E C O N O M I C S   B U L L E T I N

Maximum likelihood seasonal cointegration tests for daily data

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Abstract

In this paper we propose an extension of the maximum likelihood seasonal cointegration procedure developed by Lee (1992) for daily time series. We compute the finite sample critical values of the associated test statistics in daily seasonal time series.

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1 Introduction

The cointegration analysis and the related issue of existence of long-run equilibrium relationships among macroeconomic variables have often realized from seasonally adjusted data. As is pointed out in the literature, seasonal adjustment might lead to mistaken inference on economic relationships, and it causes a significant loss of valuable information on seasonal behavior in economic time series. That might imply the non-invertibility of seasonally adjusted series, and then the impossibility of autoregressive vector representation, and the none detection of existant cointegration relations[1]. Therefore, Ghysels *et al.* (1993) and Ericsson *et al.* (1994) suggested to use seasonal cointegration from seasonally unadjusted data to reveal the long-run dynamics.

From the preliminary work of Engle *et al.* (1989), a number of seasonal cointegration techniques have been developed by, for example, Hylleberg *et al.* (1990), Engle *et al.* (1993), Joyeux (1992) and Cubbada (1995). An approach based on the maximum likelihood method was proposed by Lee (1992) who extented the Johansen (1988) procedure to the seasonal case, which allow to analyse the multivariate systems. Lee developed tests for cointegration and seasonal cointegration for nonstationary time series which have unit roots at seasonal frequencies as well as at the zero frequency. The method is illustrated for time series consisting of quarterly observations. In this paper we propose an extension of this seasonal cointegration method to daily data because these data are important for economic analysis. For instance, the data periods for examining daily data, such as stock and exchange rate markets will be the five days of the week. We also provide the finite sample critical values for non-seasonal and seasonal cointegration.

2 Seasonal cointegration for daily data

Let x_t a n-dimensional vector generated by the following autoregressive vector model:

$$x_t = \Gamma_1 x_{t-1} + \Gamma_2 x_{t-2} + \ldots + \Gamma_p x_{t-p} + \varepsilon_t$$

where ε_t is a n-dimensional white noise vector process with mean zero and variance matrix Ω, $x_t = (x_{1t}, \ldots, x_{nt})$ are fixed for $t \leq 0$, and p is lag length. Since we allow the process x_t have unit roots at zero and seasonal frequencies, the determinant $\mid \Gamma(z) \mid$ of the matrix polynomial $(I_k - \Gamma_1 z - \ldots - \Gamma_p z^p)$ will have roots on the unit circle.

[1]See, for example, Ghysels and Perron (1993), Lee and Siklos (1997), and Franses and McAleer (1998).

Assuming that $\Delta_5 x_t$ is stationary, the seasonal error correction model (SECM) is defined as:

$$\begin{aligned}\Delta_5 x_t &= \pi_1 y_{1,t-1} + \pi_2 y_{2,t-2} + \pi_3 y_{2,t-1} \\ &\quad + \pi_4 y_{4,t-2} + \pi_5 y_{4,t-1} + \sum_{j=1}^{p-5} \phi_j \Delta_5 x_{t-j} + \mu_t + \varepsilon_t \end{aligned} \tag{1}$$

with

$$\begin{aligned}\Delta_5 &= (1 - B^5) \\ y_{1,t} &= \Big(1 + \sum_{j=1}^{4} B^j\Big) x_t \\ y_{2,t} &= (1-B)(1 - 2B\cos\frac{4\pi}{5} + B^2) x_t \\ y_{4,t} &= (1-B)(1 - 2B\cos\frac{2\pi}{5} + B^2) x_t\end{aligned}$$

with μ_t are deterministic components including a combination of an intercept, trend and seasonal dummy variables, π_i and ϕ_j ($i = 1, \ldots, 5$ and $j = 1, \ldots, p-5$) are matrices of parameters, and ε_t is i.i.d. $N_k(0, \Omega)$. While the regressor $y_{1,t}$ possesses a unit root at the zero frequency and all the seasonal unit roots are removed, the regressors $y_{2,t}$ and $y_{4,t}$ maintain complex conjugate roots at the corresponding frequency ($\frac{2\pi}{5}$ and $\frac{4\pi}{5}$, respectively) and eliminate the non-seasonal unit root as well as other complex conjugate roots.

As suggested by Lee (1992), we assume that the restrictions $\pi_{2j+1} = 0$ with $j = 1, 2$ with little effect on the test π_{2j} for seasonal cointegration[2], and using the algebraic expression of $\cos(\frac{2\pi}{5})$ and $\cos(\frac{4\pi}{5})$, we can rewrite the SECM as:

$$\begin{aligned}\Delta_5 x_t &= \pi_1 y_{1,t-1} + \pi_2 y_{2,t-2} + \pi_4 y_{4,t-2} \\ &\quad + \sum_{j=1}^{p-5} \phi_j \Delta_5 x_{t-j} + \mu_t + \varepsilon_t\end{aligned} \tag{2}$$

with

$$\begin{aligned} y_{2,t} &= (1-B)[1 - (-1-\sqrt{5})(B/2) + B^2] x_t \\ y_{4,t} &= (1-B)[1 - (-1+\sqrt{5})(B/2) + B^2] x_t\end{aligned}$$

Since the coefficient matrices π_k ($k = 1, 2, 4$) may convey the information concerning the long-run behavior of the series, we need to investigate the properties

[2]If this restriction is relaxed one cannot apply the estimation method that uses canonical correlations.

of these matrices in order to determinate whether or not the components of x_t are seasonally cointegrated in presence of unit roots at other frequencies. Note that if the matrix π_k has full rank (rank(π_k)=r_k=n), then the series does not contain unit roots at the corresponding frequency k. If the rank of π_k is zero, seasonal cointegration relationship at that frequency does not exist. Finally, if $0 \leq r_k < n$, it can be shown that $\pi_k = \alpha_k \beta_k'$ for suitable $n \times r_k$ matrices α_k and β_k such that $\beta_k' y_{k,t-1}$ is stationary even though $y_{1,t-1}$ itself is nonstationary.

The ranks r_k, $k = 1, 2, 4$, of the matrices $\pi_k = \alpha_k \beta_k'$ are determined by sequences of test statistics formed from the ordered sequence of conditional canonical correlations $\hat{\lambda}_1 \geq \ldots \geq \hat{\lambda}_k \geq \ldots \geq \hat{\lambda}_n$, which are eigenvalues (see Lee, 1992).

We may consider two variants of likelihood ratio (LR) test statistics. The trace test statistic:

$$\xi_k = -T \sum_{j=n-k+1}^{n} \ln(1 - \hat{\lambda}_j) \tag{3}$$

is formed from the k smallest canonical roots, where T is the number of effective observations. We test the null hypothesis $r_i \leq n - k$ against the alternative $r_i > n - k$.

Alternatively, the eigenvalues or λ_{max} statistic:

$$\lambda_k = -T \ln(1 - \hat{\lambda}_{n-k+1}) \tag{4}$$

is calculated from one canonical root, and we test the null hypothesis $r_i \leq n-k$ against the alternative $r_i = n - k + 1$. Note that for $k = 1$, the two statistics are equivalent. In the rest of the paper, we will focus on the eigenvalues statistics, λ_k, for convenience.

3 Asymptotic distributions and critical values

The asymptotic distributions of the test statistics for the daily seasonal cointegration are derived from those of Johansen (1988), Lee (1992) and Lee and Siklos (1995). Tables 1 to 3 display the finite sample critical values of the likelihood ratio (LR) statistics for cointegration and seasonal cointegration at each frequency: zero frequency ($\theta = 0$) and seasonal frequencies ($\theta = 2\pi/5$ and $4\pi/5$). We give the 1%, 5% and 10% acceptance regions of the critical values for each LR statistics. The finite sample critical values of test statistics were obtained by using 30,000 Monte Carlo simulations, for various sample sizes $T = 130$, 260 and 520, and for $(n - r) = 1$, 2 and 3. They are based on the regression model (1) with a combination of an intercept, a linear trend and seasonal dummies. The data generating processes are given by $\Delta_5 x_t = \varepsilon_t$ $(t = 1, 2, \ldots, T)$ with $\varepsilon_t \sim i.i.d.N(0, I_k)$, $k = 1$, 2 and 3, generated using GAUSS 386.

As shown by Lee and Siklos (1995), the distributions at seasonal frequencies are not affected by including intercept and/or trend terms. On the other hand, these distributions are affected when seasonal dummies are included in the regression model. For the zero frequency, the distribution is unaffected by additional seasonal dummies, but is affected by intercept and trend terms.

4 Conclusion and future research

In this paper we proposed an extension of the maximum likelihood cointegration procedure developed by Lee (1992) for daily time series. The critical values of the associated test statistics have been computed for various sample sizes and levels of seasonal cointegration.

In further research we intend to apply this seasonal cointegration procedure to daily stock exchange and exchange rate series in order to reveal the existence of seasonal long-run relationships.

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Table 1: Critical values for daily seasonal cointegration $(n-r)$=1.

		$\theta=0$			$\theta=2\pi/5$			$\theta=4\pi/5$		
	T	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
nc,nd,nt	130	2.92	4.06	6.81	3.01	4.22	7.01	3.04	4.16	7.02
	260	2.97	4.13	6.97	2.97	4.13	6.90	3.00	4.14	6.99
	520	2.97	4.16	6.86	2.98	4.14	6.85	3.01	4.15	7.10
c,nd,nt	130	6.50	8.07	11.60	3.01	4.24	7.02	3.04	4.19	7.06
	260	6.54	8.06	11.84	2.96	4.13	6.93	3.01	4.14	6.98
	520	6.64	8.21	11.64	2.98	4.14	6.85	3.01	4.15	7.10
c,nd,t	130	9.70	11.50	15.60	3.05	4.29	7.09	3.07	4.22	7.14
	260	9.71	11.58	15.90	2.98	4.15	6.94	3.03	4.16	6.99
	520	9.76	11.57	15.69	2.99	4.15	6.87	3.01	4.15	7.11
c,d,nt	130	3.01	4.15	7.11	9.4	11.35	15.66	9.43	11.36	15.37
	260	6.57	8.11	11.95	9.37	11.30	15.36	9.25	11.10	15.05
	520	6.65	8.24	11.66	9.30	11.18	15.03	9.37	11.21	15.21
c,d,t	130	9.80	11.66	15.84	9.49	11.43	15.78	9.51	11.44	15.50
	260	9.76	11.64	16.03	9.39	11.33	15.42	9.28	11.15	15.06
	520	9.80	11.57	15.69	9.30	11.21	15.09	9.38	11.22	15.24

Critical values are based on 30,000 replications using the regression model with (no) constant ((n)c), (no) seasonal dummies ((n)d) and (no) trend ((n)t). The data generating process is obtained from $(1-B^5)x_t=\varepsilon_t$ $(t=1,2,\ldots,T)$, with $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_T \sim i.i.d.N(0,1)$.

Table 2: Critical values for daily seasonal cointegration $(n-r)$=2.

		$\theta=0$			$\theta=2\pi/5$			$\theta=4\pi/5$		
	T	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
nc,nd,nt	130	9.54	11.31	15.36	8.83	10.66	14.36	8.80	10.50	14.43
	260	9.47	11.22	14.88	8.79	10.61	14.29	8.79	10.59	14.45
	520	9.47	11.18	14.96	8.81	10.59	14.60	8.76	10.51	14.48
c,nd,nt	130	13.22	15.29	19.55	8.85	10.69	14.39	8.80	10.49	14.37
	260	13.08	15.08	19.21	8.80	10.60	14.32	8.81	10.58	14.45
	520	13.05	15.01	19.47	8.81	10.59	14.65	8.76	10.50	14.49
c,nd,t	130	16.59	18.69	23.32	8.90	10.73	14.54	8.83	10.55	14.43
	260	16.44	18.59	23.54	8.81	10.63	14.33	8.81	10.58	14.52
	520	16.41	18.55	23.25	8.82	10.60	14.63	8.77	10.51	14.52
c,d,nt	130	13.35	15.44	19.76	15.94	18.20	23.12	15.97	18.14	23.18
	260	13.10	15.11	19.40	15.84	18.01	22.64	15.92	18.18	22.77
	520	13.07	15.03	19.50	15.86	18.01	22.65	15.83	17.90	22.42
c,d,t	130	16.77	18.94	23.59	16.08	18.37	23.35	16.05	18.32	23.26
	260	16.49	18.68	23.71	15.89	18.06	22.71	15.95	18.22	22.85
	520	16.43	18.60	23.27	15.90	18.05	22.67	15.84	17.95	22.46

Critical values are based on 30,000 replications using the regression model with (no) constant ((n)c), (no) seasonal dummies ((n)d) and (no) trend ((n)t). The data generating process is obtained from $(1-B^5)x_t=\varepsilon_t$ $(t=1,2,\ldots,T)$, with $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_T \sim i.i.d.N(0,I_2)$.

Table 3: Critical values for daily seasonal cointegration $(n-r)$=3.

		$\theta=0$			$\theta=2\pi/5$			$\theta=4\pi/5$		
	T	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
nc,nd,nt	130	16.06	18.21	22.69	15.32	17.39	21.78	15.29	17.39	21.96
	260	15.82	17.91	22.39	15.09	17.21	21.77	15.02	17.10	21.54
	520	15.79	17.89	22.32	14.99	17.09	21.45	15.07	17.24	21.50
c,nd,nt	130	19.75	21.94	26.76	15.35	17.45	21.95	15.31	17.41	21.94
	260	19.43	21.77	26.78	15.10	17.22	21.76	15.03	17.10	21.56
	520	19.34	21.53	26.50	14.99	17.11	21.46	15.08	17.27	21.53
c,nd,t	130	23.09	25.50	30.77	15.39	17.52	22.05	15.41	17.45	22.03
	260	22.84	25.23	30.60	15.15	17.22	21.80	15.05	17.13	21.63
	520	22.74	25.15	30.43	15.01	17.11	21.48	15.09	17.26	21.56
c,d,nt	130	19.92	22.18	27.16	22.65	25.24	30.58	22.72	25.15	30.40
	260	19.51	21.82	26.88	22.30	24.73	29.60	22.25	24.64	29.80
	520	19.35	21.56	26.56	21.99	24.45	29.50	22.13	24.58	29.49
c,d,t	130	23.34	25.73	31.17	22.81	25.34	30.72	22.81	25.29	30.44
	260	22.88	25.34	30.54	22.37	24.76	29.78	22.30	24.73	29.80
	520	22.76	25.17	30.44	22.03	24.44	29.55	22.16	24.59	29.51

Critical values are based on 30,000 replications using the regression model with (no) constant ((n)c), (no) seasonal dummies ((n)d) and (no) trend ((n)t). The data generating process is obtained from $(1-B^5)x_t=\varepsilon_t$ $(t=1,2,\ldots,T)$, with $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_T \sim i.i.d.N(0,I_3)$.