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AR Versus MA Disturbance Terms

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Abstract

We show how several models with moving average errors can be easily rewritten as models with autoregressive errors, thereby simplifying inference.

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In section 2 of their survey article Nicholls, Pagan and Terrell (NPT) (1975) present three examples illustrating how single equation time series models can have moving average (MA) disturbances. The purpose of this note is to show how alternative, plausible assumptions, utilizing autoregressive (AR) errors, can result in these same models having formulations with white noise errors which lead to simpler inferences.

The first case NPT consider is the rational distributed lag model of Jorgenson (1966) which they write as

$$y_t = \frac{C(L)}{B(L)} x_t + \varepsilon_t, \tag{1}$$

where: $C(L)$ and $B(L)$ are polynomials in the lag operator L, $B(L)$ is invertible, x_t is an exogenous variable and ε_t is white noise. This leads to

$$B(L) y_t = C(L) x_t + B(L) \varepsilon_t. \tag{2}$$

which has the MA disturbance $B(L)\varepsilon_t$.

An alternative specification of this model, considered by Carter and Zellner (1996, 2002), replaces the white noise error ε_t in (1) with an AR error v_t of the form

$$B(L) R(L) v_t = \varepsilon_t. \tag{3}$$

Since, as NPT point out, B(L) and C(L) are "low order polynomials" the polynomial $R(L)$ is introduced to capture the dynamic behavior of v_t in excess of that in $B(L)$. This formulation leads to

$$B(L) R(L) y_t = R(L) C(L) x_t + \varepsilon_t. \tag{4}$$

This is an autoregressive distributed lag (ARDL) model with nonlinear restrictions on the coefficients and a white noise error which is easy to estimate. Carter and Zellner (1996, 2002) consider identification and inference for this model.

In their second example NPT consider the case in which the regressand y_t is unobserved but is assumed to be determined by

$$\beta(L) y_t = \gamma(L) x_t + \varepsilon_t. \tag{5}$$

If y_t is related to an observed variable z_t by $z_t = y_t + \eta_t$, with η_t white noise, then the model for z_t is

$$\beta(L) z_t = \gamma(L) x_t + \varepsilon_t + \beta(L) \eta_t \tag{6}$$

which has an MA disturbance.

Now assume an alternative form of relationship between y_t and z_t

$$\beta(L)[z_t - y_t] = \eta_t. \tag{7}$$

Equation (7) specifies that the difference between the observed and unobserved variables is white noise only if the lag structure of the regressand is taken into account. This leads to

$$\beta(L)z_t = \gamma(L)x_t + \varepsilon_t + \eta_t, \tag{8}$$

which is an ARDL model with white noise error.

More generally, the white noise disturbance in (5) may be replaced by the AR error $\rho(L)u_t = \varepsilon_t$. Now specify that in order for the difference between the observed and unobserved regressands to be white noise account must be taken of the lag structures of both the regressand y_t and the error u_t:

$$\rho(L)\beta(L)[z_t - y_t] = \eta_t. \tag{9}$$

This leads to

$$\rho(L)\beta(L)z_t = \rho(L)\gamma(L)x_t + \varepsilon_t + \eta_t. \tag{10}$$

which is, again, an ARDL model with white noise error.

The third example in NPT features an unobserved expectation S_t^* as the regressor in a linear model explaining a variable I_t:

$$I_t = a + bS_t^* + \varepsilon_t. \tag{11}$$

They assume that the expectation S_t^* is given by

$$S_t^* = \frac{C(L)}{B(L)}x_t \tag{12}$$

which leads to the model

$$B(L)I_t = B(1)a + bC(L)x_t + B(L)\varepsilon_t \tag{13}$$

having an MA disturbance.

However, if instead of (12) we assume $S_t^* = D(L)x_t$, with $D(L)$ being fairly long, we obtain

$$I_t = a + bD(L)x_t + \varepsilon_t \tag{14}$$

which has a white noise error. More generally, let x_t^* be the unobserved regressor in

$$\phi(L)y_t = \alpha + \beta x_t^* + u_t \tag{15}$$

where $\rho(L)u_t = \varepsilon_t$. Then if, instead of an ARMA structure for x_t^*, we assume $x_t^* = \delta(L)x_t$ we obtain

$$\rho(L)\phi(L)y_t = \rho(1)\alpha + \beta\rho(L)\delta(L)x_t + \varepsilon_t \tag{16}$$

which is the same form as (4).

Thus we see that the use of simple assumptions just as plausible as those leading to moving average errors can produce models with white noise errors. These simpler models can be tested against the more complicated models with moving average disturbances terms using model selection procedures such as Bayes factors, BIC and AIC. Carter and Zellner (2002) analyze some empirical examples.

References

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