E C O N O M I C S   B U L L E T I N

Further evidence on the size and power of the Bierens and Johansen cointegration procedures

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Abstract

Although both the Johansen (1991, 1994) trace test and Bierens (1997a,b) nonparametric lambda−min test for cointegration have good size properties in Monte Carlo studies by Hubrich, Lutkepohl, and Saikkonen (2001) and Boswijk, Lucas, and Taylor (2000), the Bierens test has very low power. In contrast, Bierens reports good power for his procedure. Meanwhile, Hubrich et al. and Boswijk et al. do not include Bierens' companion method for estimating the number of cointegrating vectors, nor do they investigate the effect of serial correlation on Bierens' test. In the present paper, inclusion of the estimation step does not significantly degrade size of the Bierens procedure, even with serial correlation, but power is not improved. Serial correlation does degrade the size of the Johansen test, but it remains superior. Analysis of Bierens' (1997b) Monte Carlo results suggests that their indication of high power reflects the test's lack of scale invariance.

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1. Introduction

Because many time series are thought to possess unit roots but move together in equilibrium relationships, cointegration procedures have become widely used in empirical applications. One of the most often used is that of Johansen (1988, 1991, 1994) and Johansen and Juselius (1990). One of their procedure's attractions is its ability to detect and estimate multiple cointegrating vectors. However, the Johansen procedure, like many others, requires estimation of various structural and nuisance parameters. For example, a vector autoregressive (VAR) lag order must be specified and the lag parameters estimated. To get around this problem, Bierens (1997a,b) has proposed a nonparametric procedure that can also detect and estimate multiple vectors. No lag structure or deterministic terms need be estimated. This would seem to provide a valuable supplement to other procedures such as Johansen's.

In their review of systems cointegration tests, Hubrich, Lütkepohl, and Saikkonen (2001) conclude from Monte Carlo methods that the Johansen and the Bierens tests both have good size properties, and that the Johansen test has good power. Unfortunately, they also conclude that the Bierens test has little power in typical sample sizes. Boswijk, Lucas, and Taylor (2000) reach the same conclusion. However, these findings are perplexing and incomplete. They are perplexing because a Monte Carlo analysis by Bierens (1997b) indicates good power, and they are incomplete because both Hubrich *et al.* (2001) and Boswijk *et al.* (2000) examine only the first part of the Bierens testing procedure, the λ-*min* test. The λ-*min* test parallels the Johansen trace test by testing nulls of zero and other numbers of cointegrating vectors against higher numbers of vectors. But Bierens (1997b, 2002) also presents a method for estimating the number of cointegrating vectors, and suggests that this be used as a check on the λ-*min* test. It is the primary purpose of this paper to see whether the addition of this second step improves power or affects size, compared with use of the λ-*min* test alone. The secondary purpose of the paper is to see how these matters are influenced by the presence of some serial correlation in the data generating process (DGP); this is not done for the Bierens test in either Hubrich *et al*. (2001) or Boswijk *et al.* (2000). To accomplish this, in the present paper the complete Bierens procedure is applied to a number of DGP's that have zero or one cointegrating vectors and that sometimes have contemporaneous and/or serial correlation. For comparison, the results for the Johansen trace test are also evaluated. The main finding from these simulations is that, even with the estimation method included, the Bierens procedure still has very little power compared with the Johansen procedure, although the performance of the latter is degraded by the presence of serial correlation. The paper concludes with replication and further investigation of Bierens' (1997b) contrary finding of high power for his test. The investigation suggests that the high power finding reflects the Bierens procedure's lack of scale invariance. Therefore the main conclusion of lack of power is not overturned.

2. The Bierens procedure

The Bierens λ-*min* test follows from different rates of convergence of appropriately weighted means of z_t and Δz_t in the cointegrated versus non-cointegrated cases. Random matrices are constructed from the weighted means:

$$\hat{A}_m = \frac{8p^2}{n} \sum_{k=1}^{m} k^2 \left(\frac{1}{n} \sum_{t=1}^{n} \cos(2kp\,(t-0.5)/n) z_t \right) \left(\frac{1}{n} \sum_{t=1}^{n} \cos(2kp\,(t-0.5)/n) z_t \right)', \tag{1}$$

$$\hat{B}_m = 2n \sum_{k=1}^{m} \left(\frac{1}{n} \sum_{t=1}^{n} \cos(2kp\,(t-0.5)/n) \Delta z_t \right) \left(\frac{1}{n} \sum_{t=1}^{n} \cos(2kp\,(t-0.5)/n) \Delta z_t \right)', \tag{2}$$

where n is the sample size, and optimal values for m are determined with respect to the power function. In application, the choice of m depends on the number of variables, the significance level, and the number of cointegrating vectors, r, under the null (Bierens, 1997b, Table 1).

The test statistics are the ordered solutions $\hat{l}_{1,m} \geq \ldots \geq \hat{l}_{q,m}$ of the generalized eigenvalue problem

$$\det[\hat{A}_m - l\,(\hat{B}_m + n^{-2} \hat{A}_m^{-1})] = 0, \tag{3}$$

where q is the number of variables. Under cointegration with r cointegrating vectors, the r smallest solutions $(\hat{l}_{q-r+1,m}, \ldots, \hat{l}_{q,m})$ converge in distribution to (0, ... , 0). Thus, the smallest solution of (3) is the test for r = 0 versus r = 1 cointegrating vectors, the next smallest is the test for r = 1 versus r = 2, and so on. One rejects for small values of the test statistic.

Armed with the tentative outcome of the *λ-min* test, one can next move to the estimation method. One calculates the q + 1 values of the function

$$\hat{g}_m(r) = \left(\prod_{k=1}^{q-r} \hat{l}_{k,m} \right)^{-1} \left(n^{2r} \prod_{k=q-r+1}^{q} \hat{l}_{k,m} \right), \tag{4}$$

where the first term is set to 1 for r = 0, the second term is set to 1 for $r = q$, and the choice of m now depends on the significance level and the number of cointegrating vectors from the *λ-min* test result. Then, "$\hat{g}_m(r)$ converges in probability to infinity if the true number of cointegrating vectors is unequal to r, and $\hat{g}_m(r) = O_p(1)$ if the true number of cointegrating vectors is indeed r" (Bierens, 1997b, 391). Thus, the estimated r is the value that minimizes (4), and the probability that this value equals the true r approaches 1.0 as the sample size approaches infinity.

Examination of (3) and (4) indicates that the estimation conclusion will often, but not necessarily always, match the test conclusion in finite samples. Thus, the estimation conclusion can confirm or cast doubt on the test conclusion, providing the "check" that Bierens (1997b) mentions. Since it is the power of the Bierens procedure that is at issue, a concrete way to implement the "check" idea would be to adopt a decision rule that chooses the higher number of vectors indicated by the two approaches (when they differ). For example, suppose that there are three variables and that the true number of cointegrating vectors is one. There will be a tendency for $\hat{l}_{3,m}$ to be small, but for the *λ-min* test to reject r = 0, it is necessary for $\hat{l}_{3,m}$ to be less than the critical value. Suppose this does not occur in a given sample. The estimation approach could still correctly indicate one vector if $\hat{l}_{3,m}$ were nevertheless small enough that $n^2 l_{3,m}^2 < 1$.[1] Thus, there is the possibility of a power gain with this decision rule. Of course, a size adjustment may also be needed, because we may also be rejecting $r = 0$ more often when the number of vectors is truly zero than with the *λ-min* test alone. Without such an adjustment, an apparent power gain might only reflect size distortion.

3. The corresponding Johansen trace test

The Johansen test is so well known now that there is no need to repeat its details. But it does require several specification decisions not needed for the Bierens procedure. One is the deterministic trend specification. I use the version where the data are allowed to be trend stationary; that is, a deterministic trend is allowed in any cointegrating vectors and drift is allowed in the first differences. This corresponds to Case 2, p. 6, and Table B4, p. 81, in Hansen and Juselius (1995) and test *LR*+ in Hubrich *et al.* (2001). This choice follows from the fact that the Bierens test allows for these same deterministic trends.

The second decision for the Johansen test is its VAR lag order. Hubrich *et al*. (2001) simply specify for the test the lag order that is in the DGP. Furthermore, they focus on DGP's with lag order 1, with only a brief summary of results for lag order 2 that do not involve the Bierens test.[2] Boswijk *et al.* (2000) also use only lag order 1. This approach gives an unfair advantage to the Johansen test in a comparison with the Bierens test, because it removes from the Johansen test the cost of determining lag lengths and estimating lag parameters, a cost that the Bierens test is always, in some sense, incurring. Therefore, in the present paper the lag order for each implementation of the Johansen test is chosen by using the AIC criterion calculated from the unconstrained VAR in levels using lag orders of 2-6.[3] Since, as explained below, some of the DGP's have a lag order of 4 instead of 1, the Johansen test is given a meaningful lag-order selection challenge. Moreover, in view of the likelihood of longer lag lengths in the Johansen test implementations, I incorporate the degree-of-freedom correction suggested by Reimers (1992) and Reinsel and Ahn (1992).[4] Although Cheung and Lai (1993) compute a more refined version, it is not too different than the basic one for the null of zero cointegrating vectors.

Finally, Johansen's trace test and not the maximum eigenvalue test is used. This matches Hubrich *et al*. (2001), and also follows from the finding of Doornik, Hendry, and Nielson (1999) that the latter test is inconsistent.

4. The Data Generating Processes

The DGP's are quite similar to those in Hubrich *et al*. (2001), which in turn are based on some in Toda (1994). In the present paper, they are variations of

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[1] This condition ensures that $\hat{g}_m(1) < \hat{g}_m(0)$, so that $r = 1$ is chosen over $r = 0$. For $r = 1$ to also be chosen over $r = 2$, it must also be that $n^2 l_{2,m}^2 > 1$, which ensures that $\hat{g}_m(1) < \hat{g}_m(2)$. Suppose this is the case. Then $\hat{g}_m(2) \leq \hat{g}_m(3)$ because $l_{1,m}^2 \geq l_{2,m}^2$, and thus $\hat{g}_m(1) < \hat{g}_m(3)$ and $r = 3$ will not be chosen either.

[2] Following, for example, Johansen and Juselius (1990), the lag order is expressed in terms of levels rather than first differences..

[3] It is unlikely that any actual empirical situation would call for a lag order of 1.

[4] With q as the number of variables, p as the lag order, and n as the sample size, the correction is to multiply the trace statistics by $(n - pq) / n$ (where n is now the sample size after conditioning on lags).

$$z_t = \begin{bmatrix} \psi & 0 \\ 0 & I_{q-1} \end{bmatrix} z_{t-1} + u_t, \qquad u_t = v_t + \rho\, u_{t-3}, \qquad v_t \sim iidN\left(0, \begin{bmatrix} 1 & \Theta \\ \Theta' & I_{q-1} \end{bmatrix}\right). \tag{5}$$

The important addition here to what Hubrich ***et al.*** (2001) and Boswijk ***et al.*** (2000) implement is the allowance for serial correlation and longer VAR lags by including ρ. This is set at either 0 or 0.25. The 0.25 value combined with the lag order of 3 for u_t (giving a lag order of 4 in levels for the VAR) creates a presumably more realistic challenge for the two cointegration tests than present in the work of Hubrich ***et al***. (2001) and Boswijk ***et al.*** (2000). Thus, we can find out whether in these circumstances the Johansen test (with the AIC lag selection criterion) still has an advantage over the Bierens test.

Additionally, the number of variables, q, in the various systems ranges from two through four. The values for ψ are 1, 0.9, 0.8, 0.6, 0.3, and 0.0. The number of cointegrating vectors, r, is zero or one. If $\psi = 1$, there are zero cointegrating vectors; otherwise there is one. Next, the row vector Θ is either all zeros, or has one nonzero value of 0.8 for its left-hand element (this value is used in Hubrich, ***et al***., 2001). The sample sizes investigated are n = 100 and 200.[5] The number of replications is 2000 per DGP and sample size; this would estimate the rejection proportion under the null within ± 0.01 with 95 percent confidence (if the true proportion is in the neighborhood of 0.05).[6]

5. Results

Tables I and II contain the rejection proportions for the null of $r = 0$ for the various tests, DGP's, and sample sizes.[7] The critical values used are for the 0.05 level of significance, from Bierens (1997b) and Hansen and Juselius (1995). For each DGP and sample size, there are two Bierens results. The first is the proportion of rejections of zero cointegrating vectors from the ***λ-min*** test alone, and the second is the proportion of rejections from ***either*** the ***λ-min*** test or the estimation result (for the estimation result, this means estimating one or two vectors). This corresponds to the possibly power-enhancing decision rule discussed in section 2. Thus, the rejection proportion will always be at least as high as for the ***λ-min*** test alone, and there could be size distortion as well as power improvement.

The results confirm a number of observations made by Hubrich ***et al***. (2001) and Boswijk ***et al.*** (2000). With $\rho = 0$ and $n = 100$, both the ***λ-min*** test and the ***LR+*** tests are approximately correctly sized (the rejection proportions under $\psi = 1.0$ are approximately 0.05), and the Bierens test is a bit more conservative. Also confirmed is that, as ψ falls, the Johansen test has vastly

[5] Hubrich ***et al.*** (2001) investigate two- and three- but not four-variable systems. On the other hand, they also investigate the hypothesis of one versus two cointegrating vectors, which, as mentioned earlier in the paper, I do not address. This means that they sometimes include two cointegrating vectors in their three-variable DGP's. Hubrich ***et al.*** (2001) also include deterministic trends in some DGP's. They find that the presence of an actual trend in the DGP does not cause significantly mis-sized results for the Johansen test when the test is specified for this possibility, but certainly does when it is not. This issue is not a focus of the present paper. Boswijk ***et al***. (2000) consider only two-variable systems, but include a sample size of 1000 in addition to 100.

[6] The possible influence of initial values is controlled for generating 50 pre-sample values in each replication.

[7] All tests and Monte Carlo procedure were programmed in TSP 4.5. The Bierens procedure can be also be found in Professor Bierens' own econometrics program, EasyReg International (Bierens, 2002), which can be downloaded from http://econ.la.psu.edu/~hbierens/EASYREG.HTM. The Johansen procedure can be found in CATS in RATS (see Hansen and Juselius, 1995).

superior power compared to the Bierens *λ-min* test. In fact, the power of the *λ-min* test is essentially trivial, except perhaps when ψ is zero, which would be the extreme case of the error correction parameter restoring the equilibrium relationship fully in one time period. Moving on to the case of n = 200 (not reported by Hubrich *et al.*, 2001), power for *λ-min* is not much improved.[8] Finally, for either sample size, some contemporaneous error correlation from $\theta = 0.8$ raises the power of ***LR+***, but has no significant effect on the *λ-min* test.

Now let us consider the addition of the Bierens estimation method to the *λ-min* test (still with $\rho = 0$). With $\psi = 1.0$ and $n = 100$, there are no additional rejections for the two-variable models, but the combined procedure becomes a bit oversized as the number of variables increases. However, this tendency is absent when $n = 200$. Unfortunately, as ψ declines there is no improvement in power for either sample size under the combined procedure, except some that corresponds to the size distortion with $n = 100$.

Incorporating serial correlation into the DGP's generates several additional findings. First, the rejection proportions under zero cointegrating vectors remain essentially unchanged for the Bierens procedures. Thus, the Bierens nonparametric correction for serial correlation performs well in this respect. Unfortunately, this is offset by the power remaining as abysmal as before. Meanwhile, the Johansen ***LR+*** test becomes somewhat oversized, particularly with larger numbers of variables and lags, and at the same time power is compromised (even without size adjustment). Consequently, the superiority of the Johansen test over the Bierens test is tempered as compared with the presumably less realistic cases without serial correlation.

The oversized characteristic of the Johansen test under serial correlation, particularly evident in the lower sample-size cases of Table I, is primarily the result of the AIC frequently picking too low a lag order. Consider Table I's four-variable cases with serial correlation, which are the cases where the size distortion is worst. On average, the AIC picks too low a lag order (i.e., less than 4) in about sixty percent of the replications. To allow an adjusted comparison of these cases with the Bierens procedure, I re-calculate rejection proportions in two alternative ways for several of these cases. The first alternative is to simply size-adjust the *λ-min* and ***LR+*** test results from Table I.[9] The second is to re-run the replications of the ***LR+*** tests, always setting the lag equal to that of the DGP, which in these cases is 4, rather than using the AIC. The results are given in Table III (which also includes the Table I results for the same cases for ease of comparison). It can be seen that always choosing lag order 4 substantially diminishes the size distortion of ***LR+***, but the effect on power is different than in the size-adjusted calculations. Anyway, in either case the power superiority of the Johansen test over the Bierens test remains, although it is less striking.[10]

Another matter relevant to size distortion in the Johansen test is the degree of freedom correction. This becomes significant when more variables and longer lags are specified. As an example of its impact on size distortion under the null of no cointegration, I re-calculate without using the correction the ***LR+*** rejection proportions for the cases where $n = 100$, $q = 4$, and

[8] The effects of increasing sample size reported by Boswijk *et al.* (2000) are not easily compared with those here, because they have set up their DGP's so that the error correction parameter (if nonzero) declines in size as the sample increases in size.

[9] The Bierens estimation result does not lend itself to size adjustment, so the results of the complete Bierens procedure are not size-adjusted in Table III. For the two size-adjusted tests, the critical values for a given DGP are the 5th percentile for *λ-min* and the 95th percentile for ***LR+*** of the simulated distributions obtained with the DGP's ρ and θ values under $\psi = 1$ (and for the same sample size).

[10] Given the very low power of the Bierens test for the null of zero cointegrating vectors, I do not investigate performance with respect to the nulls of one or more vectors.

$\psi = 1.0$. The four values rise from 0.055, 0.061, 0.145, and 0.147 (in Table I) to 0.159, 0.156, 0.401, and 0.389, all far in excess of the desired value of 0.05. Thus, the correction seems useful, in contrast with the concern expressed about it in Doornik *et al.* (1999).[11]

6. Analysis of the Bierens simulations

Bierens (1997b) provides some simulation evidence in which his nonparametric procedure actually appears to have *more* power than the Johansen test (in the form of the maximum eigenvalue test). This obviously contradicts the evidence in Hubrich et al. (2001), Boswijk et al. (2000), and the present paper, and thus calls out for an explanation. Bierens (1997b) begins his empirical and Monte Carlo analysis by applying his procedure and the Johansen test to annual U.S. GNP and wages, both in logs, 1909-1988.[12] He concludes from the *λ-min* test, the non-parametric estimation procedure, and the Johansen test that there is one cointegrating vector. He then simulates the behavior of these procedures, where the DGP is an error correction model (ECM) estimated from the data. The ECM has one cointegrating vector (with a time trend) and eight lags in levels (seven in first differences). The residuals are randomly drawn from a bi-variate normal distribution with a covariance matrix equal to that of the estimated errors of the ECM. The initial values of the simulated data sets are the first eight values of the actual data. Using the 0.10 level of significance, Bierens reports that the nonparametric approach has a rejection proportion for the null of zero vectors of 0.904 (but does not distinguish what role, if any, the estimation step plays in this result). He then reports that the Johansen maximum eigenvalue test (with eight lags) has a rejection proportion of 0.854.

My replication of Bierens' Monte Carlo study generates similar numbers; in fact, I find even higher powers. The rejection proportions (at the 0.10 level) are 0.943 (*λ-min*), 0.986 (*λ-min*-estimation), and 0.961 (*LR+*).[13] However, let us now use the same data to estimate an eight-lag VAR with no cointegration (a seven-lag first difference VAR), using this for the DGP. Now the rejection proportions are 0.870 (*λ-min*), 0.953 (*λ-min*-estimation), and 0.068 (*LR+*). Thus, it turns out that the Bierens procedure in this case is grossly oversized and so the power calculations for it are not meaningful. The reason for this is probably the lack of scale invariance for the Bierens procedure. Although this property is noted in Bierens (1997a) and Bierens (2002), his solution of using data in logs is clearly inadequate in the present case because the GNP and wage data are already in logs. A second solution given by Bierens (2002) involves standardizing the data prior to running the tests. With this applied to the present data set, the Bierens rejection proportions (at the 0.10 level) with zero vectors in the DGP become 0.131 (*λ-min*), 0.205 (*λ-min*-estimation), while with one vector they become 0.145 (*λ-min*), 0.211 (*λ-min*-estimation). Thus, size distortion is greatly reduced, but there is also no significant size-adjusted power. The conclusions of section 5 are confirmed.

7. Concluding summary

The Monte Carlo analysis presented in this paper confirms the findings of Hubrich *et al.* (2001) that the Johansen (1991, 1994) and Bierens (1997a,b) tests generally have good size properties

[11] Further evidence in favor of the correction is in Cushman, Lee, and Thorgeirsson (1996) and Cushman(2000).
[12] This data set is available in Bierens (2002).
[13] As in the rest of the paper, I use the Johansen trace test rather than the maximum eigenvalue test, and I have applied the degree-of-freedom correction. Bierens (1997b) does not apply the degree-of-freedom correction.

while the Bierens (1997a,b) nonparametric *λ-min* cointegration test has little power. The primary finding here is that, while size properties remain good, power is not improved by supplementing the *λ-min* test with Bierens' (1997b) companion method for estimating the number of cointegrating vectors. An additional finding is that the size and power properties are not noticeably influenced by the presence of some autoregressive serial correlation in the data generating process. On the other hand, serial correlation degrades the performance of the Johansen trace test. It nevertheless remains superior to the Bierens procedures under these conditions. Finally, Bierens (1997b) own report of good power for his tests appears to be the consequence of lack of scale invariance.

Table I: Rejection proportions by test and DGP, n = 100

		$\rho = .00 \quad \theta = .0$			$\rho = .00 \quad \theta = .8$			$\rho = .25 \quad \theta = .0$			$\rho = .25 \quad \theta = .8$		
q	ψ	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*
2	1.0	.047	.047	.076	.052	.052	.064	.055	.055	.088	.044	.044	.080
2	.9	.051	.051	.114	.060	.060	.335	.053	.053	.095	.049	.049	.293
2	.8	.059	.059	.239	.070	.070	.766	.057	.057	.148	.067	.067	.511
2	.6	.072	.072	.673	.085	.085	.977	.071	.071	.320	.078	.078	.684
2	.3	.092	.092	.959	.098	.098	.989	.077	.077	.564	.091	.091	.749
2	.0	.138	.138	.985	.138	.138	.989	.088	.088	.652	.094	.094	.776
3	1.0	.047	.056	.064	.049	.059	.064	.046	.058	.108	.042	.051	.102
3	.9	.054	.063	.067	.046	.058	.227	.040	.049	.102	.057	.066	.283
3	.8	.060	.070	.138	.057	.067	.527	.056	.068	.124	.049	.057	.460
3	.6	.068	.079	.407	.058	.071	.935	.052	.063	.235	.068	.080	.646
3	.3	.078	.092	.840	.084	.102	.983	.066	.073	.513	.068	.085	.713
3	.0	.082	.096	.971	.091	.100	.987	.082	.095	.650	.075	.094	.717
4	1.0	.055	.100	.055	.053	.099	.061	.047	.085	.145	.049	.091	.147
4	.9	.064	.105	.070	.054	.097	.188	.056	.092	.115	.055	.094	.277
4	.8	.053	.093	.099	.054	.099	.391	.056	.110	.144	.054	.091	.432
4	.6	.060	.110	.241	.059	.104	.786	.065	.100	.202	.053	.092	.627
4	.3	.063	.114	.654	.061	.112	.963	.055	.102	.441	.065	.111	.747
4	.0	.071	.119	.883	.071	.127	.981	.061	.102	.679	.052	.100	.756

Note: The ***λ-min*** column gives the proportion of the 2000 replications with a rejection from the Bierens ***λ-min*** test, the *λ/E* column gives the proportion with a rejection from either the ***λ-min*** or Bierens estimation result, and the *LR+* column gives the proportion with a rejection from the Johansen test with linear trends.

Table II: Rejection proportions by test and DGP, *n* = 200

		$\rho = .00 \quad \theta = .0$			$\rho = .00 \quad \theta = .8$			$\rho = .25 \quad \theta = .0$			$\rho = .25 \quad \theta = .8$		
q	ψ	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*	*λ-min*	*λ/E*	*LR+*
2	1.0	.039	.039	.051	.050	.050	.062	.049	.049	.069	.035	.035	.067
2	.9	.062	.062	.265	.063	.063	.869	.060	.060	.192	.058	.058	.581
2	.8	.069	.069	.769	.076	.076	.999	.069	.069	.450	.075	.075	.871
2	.6	.089	.089	.994	.110	.110	1.00	.071	.071	.816	.085	.085	.969
2	.3	.141	.141	1.00	.145	.145	1.00	.099	.099	.952	.115	.115	.993
2	.0	.189	.189	1.00	.199	.199	1.00	.146	.146	.984	.154	.154	.995
3	1.0	.054	.054	.058	.047	.047	.057	.054	.054	.075	.047	.047	.078
3	.9	.048	.048	.157	.049	.049	.670	.063	.063	.147	.058	.058	.454
3	.8	.060	.060	.518	.061	.061	.987	.055	.055	.275	.060	.060	.694
3	.6	.062	.062	.978	.079	.079	1.00	.062	.062	.613	.062	.062	.868
3	.3	.084	.084	1.00	.100	.100	1.00	.069	.069	.825	.080	.080	.927
3	.0	.118	.118	.999	.139	.139	1.00	.087	.087	.901	.086	.086	.943
4	1.0	.060	.060	.064	.061	.061	.060	.057	.057	.090	.052	.052	.090
4	.9	.054	.054	.116	.053	.053	.529	.058	.058	.122	.059	.059	.377
4	.8	.046	.046	.347	.064	.064	.948	.055	.055	.210	.059	.059	.585
4	.6	.078	.078	.900	.061	.061	1.00	.062	.062	.463	.064	.064	.738
4	.3	.075	.075	.999	.077	.077	1.00	.067	.067	.697	.067	.067	.824
4	.0	.085	.085	1.00	.095	.095	1.00	.074	.074	.783	.083	.083	.864

Note: See note to Table I.

Table III: Some alternative rejection proportions, *n* = 100

		$\rho = .25 \quad \theta = .0$					$\rho = .25 \quad \theta = .8$				
q	ψ	*λ-min* (*from Table 1*)	*LR+* (*from Table 1*)	*λ-min* (*adj.*)	*LR+* (*adj.*)	*LR+* (*4*)	*λ-min* (*from Table 1*)	*LR+* (*from Table 1*)	*λ-min* (*adj.*)	*LR+* (*adj.*)	*LR+* (*4*)
4	1.0	.047	.145	.050	.050	.074	.049	.147	.050	.050	.083
4	.6	.065	.202	.071	.078	.116	.053	.627	.055	.442	.188
4	.3	.055	.441	.061	.252	.135	.065	.747	.067	.627	.204

Note: See note to Table I. In addition, “*(adj.)*” indicates size-adjusted, and “*(4)*” indicates the use of lag order 4 in the Johansen test.

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