E c o n o m i c s   B u l l e t i n

Reputation by imitation: an evolutionary chain−store game with strategic matching

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Abstract

This paper shows how strategic matching generates reputation−building behavior in an evolutionary chain−store game. Strategic matching means the possibility for an entrant to choose in a strategic way the local market into which it will move. Players are boundedly rational and follow behavioral rules simply requiring that the frequency of any strategy enjoying the highest payoff should never decrease. The model shows how strategic matching, in preventing the random entries in markets of fighting monopolists, reinforces the reputation effects. Under some conditions, the Nash equilibrium with reputation effects emerges as the long−run equilibrium of the evolutionary chain−store game. Using the bounded rationality set−up offered by evolutionary game theory, the paper follows Selten (1978)'s intuition underlying the necessity of a limited rationality approach in order to capture reputation effects.

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1 Introduction

One of the simplest ways to analyze the *reputation phenomenon* is to consider a game in which a single long-run player faces a finite or infinite sequence of short-run opponents, each of whom plays only once. The question is whether the long-run player is willing to incur losses today in order to influence the behavior of future opponents. A common intuition is that the strategy chosen by the long-run player in the first periods is the one which yields the highest future payoff, provided that short-run losses are outweighed by long-run gains. However, it has proven difficult to model such phenomena in finite horizon.

Selten (1978) proposes a game, called the *chain-store game*, where a monopolist (a chain-store) faces a set of potential competitors deciding subsequently whether or not to enter a local monopolist's market. Under the *common knowledge assumption*, even though it is a Nash equilibrium for every potential entrant to stay out and for the monopolist to fight each one of them, Selten pointed out that the reputation effect does not emerge as a rational behavior in the game[1]. This counter-intuitive result has been called the *chain-store paradox* because, even though the monopoly can build a reputation for fighting, the perfectness criterium does not select this strategy. Selten (1978) concludes by arguing that, if reputation effects are observed in reality, then we need a limited rationality approach in order to reduce discrepancies between theoretic game analysis and human behavior.

Following Rosenthal (1981), Kreps and Wilson (1982) and Milgrom and Roberts (1982) show that the monopolist can deter entrants if the complete information assumption is relaxed. They modify the chain-store game by assuming some doubt on the side of the entrants about the monopolist's payoffs[2]. It is worth noting that one of the interpretations proposed by Milgrom and Roberts (1982) for justifying the informational asymmetry involves « the entrants allowing that the firm is not behaving as a fully rational game theorist ».

In this paper I use evolutionary game theory to model the reputation effects emerging in the long-run as the best strategy to imitate. It is thus a limited rationality approach following Selten's intuition and Milgrom and Roberts (1982)'s justification of uncertainty. In the evolutionary framework, (i) players are assumed to be boundedly rational agents using the past experience and simple behavioral rules for elaborating a strategy[3] and (ii) there are several chain-store games played simultaneously and over again.

From (i) and (ii) two departures have to be considered in regard to the original Selten's game. First, incumbent firms can now learn from experience about the other monopolists' choices; strategies emerge from a trail-and-error learning process instead of introspective-type arguments[4]. Second, and due to the multiplicity of games, evolutionary models have to make some assumptions as to how players meet in each stage game. The literature offers various specifications regarding the matching mechanism and, as Oechssler (1997) and Robson and Vega-Redondo (1996) showed, evolutionary models are quite sensitive to the specification of the matching process. In a general way, one may define two models of matching: the *fully global selection* model and the *group selection* model. In the former, interaction takes place within the entire population, where individuals are randomly matched to play a bilateral game[5]. In contrast, the second model of matching assumes that interaction takes place within

[1] This equilibrium is not subgame-perfect. See Selten (1975) and Selten (1978).

[2] However, as suggested by the authors themselves, models with incomplete information present some drawbacks: "By cleverly choosing the nature of the small uncertainty (precisely its support), one can get out of a game-theoretic analysis whatever one wishes" (Kreps and Wilson (1982)). On this point see also Fudenberg and Maskin (1986). In an alternative way, Massò (1992) and Trockel (1986) argue that the sequentiality of Selten's game, i.e. the order in which the players enter the game, may have consequences for the analysis in terms of sequential equilibrium. This assumption allows to avoid problems inherent to the incomplete information assumption. Both papers uses models with imperfect information and show that there exist a set of sequential equilibria where reputation and entry deterrence are possible.

[3] In this setup, the game and players rationality are not common knowledge.

[4] On the point of departure between evolutionary game theory and the introspective approach, see Kandori, Mailath and Rob (1993) and Samuelson (1997).

[5] For more details on this class of matching mechanisms, see Fudenberg and Levine (1998).

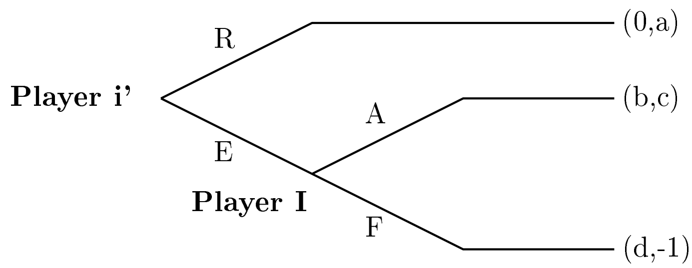
relatively small subpopulations, where there is infrequent migration between subpopulations[6]. Both models share the assumption that the matching mechanism relies on a perfectly random process, removing *strategic considerations* like group or partner choice. Recently, however, some evolutionary models have included the possibility of *strategic considerations* in the matching process. For instance, Oechssler (1997) studies the coordination problem in a population partitioned into groups, where players can, not only choose which action to take in the game, but also which group they want to join. Similarly, Bergstrom (2002*a*) explores the possibility of partner choice in a multiplayer prisoners' dilemma game with voluntary matching.

In the same spirit, the evolutionary chain-store game presented here considers the possibility of *strategic matching* on the side of competitors. The intuition sustaining this assumption rests on the multiplicity of chain-store games offered to the competitors by the evolutionary set-up. Due to this multiplicity, the choice of a particular monopolist becomes available to the potential competitors. This is what I call *strategic matching*: the possibility for an entrant to choose in a strategic way the local market into which it will move, according to the past decisions of the monopolists. The strategic matching assumption plays a key role in the evolutionary analysis of the chain-store game. It permits to select the Nash equilibrium with reputation effects as the *long-run equilibrium* of the game. Thus, the model can produce a clear-cut selection between equilibria based on the strategic matching assumption.

The rest of the paper is organized as follows: Section 2 reviews the original Selten's chain-store game. Section 3 presents the evolutionary model with strategic matching. Section 4 examines the selection or learning mechanism of the evolutionary process, and Section 5 states the results of this mechanism in the chain-store game. Section 6 specifies the mutation mechanism and its results. Section 7 concludes the paper.

2 The original set-up

The chain-store game consider a situation where a single long-run incumbent firm I faces sequential and potential entry by a set of short-run firms denoted $C = \{1', ..., i', ..., n'\}$. Player I is operating n stores in different locations. Let i represents a store of player I. Each period, a potential competitor i', which plays only once but observes all previous play, confronts the decision problem described in the following game:



where a, $b > 0$, $d < 0$ and $-1 < c < a$. In Selten's game, each competitor i' has to choose between two actions: enter the market (Action E) or not enter (R). If player i' decide to enter, the firm I has to decide for its store i whether it fights the entrant (F) or it acquiesces the entry (A).

With the assumption that the game and the rationality of all the firms are common knowledge, Selten (1978) points out that the only subgame-perfect equilibrium of the game corresponds to the situation where each entrant chooses "E" and the monopolist accepts the entry. Thus, the reputation effects (the Nash equilibrium in which the incumbent firm fights the first entries in order to deter the next ones) do not emerge as a rational behavior in the game.

[6] Biologists call this class of models *Haystack models*. See Bergstrom (2002*b*).

3 An evolutionary model with strategic matching

This section inserts the chain-store game into an evolutionary setup. I use the model developed in Robson and Vega-Redondo (1996) to which I add the possibility of *strategic matching*.

Assume that there are large but finite sets of competitors and monopolists; denote these sets respectively $C = \{1', ..., i', ..., n'\}$ and $M = \{1, ..., I, ..., N\}$, with $n' = Nn$. Player I represents a chain-store as described in Section 2. The stores locations of each player I are indicated by $\lambda_{iI} = (x_{iI}, y_{iI})$ where $\lambda_{iI} \in \mathbb{R}^2$. Let $\lambda_I = \{\lambda_{1I}, ..., \lambda_{iI}, ..., \lambda_{nI}\}$ be the locations profile of I with $\lambda_{iI} \neq \lambda_{jI}$ $\forall i, j$. Then, $\Delta = \{\lambda_I\}_{I \in M}$ is the set of all possible market points.

As in the original game, monopolists face a set of potential competitors. Time is measured discretely and indexed by $t = 1, 2, 3....$ At period t, player $i' \in C$ is paired with a market point $\lambda_{iI} \in \Delta$ to play the one-shot chain-store game described in Section 2. The matching takes place according to the following matching process. With probability θ, each competitor $i' \in C$ chooses in a strategic way (i.e., according to its expectations about the monopolist's strategy) a market point $\lambda_{iI} \in \Delta$. This is called *strategic matching*[7]. With the complementary probability $1 - \theta$, each competitor $i' \in C$ is randomly matched with a market point $\lambda_{iI} \in \Delta$. This is called *random matching*. Notice that monopolists cannot choose a particular competitor and then, from their point of view, the matching only follows a random process.

The sets $C = \{1', ..., i', ..., n'\}$ and $M = \{1, ..., I, ..., N\}$ evolve in the long-run according to entry and exit of firms. A chain-store game ends when each market point $\lambda_{iI} \in \lambda_{iI}$ has been entered. In this case, I assume that player I is replaced with a new chain-store in M so that the size of the set remains stable. Similarly, competitors regularly leave the game and are replaced with new players.[8]

In an evolutionary model, we are interested by the description of the strategic behavior adopted by the players in the long-run. Let $z_t = (z_t^c, z_t^m) \in Z \equiv \{z = (z^c, z^m) : 0 \leq z^c \leq n',\ 0 \leq z^m \leq N\}$ be the state at t of the evolutionary dynamics, where z_t^c and z_t^m represent respectively the number of players using R in C and using F in M. For convenience, states $z_1 = (n', N)$ and $z_2 = (0, 0)$ will be directly written $z_1 = (R, F)$ and $z_2 = (E, A)$. The evolution of state z_t is realized via two different mechanisms: selection and mutation. In the next section, I present the selection mechanism. Mutations are considered in Section 6.

4 The selection mechanism

The selection component of the dynamics describes how players choose their strategies. They are assumed to be myopic[9] and adaptive. Thus, agents do not form expectations about the future course of play and simply take into account the decisions made in the past to determine their strategies. This means that changing from one strategy to another is dictated by such considerations as: How well do I perform compared to the other players? What is the strategy used by the most successful player or what is the strategy with the highest average payoff?

Let $e_t = (e_{1't}, ..., e_{i't}, ..., e_{n't})$ be the strategy-profile of the competitors at period t, and denote by $\pi_{C,t} = (\pi_{1't}, ..., \pi_{i't}, ..., \pi_{n't})$ the corresponding payoff-profile. In the same way, $m_t = (m_{1t}, ..., m_{It}, ..., m_{nt})$ represents the strategy-profile of the incumbent firms at period t, with $m_{It} \in \{A, F\}$. It is further assumed that each store i follows the strategy chosen by player I. If player I faces no effective entry at t then its strategy at t is the last one used when confronted with an entry.

When strategic matching is available (probability θ), player $i' \in C$ observes m_{t-1} and computes a myopic best reply. His strategy consists of choosing an action $a_{i't} \in \{E, R\}$ and

[7]See Section 1.

[8] This represents an economic interpretation of mutation phenomenon. See Canning (1989) and Section 6.

[9]On the justification of the myopic assumption, see the end of this section.

of selecting a market point $\lambda_{iI} \in \Delta$.[10] Formally, player i''s strategy is a couple $e_{i't} = (a_{i't}, \lambda_{iI})$. Let $\pi_{i'}(e_{i'}, m_I, n_i') \in \mathbb{R}$ be the payoff function of player i', with n_i' the number of competitors choosing λ_{iI}. I assume that $\pi_{i'}((E, \lambda_{iI}), m_I, n_i') \leq d$ if $n_i' > 0$. This means that the entry of more than one entrant in the same market $\lambda_{iI} \in \Delta$ removes the expected gain of the monopolist's acquiescence for the entrants.

In period t, player $i' \in C$ selects a strategy $e_{i't} = (a_{i't}, \lambda_{iI})$ that satisfies the behavioral rule

$$e_{it} \in argmax_{a,\lambda} \ \pi_{i'}(e_{i't}, m_{It-1}, n_i'). \tag{1}$$

In this program, player i' firstly chooses a myopic best reply to each of the opponent strategies in m_{t-1}; namely player i' constructs all the possible couples $(a_{i't}, \lambda_{iI})$; he then chooses among them the one maximizing its payoff function.

With the probability $1-\theta$, player $i' \in C$ is randomly matched with a market point $\lambda_{iI} \in \Delta$. Given any $z_t = (z_t^c, z_t^m) \in Z$, let $\tilde{r}_t$ be the random variable describing the matching process at time t between competitors and monopolists. A realization r_t of $\tilde{r}_t$ represents the set of numbers $r_{k,l,t} \geq 0$, with $k = R, E$ and $l = A, F$. The number of pairings between k-users and l-users at t is then $r_{k,l,t}$.[11] The support of $\tilde{r}_t$ is denoted $P(z_t)$. The random average payoffs for each strategy are given by:

$$\bar{\pi}_E(z_t, r_t) = \frac{b \ r_{E,A,t} + d \ r_{E,F,t}}{n' - z_t^c} \ , \text{ and } \ \bar{\pi}_R(z_t, r_t) = \ 0,$$

with $n' - z_t^c = r_{E,A,t} + r_{E,F,t}$. The evolution of z^c, the number of competitors choosing R, satisfies the following restrictions,

$$z_t^c = \begin{cases} \geq z_{t-1}^c & \text{if } \bar{\pi}_E(z_t, r_t) < 0, \\ \leq z_{t-1}^c & \text{if } \bar{\pi}_E(z_t, r_t) > 0, \end{cases} \tag{2}$$

with $z^c \in \{1, ..., n' - 1\}$.[12] This latter restriction ensures that the payoffs are well defined and that strict inequalities are feasible. The selection mechanism leading monopolists decisions is similar to those of competitors in the case of random matching. The average payoffs are given by:

$$\bar{\pi}_A(z_t, r_t) = \frac{a \ r_{A,R,t} + c \ r_{A,E,t}}{n(N - z_t^m)} \ , \text{ and } \ \bar{\pi}_F(z_t, r_t) = \frac{a \ r_{F,R,t} - r_{F,E,t}}{n \ z_t^m}.$$

The evolution of z^m, the number of monopolists choosing F, satisfies the following restrictions,

$$z_t^m = \begin{cases} \geq z_{t-1}^m & \text{if } \bar{\pi}_F(z_t, r_t) > \bar{\pi}_A(z_t, r_t), \\ \leq z_{t-1}^m & \text{if } \bar{\pi}_F(z_t, r_t) > \bar{\pi}_A(z_t, r_t), \end{cases} \tag{3}$$

with $z^m \in \{1, ..., N - 1\}$.

This formulation of the selection mechanism is very flexible since it means only that the frequency of any strategy enjoying the highest payoff should never decrease[13]. One possible

[10] Following Oechssler (1997), E and R are called *actions* since in our model of strategic matching competitors strategy indicates both the choice of a location and of an action.

[11] Every possible way of pairing k-users and l-users is assumed to be equally likely.

[12] In case of identical average payoffs, players behavior can be specified in any arbitrary manner, as the random choice of a strategy.

[13] It is close to the notion of *payoff monotonic* dynamics defined by Weibull (1995).

interpretation is that players' decisions are an imitation process where people learn what are good strategies by observing what has worked well for other people.

What supports the different behavior assumed between « random » and « strategic » competitors? This difference relies on the information used by each category of competitors. In the case of random matching, players only use global information on the behavior of firms, that is information concerning an industry containing several incumbent firms. On the other hand, strategic competitors use more precise information concerning each firm of an industry considered as independent. Notice also that the best reply dynamics of strategic competitors needs more information than the imitative process. The latter is usually interpreted as the idea that the world is a complicated place where agents cannot calculate the best responses to their environment. Therefore one can argue that competitors do not have the same access to information, and thus do not use the same behavioral rules.

The dynamics just described allow some important inertia in strategy adjustment. Indeed, evolutionary models assume that not all agents react instantaneously to their environment but rather gradually adjust their strategy following the selection mechanism. Inertia is an important aspect of the evolutionary dynamics since it offers a good justification of myopic behavior: as players know that only a small segment of agents changes its actions, strategies that prove to be effective today are likely to remain effective in the near future. Accordingly, I do not consider that all players simultaneously adjust their strategy at each period t. Rather, I assume that there is some inertia in the learning process. Formally, each player independently with some fixed probability $\phi \in (0,1]$ receives the opportunity to update his strategy in each given period.

5 Results of the selection mechanism

In this section, I examine the paths along which behavior evolves and, in particular, the states towards which the selection process converge. As we will see below, the possibility of strategic matching has strong implications on the trajectories of the evolutionary system.

In the selection mechanism with random matching (the process (2)-(3)), extinct strategies are required to remain extinct, i.e., mutation is needed to introduce a new strategy[14]. Thus,

Observation 1 *States $z'_1 \in Z_1 \equiv \{(R, z^m) : 0 \leq z^m \leq N\}$, $z_2 = (E, A)$, and $z_3 = (E, F)$ are stationary states of the selection dynamics (2)-(3).*

This means that, without perturbations or mutations, once the learning process (2)-(3) has reached one of the states given in Observation 1, it remains forever. On the other hand, all states $z'_2 \in Z_2 \equiv \{(z^c, z^m) : 0 < z^c < n',\ 0 \leq z^m \leq N\}$ are not stationary states. In order to see this, consider the 3 possible conditions under which states z'_2 are stationary states: (1) $\bar{\pi}_E(z_t, r_t) = 0$ and $z^m = 0$ or (2) $\bar{\pi}_E(z_t, r_t) = 0$ and $z^m = N$ or (3) $\bar{\pi}_E(z_t, r_t) = 0$ and $\bar{\pi}_A(z_t, r_t) = \bar{\pi}_F(z_t, r_t)$. However, only a particular number of pairings between E-users and A-users ensures that $\bar{\pi}_E(z_t, r_t) = 0$. Formally, it is necessary that $r_{E,A} = -d(n' - z^m)/b - d$ for $\bar{\pi}_E(z_t, r_t) = 0$. As $r_{E,A}$ is the realization of a random variable, one cannot observe $r_{E,A,t} = -d(n' - z^m)/b - d$ for all periods $\tau \geq t$. Moreover, we can state the following:

Proposition 1 *Selection dynamics (2)-(3) converges to states $z_2 = (E, A)$ or $z'_1 \in Z_1 \equiv \{(R, z^m) : 0 \leq z^m \leq N\}$ from all states $z'_2 \in Z_2$.*

Proof. See the appendix.

[14] This constitutes a point of departure between the Kandori, Mailath and Rob (1993)'s model and the assumption made by Robson and Vega-Redondo (1996), which in turn is similar to the biologists formulation of evolutionary dynamics.

Proposition 1 shows that reputation effects may exist in the evolutionary chain-store game with purely random matching. The first reason is that in an evolutionary setup (and contrary to the standard setup) the monopolist does not condition its strategy on the number of stores left. Second, entrants are myopic players considering that a monopolist who fought before will fight again. They do not try to influence the future play of their opponents. Thus, Subgame perfection may not be attained here.

When strategic matching is considered, the selection mechanism is the process (1)-(3). In that case, the set of stationary states is reduced due to the dynamics of best reply followed by competitors.

Proposition 2 *States $z_1 = (R, F)$ and $z_2 = (E, A)$ are the only stationary states of the selection dynamics (1)-(3). Furthermore, the system deterministically converges to state $z_1 = (R, F)$ from all states with $z^m \geq 1$.*

Proof. See the appendix.

Proposition 1 means that, under the random matching assumption, the selection mechanism either converges to the subgame-perfect equilibrium $z_2 = (E, A)$ or to the set $Z_1 \equiv \{(R, z^m) : 0 \leq z^m \leq N\}$. The latter is a component of Nash equilibria expressing the strategic indifference of the monopolists when confronted with strategy R. This disappears when strategic matching is assumed (Proposition 2). In that case, the set Z_1 is reduced to the singleton $z_1 = (R, F)$. There are two forces behind this result. First, incumbent firms that choose F are protected by the strategic choice of a location from the entrants which play E during the convergence to $z_1 = (R, F)$. Second, monopolists which remain with strategy A are selected by competitors, which decreases the average payoff of A. These two forces together favor the imitation of strategies F which in turn favors the imitation of R. Therefore, indifference between A and F does not appear when the learning process reveals to competitors that R is the best strategy to play. Thus, preventing the random entries in markets of fighting incumbents strategic matching reinforces the reputation effects.

6 The selection mechanism with mutations

I turn now to the possibility of *mutation* in order to show that state $z_1 = (R, F)$ is the *long-run equilibrium* of the evolutionary chain-store with strategic matching. Our attention is thus limited to selection dynamics (1)-(3). Besides this selection mechanism, *mutation* is the other force acting on agents' strategies. It refers to a situation where an individual randomly switches to a new strategy. After the completion of the learning adjustment, each agent independently changes his strategy with a small probability ϵ. The learning process is then perturbed[15]. In economic contexts, the mutation phenomenon may be interpreted as experimentation of nonoptimal strategies in the sense of (1)-(3) or the entry of a new player who knows nothing about the game.

The dynamic process (1)-(3) combined with the mutation mechanism generates a Markov chain over the finite state space Z. The existence of a small probability $\epsilon > 0$ ensures that the process has a unique stationary distribution[16] summarizing the long-run behavior of the system, regardless of initial conditions. The latter characteristic of the model is particularly interesting when the selection mechanism had several absorbing states, since it allows a selection to be made between them. Our goal is to find the *long-run equilibrium* of the game assuming that $\epsilon \to 0$. In the case of the evolutionary chain-store game, we have to compute the number of mutations required in the transitions between the stationary states of the selection mechanism. The long-run equilibrium is simply the one requiring the fewest

[15] As mentioned by Samuelson (1997), mutation is a residual capturing whatever has been excluded when modeling selection.

[16] See Kandori, Mailath and Rob (1993) or Young (1993).

mutations[17].

Proposition 3 *The number of mutations required to move the system from $z_1 = (R, F)$ to $z_2 = (E, A)$ is at least two when selection dynamics (1)-(3) is considered.*

Proof. See the appendix.

From Propositions 2 and 3, it immediately follows

Corollary 1 *State $z_1 = (R, F)$ is the long-run equilibrium of the evolutionary chain-store game with strategic matching.*

Proposition 2 informs us that one mutation is sufficient for convergence of the system to $z_1 = (R, F)$. As transition from $z_1 = (R, F)$ to $z_2 = (E, A)$ requires at least two mutations, state $z_1 = (R, F)$ is the long-run equilibrium of the evolutionary chain-store game when the choice of a monopolist is viewed as a component of the entrants' strategy. It is worth noting that state $z_2 = (E, A)$ is the long-run equilibrium of the evolutionary chain-store game with random matching. This is due to the absence of the two forces sustaining the result in proposition 2.

7 Conclusion

This paper proposed an evolutionary chain-store game with *strategic matching*. The model considers potential competitors which have to not only choose whether or not to enter, but also the local market into which they will move. This is what I call *strategic matching*. The model show how strategic matching can favor « fighting monopolists », and thus how the reputation effects emerge in the long-run as the best strategy to imitate. Using evolutionary game theory, the paper follows Selten (1978)'s intuition underlying the necessity of a limited rationality approach in order to reduce discrepancies between theoretic game analysis and human behavior.

Appendix

Proof of Proposition 1. Assume that $0 < z^c < n'$ (both strategies R and E are present in C) and that $\bar{\pi}_E(z_t, r_t) < 0$. Then, strategy R is imitated whatever the strategic choice of incumbents, since R realizes 0 against A and F. On the side of the incumbents, the increase of R-users can create an indifference between the options A and F depending on the speed of adjustment of the system (i.e., the level of $\phi_{i'}$ and ϕ_I). This may lead the system to the set $Z_1 \equiv \{(R, z^m) : 0 \leq z^m \leq N\}$ which includes $z_1 = (R, F)$. Assume now that $\bar{\pi}_E(z_t, r_t) > 0$ and at the same time $\bar{\pi}_A(z_t, r_t) > \bar{\pi}_F(z_t, r_t)$. As more E-users sustains more A-users (and reciprocally), the system reaches the state $z_2 = (E, A)$. In the situation where $\bar{\pi}_A(z_t, r_t) < \bar{\pi}_F(z_t, r_t)$, more F-users prevents the increase of the E-users (and reciprocally), moving the system away from $z_2 = (E, A)$.

Proof of Proposition 2. Assume first that $0 < z^m < N$. Then, at period t, competitors have a unique best reply to each of the two incumbent strategies (to enter if the monopolist accepts and to give up if it fights). Due to the strategic matching, during the subsequent periods $\tau \geq t$ the best-reply mechanism ensures that $\bar{\pi}_A(z_t, r_t) = c$ whereas $\bar{\pi}_F(z_t, r_t) = a$. As $c < a$, one observes that $z^m_\tau > z^m_t$, that is to say strategy F is imitated by monopolists constraining competitors to choose R. This result also holds for $z^m = N$, and then it can be stated that the selection dynamics (1)-(3) converges deterministically to state $z_1 = (R, F)$ from all $z^m \geq 1$.

Consider now that $z^m_t = 0$. Thus, the only best reply is E. As the monopolists dynamics selection cannot generate itself new strategies (extinct strategies remain extinct), state $z_2 = (E, A)$ is observed during the subsequent periods.

[17] For more details on this result, see Kandori, Mailath and Rob (1993) or Samuelson (1997) or Vega-Redondo (1996).

Proof of Proposition 3. We have to show that one mutation is not sufficient to move the dynamics from $z_1 = (R, F)$ to $z_2 = (E, A)$. Assume first that the system is in $z_1 = (R, F)$ and consider one mutation $j' \in C$ playing E. As all monopolists play F, we observe that $\bar{\pi}_E(z_t, r_t) > \bar{\pi}_R(z_t, r_t)$. Consider now one mutation $J \in M$ playing A. Then, strategic competitors choose the locations of mutant $J \in M$ which ensures that $\bar{\pi}_A(z_t, r_t) = c$ whereas $\bar{\pi}_F(z_t, r_t) = a$. As $c < a$, monopolist J returns to strategy F constraining competitors to choose R.

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