E C O N O M I C S   B U L L E T I N

Asymmetric cycles in unobserved components models

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Abstract

A class of structural time series models with an asymmetric cyclical component is presented and used in order to test for asymmetry in economic time series. The asymmetric cycle is defined as a sine−cosine wave where the frequency of the cycle depends on past observations of the stochastic process being modelled. Due to the conditional Gaussianity of the model, Kalman filtering techniques can be used in the estimation of the parameters, and a standard test for equality of cyclical frequency can be used as a symmetry test. Applying the test to US economic time series reveals strong cyclical asymmetries in unemployment and industrial production, and no significant deviation from symmetry in GDP. The test is also applied to industrial production data in EU countries, with mixed results.

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1 Introduction

Although the discussion on whether business cycle variables are symmetric or, on the contrary, present some type of asymmetry can be traced back to Mitchell (1927) and Keynes (1936), it was the influential article of Neftci (1984) that placed the subject in a purely quantitative setting. Using finite Markov chain methods, Neftci (1984) found evidence of asymmetric behaviour in unemployment rate data for the US in the sense that increases in unemployment are steeper and shorter-lived than decreases. Further evidence on this type of asymmetry, named *steepness* by Sichel (1993), is given by DeLong and Summers (1986) on unemployment rates, Hamilton (1989), which presents evidence of asymmetries in US GNP growth rates and Sichel (1993), which introduces a new type of asymmetry, *deepness*, that defines the fact that business cycle troughs are lower than peaks are high. Neftci's methodology was applied to US GNP, investment and productivity by Falk (1986), rendering less convincing results on the asymmetry hypothesis.

This paper presents a simple and flexible method to model and test for cyclical asymmetry, based on the decomposition of a stochastic process into unobserved components. The potential asymmetry is modelled by allowing the frequency of the cyclical component of the series to shift depending upon past realizations of the observed process. The conditional Gaussianity of the model allows for the use of Kalman filtering in order to estimate the parameters, and a simple test for asymmetry can be constructed using the test statistic for testing the null hypothesis of equal cyclical frequency across regimes. The method is used in order to test for asymmetry in US data (unemployment rate, industrial production and GDP) and data corresponding to EU countries (industrial production). While US unemployment and industrial production present strong evidence of cyclical asymmetry, we cannot reject the null of symmetry for real GDP at any sensible significance level. The results for industrial production in the EU are mixed: significant departures from symmetry are found in France, Germany and UK, and to a lesser extent in Spain and Sweden.

The paper is organized as follows. Section two presents the trend plus asymmetric cycle model, section three comments on Kalman filtering and estimation of the parameters in the structural time series model with an asymmetric cycle. Section four tests for asymmetry on US and EU data. Section five concludes.

2 The trend plus asymmetric cycle model

The *trend plus asymmetric cycle* model arises as a straightforward generalization of the trend plus cycle model developed by Harvey (1989) by allowing the frequency of the cyclical component to vary depending on past realizations of the process itself. Assume that the process of interest (y_t) can be decomposed in an additive fashion into a trend component (μ_t), a cycle component (ψ_t) and an irregular component (ϵ_t), where the trend captures long-term movements of the series, the frequency of the cyclical component depends on past realizations of y_t and the irregular component, assumed to be white noise, comprises the rest of the movements of the series which are not captured by either the trend or the cyclical component.[1] That is,

$$y_t = \mu_t + \psi_t + \epsilon_t, \qquad \epsilon_t \sim \text{NID}(0, \sigma_\epsilon^2) \tag{1}$$

The trend component is specified in its most general form as follows,

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \upsilon_t, \qquad & \upsilon_t \sim \text{NID}(0, \sigma_\upsilon^2), \quad (2)\\ \beta_t &= \beta_{t-1} + \xi_t, \qquad & \xi_t \sim \text{NID}(0, \sigma_\xi^2), \quad (3) \end{aligned}$$

where υ_t and ξ_t are disturbances uncorrelated mutually and with the irregular component, ϵ_t. It can be easily noticed that such a specification of the trend nests as special cases the linear time trend model (if σ_ξ^2=0 and σ_υ^2=0), the random walk with drift (if σ_ξ^2=0 and σ_υ^2 >0) and the smooth trend model (if σ_ξ^2 >0 and σ_υ^2=0).
The asymmetric cyclical component ψ_t is modelled as a stochastic sine-cosine wave with a regime-dependent frequency,

$$\begin{aligned} \psi_t &= \rho \left\{ I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2 \right\} \psi_{t-1} + \\ &+ \rho \left\{ I(\{y_\tau\}_{\tau=1}^{t-1})\sin\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\sin\lambda_2 \right\} \psi_{t-1}^* + \omega_t, \quad (4)\\ \psi_t^* &= \rho \left\{ -I(\{y_\tau\}_{\tau=1}^{t-1})\sin\lambda_1 - [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\sin\lambda_2 \right\} \psi_{t-1} + \\ &+ \rho \left\{ I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2 \right\} \psi_{t-1}^* + \omega_t^*, \quad (5) \end{aligned}$$

where ψ_t^* appears by construction, $\rho \in [0, 1)$ is a damping factor, λ_1 and λ_2 are the frequencies of the cycle in the two possible regimes ($\lambda_1 \in [0, \pi]$, $\lambda_2 \in [0, \pi]$), ω_t and ω_t^* are iid normally distributed disturbances, mutually uncorrelated and with equal, fixed variance σ_ω^2, $I(\{y_\tau\}_{\tau=1}^{t-1})$ is an indicator function taking value one if a given function of the realized values, $f(\{y_\tau\}_{\tau=1}^{t-1})$ is positive and zero otherwise, that is,

$$I(\{y_\tau\}_{\tau=1}^{t-1}) = \begin{cases} 1 & \text{if } f(\{y_\tau\}_{\tau=1}^{t-1}) > 0, \\ 0 & \text{if } f(\{y_\tau\}_{\tau=1}^{t-1}) \leq 0. \end{cases} \tag{6}$$

Stating equations (4) and (5) in matrix form,

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \mathbf{A}(\{y_\tau\}_{\tau=1}^{t-1}) \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix}, \tag{7}$$

where the $\mathbf{A}(\{y_\tau\}_{\tau=1}^{t-1})$ is given by

$$\begin{bmatrix} I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2 & I(\{y_\tau\}_{\tau=1}^{t-1})\sin\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\sin\lambda_2 \\ -I(\{y_\tau\}_{\tau=1}^{t-1})\sin\lambda_1 - [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\sin\lambda_2 & I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2 \end{bmatrix}.$$

If $\lambda_1 = \lambda_2 = \lambda$, the model boils down to the symmetric trend plus cycle model developed by Harvey (1985, 1989), with the cycle component defined by

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix}.$$

[1] A seasonal component can be introduced in a straightforward fashion. In order to keep the model simple and due to the fact that the empirical applications will be performed on seasonally adjusted data, no seasonal component is included in the exposition of the model.

We may be interested in modelling cyclical asymmetries respective to whether our process is in an expansive or recessive phase (in terms of positive or negative growth of y_t). In this case, the f function to use in the indicator is $f(\{y_\tau\}_{\tau=1}^{t-1}) = y_{t-1} - y_{t-2}$, so that the indicator function takes value one if positive growth was observed in the last period, and zero otherwise . The same way, we could be interested in modelling accumulation effects in cyclical behaviour, specifying different cyclical frequency for levels of y_t exceeding some known level $\bar{y}$. In this case, a sensible function to use would be $f(\{y_\tau\}_{\tau=1}^{t-1}) = y_{t-1} - \bar{y}$. While the derivations of the estimators and testing procedures will be done for a generic $f(\cdot)$ function, in the empirical applications we will stick to certain types of asymmetries. It can be easily proved that the cyclical component is a threshold autoregressive-moving average - TARMA(2;2,1) - process [see, e.g. Tong (1983, 1990)], where the threshold variable is $f(\{y_\tau\}_{\tau=1}^{t-1})$. Notice from (7) that the cyclical component may be expressed as

$$\left[\mathbf{I} - \rho\mathbf{A}(\{y_\tau\}_{\tau=1}^{t-1})L\right] \begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix}, = \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix},$$

where L is the lag operator, such that $L^f x_t = x_{t-f}$. Just by computing the inverse of $\left[\mathbf{I} - \rho\mathbf{A}(\{y_\tau\}_{\tau=1}^{t-1})L\right]$, the resulting expression for ψ_t is

$$\psi_t = \frac{1 - \rho\left\{I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2\right\} L\omega_t}{1 - 2\rho\left\{I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2\right\} L + \rho^2 L^2} - \\ - \frac{\rho\left\{I(\{y_\tau\}_{\tau=1}^{t-1})\sin\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\sin\lambda_2\right\} L\omega_t^*}{1 - 2\rho\left\{I(\{y_\tau\}_{\tau=1}^{t-1})\cos\lambda_1 + [1 - I(\{y_\tau\}_{\tau=1}^{t-1})]\cos\lambda_2\right\} L + \rho^2 L^2},$$

which is a TARMA(2;2,1) model with $f(\{y_\tau\}_{\tau=1}^{t-1})$ as threshold variable, where both the first autoregressive and the moving average parameter depend on whether the value of $f(\{y_\tau\}_{\tau=1}^{t-1})$ is positive or negative. If $\rho \in [0, 1)$, each one of the regimes is stationary, which suffices for the ergodicity of the TARMA process. However, this condition could be relaxed depending on the properties of $f(\{y_\tau\}_{\tau=1}^{t-1})$ and the trend component. Notice that the roots of the autoregressive roots of each regime are constrained to be complex, leading to regime-specific pseudo-cyclical behaviour. In the symmetric case, the cycle is an ARMA(2,1) process where the autoregressive roots are constrained to be complex [see Harvey (1985)].

Instead of specifying two different unobserved dynamic processes for the series y_t, as in the case of the trend plus asymmetric cycle model, the cycle can be incorporated directly to the trend. The resulting (*asymmetric cyclical trend*) model has the following dynamic specification,

$$y_t = \mu_t + \epsilon_t \tag{8}$$
$$\mu_t = \mu_{t-1} + \psi_{t-1} + \beta_{t-1} + \upsilon_t, \tag{9}$$

with β_t and the cyclical component defined as in (3) and (7), respectively.

3 Kalman filtering and estimation

Both the trend plus asymmetric cycle and the asymmetric cyclical trend model are conditionally Gaussian, that is, given observations up to and including y_{t-1}, y_t is normally distributed for all $t > 0$. This allows us to use Kalman filtering in order to set algorithms for the maximum likelihood (ML) estimation of the unknown parameters of the model. The first step that needs to be taken is to formulate the model in state space form. The measurement and transition equations are given by

$$\begin{aligned} y_t &= z'\alpha_t + \epsilon_t, && (10)\\ \alpha_t &= \mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1})\alpha_{t-1} + \chi_t, && (11) \end{aligned}$$

where α_t, the state vector, is $(\mu_t\, \beta_t\, \psi_t\, \psi_t^*)'$, and $\chi_t = (\upsilon_t\, \xi_t\, \omega_t\, \omega_t^*)'$. For the case of the trend plus asymmetric cycle model, z=(1 0 1 0), and for the asymmetric cyclical trend z=(1 0 0 0). On the other hand, $\mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1})$ is given by

$$\mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1}) = \begin{pmatrix} \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \rho\mathbf{A}(\{y_\tau\}_{\tau=1}^{t-1}) \end{pmatrix} \qquad (12)$$

for the case of the trend plus asymmetric cycle model, and by the same matrix with a third one in the first row for the asymmetric cyclical trend case. The parameter vector of interest is $\phi = (\sigma_\epsilon^2\, \sigma_\upsilon^2\, \sigma_\xi^2\, \sigma_\omega^2\, \rho\, \lambda_1\, \lambda_2)'$, which can be estimated by maximizing the log-likelihood function,

$$\log L = -T/2(\log 2\pi) - 1/2\sum_{t=1}^{T}\log f_t - 1/2\sum_{t=1}^{T} v_t^2/f_t,$$

where T is the total number of available observations on y_t, v_t is the prediction error when using the estimated value of y_t, $\hat{y}_t$, and f_t is defined as

$$f_t = z'\mathbf{P}_{t|t-1}z + \sigma_\epsilon^2,$$

where $\mathbf{P}_{t|t-1}$ is the covariance matrix of the estimator of α_t given information up to period $t-1$, that is,

$$\mathbf{P}_{t|t-1} = \mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1})\mathbf{P}_{t-1}\mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1}) + \mathrm{diag}(\sigma_\upsilon^2\, \sigma_\xi^2\, \sigma_\omega^2\, \sigma_\omega^2),$$

which can be written as the following recursion

$$\mathbf{P}_{t|t-1} = \mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1})[\mathbf{P}_{t-1|t-2} - f_t^{-1}\mathbf{P}_{t-1|t-2}z'z\mathbf{P}_{t-1|t-2}]\mathbf{T}(\{y_\tau\}_{\tau=1}^{t-1}) + \mathrm{diag}(\sigma_\upsilon^2\, \sigma_\xi^2\, \sigma_\omega^2\, \sigma_\omega^2).$$

Reparametrizing the vector ϕ of unknown parameters in such a way that each element of it is expressed relative to the variance of the error process in the measurement equation, σ_ϵ^2, allows us to concentrate this parameter out of the log-likelihood and estimate the rest of the parameters in ϕ by maximizing the concentrated log-likelihood. Different algorithms can be used for the estimation of the unknown parameters [see Harvey (1989), pp. 126ff]. The asymptotic normality of the ML estimator of ϕ allows the construction of simple likelihood ratio, Wald or Lagrange multiplier tests for equality of the frequency parameters λ_1 and λ_2. Under some regularity conditions, which include that the true parameter vector is an interior point of the parameter space and that it is identifiable,[2] the asymptotic normality of the ML estimator of ϕ is assured, and therefore a simple test for equal frequency across cyclical regimes can be used.

4 Cyclical assymetries in US and EU data

This section presents some empirical applications of asymmetric cyclical models on economic data. For the US, three variables with quarterly periodicity are analyzed: industrial production, unemployment rate, and real GDP.[3] Sichel (1993) finds *steepness* and *deepness* asymmetry in unemployment, and only *deepness* asymmetry in industrial production, while no asymmetry is found in aggregate production. In order to account for different cyclical behaviour in the contractive and expansive economic regime, we set $f(\{y_\tau\}_{\tau=1}^{t-1}) = \Delta y_{t-1}$, that is, we define the two regimes according to whether positive or negative growth was observed in the last quarter. After trying different specifications based on the most general model given by (1)-(6), the most accurate fit was given by the model with a smooth trend ($\sigma_v^2 = 0$) plus a cycle for the three series of interest. The irregular component was found to be insignificant for unemployment and industrial production and was removed from the specification. None of the specifications reported presents significant deviations from normality and lack of autocorrelation (up to eight lags) in the residuals. The first column in Table 1 shows the estimated parameters of the cyclical component for (logged) US industrial production, together with the value of the Wald test for symmetry ($\lambda_1 = \lambda_2$) and its corresponding p-value. The null of symmetry is strongly rejected, and the frequency of the cyclical regime corresponding to expansions (λ_1) appears considerably lower than the frequency in recessions, giving evidence of higher steepness during recessions. The period corresponding to expansions in the cycle of industrial production is approximately 23.3 quarters, and in recessions it corresponds to approximately 9.6 quarters. These results contradict the evidence presented in Sichel (1993) concerning the difference in steepness across recessions and expansions. The second column in Table 1 shows the results of the asymmetric cyclical component of the (logged) unemployment rate. Notice that λ_1 is in this case the cyclical frequency corresponding to recessions (increases in unemployment). The results present strong evidence of asymmetric behaviour of unemployment rates in the US, which has an estimated period in the expansive regime of 26.2 quarters, and 13.7 quarters in the recessive regime. However, for the case of GDP, presented in the third column of Table 1, no evidence of

[2]Strict positiveness of ρ and lack of correlation between the cyclical disturbances suffice for identifiability of both the trend plus asymmetric cycle model and the asymmetric cyclical trend model.

[3]The source of the data is the International Financial Statistics Database, published by the IMF. Quarterly data was used, ranging from 1957:1 to 2001:2 for industrial production, and from 1965:1 to 1999:1 for unemployment rate and GDP.

cyclical asymmetry is found. The estimated cyclical frequency under symmetry is 0.23, corresponding to a period of approximately 7 years.
Table 2 presents the point estimates for the cyclical frequencies of industrial production in all EU countries except Luxembourg,[4] together with the Wald test statistic for symmetry and its p-value and the constraints that were used in the specification of the structural time series model for each series. The industrial production series in France, Germany and UK present strong evidence of asymmetry, with steeper recessive cycles and flatter cyclical behaviour in expansions. Weaker evidence exists for Spain and Sweden, where the Swedish case presents slightly steeper cycles in the expansion phase compared to the recessive regime. Industrial production in the rest of the countries does not present any significant deviation from cyclical symmetry. The ratio of the estimated recession frequency to the expansion frequency for those countries which present at least weak evidence of asymmetry varies between around 0.1 for France and 1.1 for Sweden.

5 Conclusions and paths of further research

A simple and flexible parametric framework for modelling and testing for cyclical asymmetries has been introduced by defining a structural time series model where the frequency of the cyclical component is made dependent on past observations of the modelled stochastic process. The simple setting allows for testing for asymmetric cyclical behaviour with respect to different reference functions which "trigger" the asymmetry ($f(\{y_\tau\}_{\tau=1}^{t-1})$). The method has been applied to US data on unemployment, industrial production and GDP and to data on industrial production for all EU countries with the exception of Luxembourg. Strong evidence of cyclical asymmetry has been found in US unemployment, US industrial production and industrial production in Germany, France and UK. Weaker evidence of asymmetry has been found in the data for industrial production in Spain and Sweden.
Given the fact that the reduced form of the model is a TARIMA(2,2,4) with $f(\{y_\tau\}_{\tau=1}^{t-1})$ as a threshold variable, a further step towards the generalization of models with asymmetric cycles could be taken by applying the theory developed for optimal testing in models where a nuisance parameter is only defined under the alternative hypothesis [see, e.g, Andrews and Ploberger (1994), and Hansen, (1996)] in order to test for asymmetry in an unobserved components setting without specifying $f(\{y_\tau\}_{\tau=1}^{t-1})$ completely, but letting it be determined endogenously from the data.

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Table 1: Asymmetric cycles in US data

	IP		**Unemployment**		**GDP**	
Coefficient	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
ρ	0.91	0.02	0.95	0.02	0.95	0.05
λ_1	0.27	0.05	0.46	0.03	0.23	0.05
λ_2	0.66	0.07	0.24	0.03	0.21	0.09
σ^2_ω	9.05×10^{-5}	1.35×10^{-5}	8×10^{-4}	1.22×10^{-4}	0.0001	5.43×10^{-5}
Test $\boldsymbol{\lambda_1 = \lambda_2}$	**33.14 (0.00)**		**180.36 (0.00)**		**0.02 (0.88)**	

The estimation of the trend plus asymmetric cycle for industrial production and unemployment was carried out under the restrictions $\sigma^2_\epsilon = \sigma^2_v = 0$. For GDP, the model included only the restriction $\sigma^2_v = 0$.

Table 2: Asymmetric cycles in EU industrial production data

	λ_1	λ_2	**Asymmetry test (*p*-value)**	**Specification**
Austria	0.19	0.17	0.20 (0.66)	$\sigma^2_v = 0$
Belgium	0.01	0.36	1.47 (0.23)	$\sigma^2_\epsilon = \sigma^2_v = 0$
Denmark	0.09	0.08	2.40 (0.12)	$\sigma^2_\xi = 0$
Finland	0.27	0.32	0.55 (0.46)	$\sigma^2_\xi = 0$
France	0.05	0.51	**43.99 (0.00)**	$\sigma^2_v = 0$
Germany	0.08	0.32	**12.80 (0.00)**	$\sigma^2_\epsilon = \sigma^2_v = 0$
Greece	0.37	0.41	0.21 (0.65)	$\sigma^2_v = 0$
Ireland	0.43	0.42	0.003 (0.96)	$\sigma^2_v = 0$
Italy	0.18	0.18	0.003 (0.95)	No constraints
Netherlands	0.28	0.28	0.003 (0.96)	No constraints
Portugal	0.40	0.40	0.000 (0.99)	$\sigma^2_\epsilon = 0$
Spain	0.37	0.45	2.77 (0.09)	$\sigma^2_v = 0$
Sweden	0.26	0.24	3.98 (0.05)	$\sigma^2_v = 0$
United Kingdom	0.09	0.44	**26.69 (0.00)**	$\sigma^2_\epsilon = \sigma^2_v = 0$