E C O N O M I C S B U L L E T I N

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Abstract

In this paper we study the characteristics of the non stationarity of the covariance structure of the S\P500 returns by analyzing the time spectral density of the data. We show that the S\P500 returns has the same characteristics as the modulate white noise process. So, some precautions must be taken before applying traditional stationary models to describe like long size financial time series.

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Nonstationarity characteristics of the S&P500 returns: An approach based on the evolutionary spectral density

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Abstract

In this paper we study the instability characteristics of the covariance structure of the S&P 500 returns. The method used in this paper is based in the decomposition of the time spectral density of the data. We show that the S&P 500 returns has the same characteristics as the modulate white noise process, more precisely the unconditional volatility of the data is time varying. Consequently, some precautions must be taken before applying traditional stationary models to describe like long size financial time series.

Keywords: Time-dependent spectral density; unconditional volatility; S&P 500 returns.
JEL: C12; C52.

1 Introduction

The hypothesis of the covariance stationarity is often required in many traditional models concerned with the description of financial returns data. For example, the traditional families of the ARCH models require the stationarity of the unconditional volatility, the well known concept of long memory process suppose the covariance stationarity of the studied process. But the stationarity of the second moment is a strong hypothesis. Loretan and Phillips(1994) investigated methods of testing the null of the unconditional variance constancy. They concluded the rejection of the null for many financial data. Starica and Micosh(2002) noted that some classical stylized facts in financial series can be explained by a variance shift in the data. These authors considered the S&P500 returns. In this paper we bring out some characteristics of the non stationarity of the S&P500 returns by analyzing the behavior of the time spectral density of the data. Like approach is recently used by Ahamada and Boutahar(2002) to define a test for covariance stationarity. More precisely we use the theory of the evolutionary spectral density of Priestley to identify a non stationary model of the S&P500 returns. This paper is organized as follows: In the first section we present the theory of the evolutionary spectral density of Priestley. In the second section we present the non stationarity test as proposed by Priestley(1969). Finally we apply the test to the S&P 500 returns before giving conclusion and remarks.

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2 Theory of evolutionary spectral density.

2.1 Definition

The theory of the evolutionary spectrum of Priestley (1965) is concerned with oscillatory processes, i.e. processes $\{X_t\}$ defined as follows:

$$X_t = \int_{-\pi}^{\pi} A_t(\omega) e^{i\omega t} dZ(\omega), \tag{1}$$

where, for each ω, the sequence $\{A_t(\omega)\}$, as function of t, has a generalized Fourier transform whose modulus has an absolute maximum at the origin. $\{Z(\omega)\}$ is an orthogonal process on $[-\pi, \pi]$ with $E[dZ(\omega)] = 0$,[1] $E[|dZ(\omega)|^2] = d\mu(\omega)$, where $\mu(\omega)$ is a measure. Without loss of generality, the evolutionary spectral density of the process $\{X_t\}$ is given by $h_t(\omega)$ and defined as follows:

$$h_t(\omega) = \frac{dH_t(\omega)}{d\omega}, \qquad -\pi \leq \omega \leq \pi, \tag{2}$$

where $dH_t(\omega) = |A_t(\omega)|^2 d\mu(\omega)$. The Priestley's evolutionary spectrum theory is particularly attractive concept since it has a physical interpretation. It encompasses most other approaches as special cases and includes many types of nonstationary processes. The instantaneous variance of $\{X_t\}$ is given by

$$\sigma_t^2 = var(X_t) = \int_{-\pi}^{\pi} h_t(\omega) d\omega. \tag{3}$$

These relations show that any modification of the covariance structure of the studied series may be captured by studying the stability of the evolutionary spectral density $h_t(\omega)$. In particular, the relation (3) shows that a modification of the variance of the process necessarily entails a time variation of $h_t(\omega)$.

2.2 Estimation of the evolutionary spectral density

An estimator of $h_t(\omega)$ at time t and frequency ω can be obtained using two windows $\{g_u\}$ and $\{w_v\}$. Without loss of generality, the estimator $\widehat{h}_t(\omega)$ is constructed as follows:

$$\widehat{h}_t(\omega) = \sum_{v \in Z} w_v \left| U_{t-v}(\omega) \right|^2, \tag{4}$$

where $U_t(\omega) = \sum_{u \in Z} g_u X_{t-u} e^{-i\omega(t-u)}$. We choose the following windows $\{g_u\}$ and $\{w_v\}$:

$$g_u = \begin{cases} 1/(2\sqrt{h\pi}), & \text{if} \quad |u| \leq h, \\ \\ 0, & \text{if} \quad |u| > h, \end{cases} \quad \text{and} \quad w_v = \begin{cases} 1/T', & \text{if} \quad |v| \leq T'/2, \\ \\ 0, & \text{if} \quad |v| > T'/2. \end{cases} \tag{5}$$

where h and T' are windows parameters. From Priestley (1988), $E(\widehat{h}_t(\omega)) \simeq h_t(\omega)$, $var(\widehat{h}_t(\omega))$ decreases when T' increases and $\forall\ (t_1, t_2)$, $\forall\ (\omega_1, \omega_2)$, $cov[\widehat{h}_{t_1}(\omega_1)\ \widehat{h}_{t_2}(\omega_2)] \approx 0$ if at least one of the following conditions (i) or (ii) is satisfied:[2]

$$(i) \quad |t_1 - t_2| \geq T', \qquad\qquad (ii) \quad |\omega_1 \pm \omega_2)| \geq \frac{\pi}{h}. \tag{6}$$

[1] This condition implies that $E(X_t) = 0$.

[2] For more details about the relations (i) and (ii) and the choice of h and T', the readers are referred to Priestley (1969, 1981).

3 Presentation of the nonstationarity test

3.1 Principle of the test

The test of non stationarity of Priestley and Rao can be used also as a test of model selection. When the stationarity hypothesis is rejected the test identify the non stationary model which describes the data. This test is based on the decomposition of the time spectral density $h_t(\omega)$ of the studied process (say $\{X_t\}$) as a sum of a time component(α_t), a frequency component(β_ω) and a time-frequency component($\gamma_{t\omega}$):

$$log(h_t(\omega)) = \mu + \alpha_t + \beta_\omega + \gamma_{t\omega} \tag{7}$$

Many cases can be noted:
Case.1: Because the spectral density of a stationary process is time independent then the process $\{X_t\}$ is stationary if $\alpha_t = \gamma_{t\omega} = 0$.
Case.2: If only $\gamma_{t\omega} = 0$ then the process $\{X_t\}$ is a modulate stationary process ,i.e. $X_t = f(t)\varepsilon_t$ where $f(t)$ is a deterministic function and ε_t is a stationary process(the spectral density of ε_t is only frequency dependent).
Case.3: If $\beta_\omega = \gamma_{t\omega} = 0$ then the spectral density $h_t(\omega)$ is only time dependent, then the process $\{X_t\}$ is a sequence of independent random process with time dependent covariance structure,i.e., the process X_t has precisely the form of modulate white noise process $X_t = f'(t)\varepsilon'_t$ where $f'(t)$ is a deterministic function but ε'_t is an *i.i.d.* process(spectral density of ε'_t is a constant).
Case.4: If $\alpha_t = \beta_\omega = \gamma_{t\omega} = 0$ then the spectral density is a constant and the process $\{X_t\}$ is a white noise since the spectral density of a white noise is a constant.

3.2 Description of the test

Let $\{X_t\}_{t=1}^T$ be data from a discrete process $\{X_t\}$ with theoretical evolutionary spectral density $h_t(\omega)$ and its estimate $\widehat{h}_t(\omega)$ (4) . Let consider a set of times $\{t_i\}_{i=1}^I$ and a set of frequencies $\{\omega_j\}_{j=1}^J$. Let $Y_{ij} = \ln\left(\widehat{h}_{t_i}(\omega_j)\right)$, and $h_{ij} = \ln\left(h_{t_i}(\omega_j)\right)$. From Priestley (1969), we have

$$Y_{ij} \approx h_{ij} + e_{ij}, \tag{8}$$

where the sequence $\{e_{ij}\}$ is approximately uncorrelated and identically normal distributed. If the windows(5) are used to estimate $h_t(\omega)$ then from Priestley(1969, 1981), we have the follows approximate value of the variance:

$$\sigma^2 = var(e_{ij}) \approx \frac{4h}{3T'} \tag{9}$$

Let decompose the time spectral density h_{ij} as follows: $h_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$ where α_i, β_j and γ_{ij} indicate respectively the time component, the frequency component and the time-frequency component of the evolutionary spectral density. Then the relation (8) becomes:

$$Y_{ij} \approx \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ij} \tag{10}$$

The relation(10) is a standard two factors analysis of variance model. Because $var(e_{ij})$ is known, we can apply the classical χ^2-tests for testing the presence of temporal effects (α_i), frequency effects(β_j) or time-frequency effects(γ_{ij}). The results of these tests are summarized in a classical table of variance analysis as follows:

Table.1: Table of two factors analysis of variance for Priestley's test.

Effect ⇓ \Statistics ⇒	Statistics	Degree of freedom of the χ^2-tests under the null of absence of effects
Temporal effects: α_i	$S_T = \frac{J}{\sigma^2}\sum_{i=1}^{I}(Y_{i.} - Y_{..})^2$	$(I-1)$
Frequency effects: β_j	$S_F = \frac{I}{\sigma^2}\sum_{j=1}^{J}(Y_{j.} - Y_{..})^2$	$(J-1)$
Interaction effects: γ_{ij}	$S_{TF} = \frac{1}{\sigma^2}\sum_{i=1}^{I}\sum_{j=1}^{J}(Y_{ij} - Y_{i.} - Y_{.j} + Y_{..})^2$	$(I-1)(J-1)$
Total	$S_{TF} = \sum_{i=1}^{I}\sum_{j=1}^{J}(Y_{ij} - Y_{..})^2$	$IJ-1$

where $Y_{i.} = \frac{1}{I}\sum_{j=1}^{J} Y_{ij}$, $Y_{.j} = \frac{1}{J}\sum_{i=1}^{I} Y_{ij}$ et $Y_{..} = \frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} Y_{ij}$. Under the null of the absence of the temporal effects, S_T is distributed as χ^2 with $(I-1)$ degree of freedom. Under the null of the absence of the frequency effects, S_F is distributed as χ^2 with $(J-1)$ degree of freedom. And under the null of the absence of the time-frequency effects, S_{TF} is distributed as χ^2 with $(J-1)(I-1)$ degree of freedom.

4 Application to the S&P 500 returns

We apply the previous test for the description of the daily returns of the S&P 500 denoting by X_t from 08/17/1993 to 07/18/2001 (size $T = 2000$). Before the application of the tests we need to fixe some parameters and to chose the set of time and the set of frequencies. For the windows parameters of the relation (5), we fixe $h = 7$ and $T' = 100$ then from(9) we have $\sigma^2 = var(e_{ij}) \approx \frac{28}{300}$. The sets of time and frequencies are chosen as follows:

$$\{t_i = 100i\}_{i=1}^{I=20} \text{ and}^3 \ \{\omega_j = \pi/20(1 + 3(j-1))\}_{j=1}^{J=7} \tag{11}$$

The following table.2 summarizes the results of the tests for the S&P 500 returns.

Table.2: Table of two factors analysis of variance for Priestley's test.

Effect ⇓ \Statistics ⇒	Statistics	Degree of freedom of the χ^2-tests under the null of absence of effects
Temporal effects: α_i	$S_T = 156,19994$	19 (critical value for $\alpha = 0.05: \ 30,1$)
Frequency effects: β_j	$S_F = 4,8218701$	6 (critical value for $\alpha = 0.05: 12,6$)
Interaction effects: γ_{ij}	$S_{TF} = 23,555906$	114 (critical value for $\alpha = 0.05: 140$)

The table.2 indicates without ambiguousness that we can reject the null of the absence of the temporal effects, more precisely that indicates the presence of temporal effects in the time spectral density of the S&P

500 returns. On the other hand, the table.2 indicates that we cannot reject the null of the absence of the frequency effects and the absence of the time-frequency effects, then there are no frequency effects and no time-frequency effects in the time spectral density of the S&P 500 returns. These results shows that the S&P 500 returns must be modelized as the case.3 (section.3.1),i.e., $X_t = f'(t)\varepsilon'_t$ where $f'(t)$ is a deterministic function and ε'_t is an *i.i.d.* process. The function $f'(t)$ may be a constant function over successive intervals. That implies that the unconditional volatility of X_t is given by the time varying deterministic function $var(X_t) = \sigma^2 f'^2(t)$ where $\sigma^2 = var(\varepsilon'_t)$. Because the unconditional volatility is not constant, the classical models of description of financial returns data cannot be applied to X_t. Preliminary estimation of $f'(t)$ is necessary before applying the traditional stationary model (as ARCH families model) to $\widehat{\varepsilon'_t} = \frac{X_t}{\widehat{f'(t)}}$. There are a lot of methods to estimate $f'(t)$ as methods based on application of filter (Hordrick Prescott) or the break point detection method based in algorithm of cumulative sums(Inclan and Tiao, 1994) if $f'(t)$ is constant over successive intervals. So, if the filtered stationary series $\widehat{\varepsilon'_t}$ is described by a stationary GARCH model then X_t is described by a GARCH model with time varying parameters(parameters are multiplied by $\widehat{f'}(t)$). The GARCH model with time varying parameters is used by Starica and Micosh(1999) to describe the S&P500 returns. So, our results confirm the Starica and Micosh approach(1999). The estimation of the time spectral density, in figure.2, indicates some evident shift from $t = 1000$ (07/31/1997). The amplitudes of spectral pick changed considerably from this date. That confirms the instability of the covariance structure of the studied data. The figure.2 seems to indicate also that the spectral density is approximately constant in the interval $t = 1, ..., 1000$(07/31/1997), so the data are covariance stationary in this interval while in the interval $t = 1001, ..., 2000$ the unconditional volatility of the process has significantly modified.

5 Conclusion

In this paper we examined the covariance structure of the S&P 500 returns by using a decomposition of the time spectral density of the data. The methods revealed that the S&P 500 returns denoted by X_t can be specified as a modulate white noise process, i.e., $X_t = f'(t)\varepsilon'_t$ where $f'(t)$ is a deterministic function and ε'_t is an *i.i.d.* process. The function $f'(t)$ my be constant over successive intervals. Then the unconditional variance is time varying,i.e., $var(X_t) = (f'(t))^2\sigma^2$ where $var(\varepsilon'_t) = \sigma^2$. So, the time varying variance must be previously estimated and extracted from the data before using traditional stationary methods(families of ARCH model, long memory process etc..) to $\widehat{\varepsilon'_t} = \frac{X_t}{\widehat{f'(t)}}$. Our results confirm the already results obtained by Loretan an Phillips(1994), Starica and Micosh(1999).

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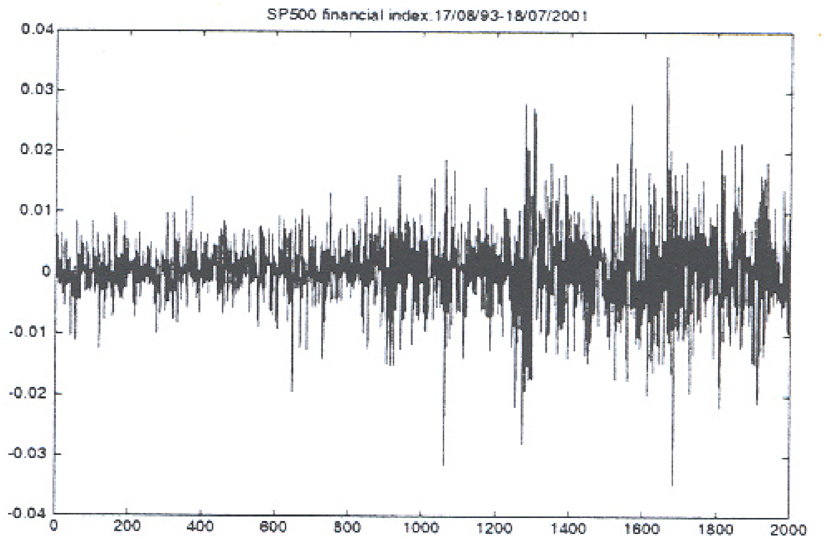

Figure 1. S&P 500 Return Series

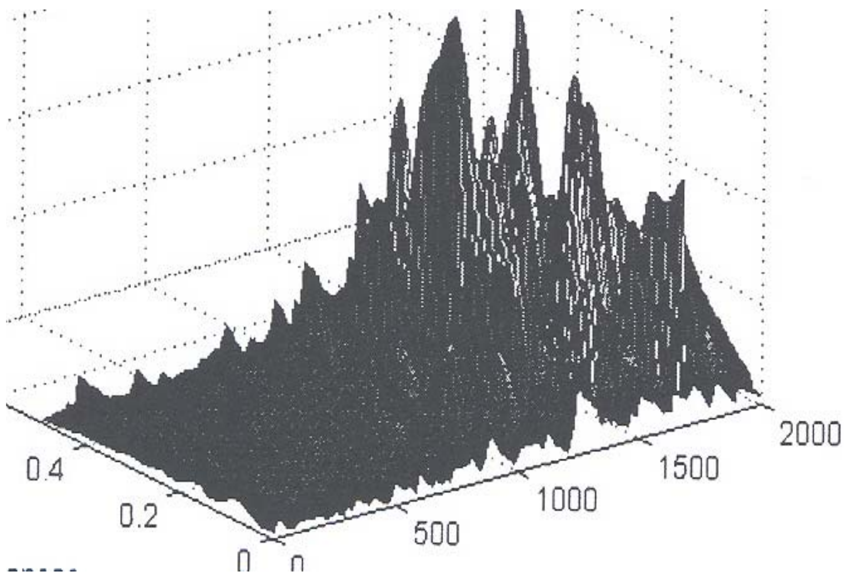

Figure 2. Estimation of the Time Spectral Density of the S&P 500 Returns. Legend. x-axis: Time(1,..,2000); y-axis: Frequencies(0; 0.5); z-axis: Time Spectral Density