335.463	339.261	343.803	349.604	358.172	365.972	374.872
14	372.283	388.526	392.597	397.447	403.580	412.966	421.208	431.355
15	427.914	445.375	449.840	455.079	461.733	471.300	480.074	489.888

and Saikkonen and Lütkepohl (2000*a*). Now, critical values for eight different percentage points and up to 15-dimensional systems are available in contrast to the former limitation of three percentage points and five-dimensional systems. Table 2 starts with $n - r_0 = 2$ instead of $n - r_0 = 1$ since the rank under the alternative hypothesis must be smaller than the dimension n. Otherwise, the alternative represents a model with n stationary time series excluding a trend in levels ($\beta'\mu_1 = 0$). This cannot occur since at least one series is assumed to contain a linear trend (compare Saikkonen and Lütkepohl, 2000*a*). Table 3 refers to the test versions taking account of a mean term only. It corresponds to Table 15.1 in Johansen (1995) but also considers 13 to 15-dimensional systems. The percentiles in Tables 1-3 will also be made available in the software program JMulTi, a Java based multiple time series software.[2]

[2]More details on this software program can be found in the internet under http://www.jmulti.de.

Comparing the new set of percentiles with the corresponding old ones we can see that there exist only minor differences in general. An exception, however, are the results for $LR_{ort}(r_0)$ with respect to $n - r_0 = 4$ and $n - r_0 = 5$ (compare Table 1 in Saikkonen and Lütkepohl 2000a and Table 2 in this note). Here, the numerical differences are much more pronounced; for example, for $n - r_0 = 5$ and the 95% percentile we have 54.280 instead of 52.06 in Saikkonen and Lütkepohl (2000a, Table 1). Since Saikkonen and Lütkepohl (2000a) do not present many details of their simulations I cannot explore the deviations in more detail. However, the percentiles with respect to $n - r_0 = 2$ and $n - r_0 = 3$ are in accordance.

The tables are to be read in the following way. Let us consider we have a six-dimensional system assuming a general linear trend and want to test the null hypothesis $r_0 = 3$ at a 5% significance level. The corresponding critical value 28.455 can be found in Table 1 for the row $n - r_0 = 3$ and the 95% percentile. The critical value is the same no matter whether a level shift is present or not. If one applies the restriction that the linear trend is orthogonal to the cointegration space we look up in Table 2 and obtain 20.819.

References

Ford, M. P., and Ford, D. J. (2001) "Investigation of GAUSS' random number generators" http://www.aptech.com/random/rndu36.pdf.

Johansen, S. (1988) "Statistical analysis of cointegration vectors" *Journal of Economic Dynamics and Control* **12**, 231–254.

Johansen, S. (1995) *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models* Oxford University Press Oxford.

Johansen, S., and Nielsen, B. (1993) "Manual for the simulation program DisCo" Institute of Mathematical Statistics, University of Copenhagen.

Lütkepohl, H., and Saikkonen, P. (2000) "Testing for the cointegration rank of a VAR process with a time trend" *Journal of Econometrics* **95**, 177–198.

Lütkepohl, H., Saikkonen, P., and Trenkler, C. (2001) "Maximum eigenvalue versus trace tests for the cointegrating rank of a VAR process" *Econometrics Journal* **4**, 287–310.

Marsaglia, G. (1996) "Diehard: a battery of tests of randomness" http://stat.fsu.edu/~geo/ Department of Statistics, Florida State University.

Marsaglia, G. (2000) "The monster, a random number generator with period over 10^{2587} as long as the previously touted longest period one" mimeo Department of Statistics, Florida State University.

Saikkonen, P., and Lütkepohl, H. (2000*a*) "Testing for the cointegrating rank of a VAR process with an intercept" *Econometric Theory* **16**, 373–406.

Saikkonen, P., and Lütkepohl, H. (2000*b*) "Testing for the cointegration rank of a VAR process with structural shifts" *Journal of Business & Economic Statistics,* **18**, 451–464.

Saikkonen, P., and Lütkepohl, H. (2000*c*) "Trend adjustment prior to testing for the cointegration rank of a VAR process" *Journal of Time Series Analysis* **21**, 435–456.

Saikkonen, P., and Luukkonen, R. (1997) "Testing cointegration in infinite vector autoregressive processes" *Journal of Econometrics* **81**, 93–126.

Vinod, H. D. (2000) "Review of GAUSS for Windows, including its numerical accuracy" *Journal of Applied Econometrics* **15**, 211–220.