E C O N O M I C S B U L L E T I N

Endogenous growth, transitional dynamics and the welfare costs of inflation

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Abstract

This paper quantifies the welfare costs of inflation in an endogenous growth setup when transitional dynamics are taken into account. We report much smaller costs than when these dynamics are omitted.

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1 Introduction

Recent studies on money and endogenous growth find contrasted welfare costs of inflation (WCOI hereafter). Gomme (1993) reports surprisingly low costs (0.03%) for an annual monetary growth of 10%. Reproducing the same exercise in a large set of endogenous growth models, Wu and Zhang (1998) find costs always higher than 1.7%. They emphasize an important "growth rate effect" in their models which is negligible in Gomme's setup. In their framework, inflation is negatively related to growth in the long-run. Hence, an increase in the inflation rate relative to its optimal level gives rise to a lower growth rate and consequently to large WCOI in the long-run.

But this argument neglects the transitional effects of inflation on growth and on welfare. The measures of the WCOI of the above mentioned papers are consequently incomplete. Our objective in this note is to provide a complete measure (including transient and long-run movements) of the WCOI in an endogenous growth model. We choose a simple Lucas-type model and show that taking transitional dynamics into account considerably reduces the growth rate effects and turns large long-run costs into small total costs (0.1%).

2 The Model

Our economy is inhabited by a large number of identical, infinitely lived households. We introduce money by requiring that transactions in the economy be financed with previously accumulated cash balances. More precisely, at date t, the representative household finances its consumption c_t and a part ε of its investment $x_{k,t}$ through beginning-of-period cash balances, inherited from the previous period m_{t-1} and a lump-sum monetary transfer τ_t issued by the monetary authority

$$c_t + \varepsilon x_{k,t} \leq \frac{m_{t-1} + \tau_t}{p_t} \tag{1}$$

p_t is the price level. $0 < \varepsilon < 1$.

After the good market closes, the representative household receives labor and capital incomes, and rearranges its portfolio. He faces the following budget constraint

$$c_t + x_{k,t} + \frac{m_t}{p_t} = w_t n_{k,t} h_{t-1} + r_t k_{t-1} + \frac{m_{t-1} + \tau_t}{p_t} \tag{2}$$

w_t is the real wage rate. r_t is the real rental price of capital during the period. $n_{k,t}$ is the fraction of time devoted to the production sector. k_{t-1} and h_{t-1} are the levels of physical and human capital available at period t respectively.

The law of motion for physical capital is

$$k_t = x_{k,t} + (1 - \delta_k)\, k_{t-1} \tag{3}$$

where $0 < \delta_k < 1$ is the depreciation rate of physical capital.

The law of motion for human capital is given by

$$h_t = (1 - \delta_h)\, h_{t-1} + B n_{h,t} h_{t-1} \tag{4}$$

where $0 < \delta_h < 1$ is the depreciation rate of human capital. $n_{h,t}$ is the fraction of time devoted to the education sector. $B > 0$ is a scale parameter.

The representative household seeks to maximize the value of discounted streams of utility

$$\sum_{t=0}^{+\infty} \beta^t u\left(c_t, 1 - n_{k,t} - n_{h,t}\right) \tag{5}$$

subject to equations (1), (2), (3) and (4). The household time endowment is normalized to one. Then $1 - n_{k,t} - n_{h,t}$ is the fraction of time devoted to leisure at date t. $0 < \beta < 1$ is the subjective discount rate.

On the production side, each firm has access to a constant returns-to-scale technology which delivers output y_t according to

$$y_t = f\left(k_{t-1}, n_{k,t} h_{t-1}\right) = A k_{t-1}^{\alpha} \left(n_{k,t} h_{t-1}\right)^{1-\alpha} \tag{6}$$

$A > 0$ is a scale parameter and $0 < \alpha < 1$. The problem of a typical firm is to maximize profits π_t

$$\pi_t = y_t - w_t\left(n_{k,t} h_{t-1}\right) + r_t k_{t-1} \tag{7}$$

through its choice of k_{t-1} and $n_{k,t} h_{t-1}$ subject to equation (6). r_t and w_t are taken as given.

Finally, the monetary authority finances its lump-sum monetary injections through the creation of money. It faces the following intra-period budget constraint

$$\tau_t = \mu m_{t-1} \tag{8}$$

where μ is the exogenously given constant growth rate of money.

3 Competitive Equilibrium

3.1 first order conditions

We derive the first order conditions for the non-optimal case:

$$(1-\varepsilon)\frac{u_{2,t}}{w_t h_{t-1}} + \varepsilon u_{1,t} = \beta\left\{\frac{u_{2,t+1}}{w_{t+1}h_t}\left[r_{t+1} + (1-\varepsilon)(1-\delta_k)\right] + u_{1,t+1}\varepsilon(1-\delta_k)\right\} \tag{9}$$

$$\frac{u_{2,t}}{p_t w_t h_{t-1}} = \beta\left\{\frac{u_{1,t+1}}{p_{t+1}}\right\} \tag{10}$$

$$\frac{h_t}{h_{t-1}} = \beta\left\{\frac{u_{2,t+1}}{u_{2,t}}\left[Bn_{k,t+1} + (Bn_{h,t+1} + 1 - \delta_h)\right]\right\} = 0 \tag{11}$$

$$c_t + k_t = y_t + (1-\delta_k)k_{t-1} \tag{12}$$

$$m_t = (1+\mu)m_{t-1} \tag{13}$$

$$r_t = A\alpha k_{t-1}^{\alpha-1}(n_{k,t}h_{t-1})^{1-\alpha} \tag{14}$$

$$w_t = A(1-\alpha)k_{t-1}^{\alpha}(n_{k,t}h_{t-1})^{-\alpha} \tag{15}$$

where $u_{1,t} = \partial u(c_t, 1-n_{k,t}-n_{h,t})/\partial c_t$ and $u_{2,t} = \partial u(c_t, 1-n_{k,t}-n_{h,t})/\partial(1-n_{k,t}-n_{h,t})$. For the dynamical system to be complete, we must add equations (4), and (1) where this last equation always binds provided that the money growth rate is high enough. As explained by Gomme (1993), the intertemporal equation (10) is an illustration of the decisions distortion implied by the cash-in-advance constraint. The Friedman rule, which guarantees optimality, is obtained when

$$\beta\left\{\frac{u_{1,t+1}}{p_{t+1}}\right\} = \frac{u_{1,t}}{p_t} \tag{16}$$

Notice that, in this case, the cash-in-advance constraint no longer binds. Replacing (16) with (10), we get the Pareto-optimal conditions.

3.2 model calibration

We give the instantaneous utility function the following specific form

$$u(c, 1 - n_k - n_h) = \frac{\left[c^{\omega}(1 - n_k - n_h)^{1-\omega}\right]^{1-\sigma}}{1 - \sigma} \quad (17)$$

where $0 < \omega < 1$ and $\sigma > 0$. $1/[1 - \omega(1 - \sigma)]$ is the intertemporal elasticity of substitution in consumption.

In our calibration exercise, we closely follow Gomme (1993) and Wu and Zhang (1998) in order to compare our results with theirs. The time period is assumed to be a quarter. The capital share parameter is set equal to 0.36. We impose $\delta_k = \delta_h = 0.025$. β is set to 0.99. Following Gomme (1993), we set $\omega = 0.2281$ and $\sigma = 3.1922$. The balanced growth rate is set to its empirical counterpart over the period 1954:1-1989:4, $g = 0.3542\%$. We impose $\mu = 0.014$, the average quarterly growth rate of U.S. M1 over the period 1959:2-1989:4. All other parameters and steady-state values are contingent on the fraction ε of investment subject to the cash-in-advance constraint.

4 Results

We now turn to the assessment of the welfare cost implied by different money growth rules relative to the optimal one. Trajectories and welfare costs are computed using the orthogonal collocation method based on Chebyshev polynomials. We compare the implications of three exercises:

(i) The economy starts on the steady-state of the optimal regime. This regime is obtained when $(1 + \mu^{opt}) = \beta (g^{opt})^{\omega(1-\sigma)}$ where g^{opt} is the optimal long-run growth rate. This equality comes directly from (16), evaluated at the deterministic steady-state. It is then a straightforward computational task to derive the total amount of welfare V^{opt}. Let c^{opt} refer to the permanent consumption flow associated with[1] V^{opt}.

(ii) The economy starts on the steady-state of the optimal regime, but the monetary authority

[1] We measure these permanent consumption flows while considering $g, n_k, n_h, \widetilde{y} (= y/h)$ at their suboptimal steady-state values.

changes its monetary policy in $t = 0$ and chooses a new constant growth rate of money[2] $\mu^* > \mu^{opt}$. Then the economy experiences a transition before asymptotically reaching a new suboptimal steady-state. The total amount of welfare is then V^* and its associated permanent consumption flow is c^*.

(iii) The economy is directly on its suboptimal steady-state implied by the monetary policy μ^*. We deduce the total amount of welfare V as well as its associated permanent consumption flow, c. In this case, it is quite easy to determine V

$$V = \frac{[c/h]^{\omega(1-\sigma)} [1 - n_k - n_h]^{(1-\omega)(1-\sigma)}}{(1-\sigma)\left[1 - \beta (1+g)^{\omega(1-\sigma)}\right]} \tag{18}$$

V depends positively on c, leisure $(1 - n_k - n_h)$ and the growth rate of human capital g.

Comparing cases (i) and (ii) gives the correct evaluation of the WCOI, measured by[3] $\lambda = 100 \times (c^{opt} - c^*)/\widetilde{y}$. λ is the welfare loss of switching from the optimal economy to a new policy μ^* at $t = 0$. In this case, transient movements between optimal and suboptimal steady-states are correctly taken into account.

Comparing cases (i) and (iii) gives the (incomplete) evaluation of the WCOI, considered by Gomme (1993) and Wu and Zhang (1998). This cost is measured by $\lambda_{ss} = 100 \times (c^{opt} - c)/\widetilde{y}$. λ_{ss} is the welfare loss of switching from the optimal steady-state (implied by μ^{opt}) to the suboptimal one (implied by μ^*), without undergoing any transient movements, which consequently are omitted from this calculation.

The residual amount of welfare costs due to transitional effects is deduced from the identity $\lambda_{trans} = \lambda - \lambda_{ss}$.

The whole set of results is reported in Table (1). With $\varepsilon = 0$, we consider three (suboptimal) monetary policies $\mu^* = 0\%, 10\%$ or 100%. Let us first consider the values of the long-run WCOI (λ_{ss}). Ignoring transitional effects, long-run WCOI are substantially higher than those of Gomme (1993) and quite close to those obtained by Wu and Zhang (1998)[4]. If we turn now to the complete evaluation of the WCOI (i.e. λ), it appears that λ is much smaller than λ_{ss}. Hence, transitional

[2] The case $\mu^* < \mu^{opt}$ is not considered. It gives rise to negative nominal interest rates.

[3] Our measures of the WCOI are expressed in terms of *income* ($\widetilde{y}$) as in Gomme (1993) and Wu and Zhang (1998).

[4] This result was to be expected since our model is very similar to that retained by Wu and Zhang (1998)

Table 1: long-run, transient and total WCOI

case	λ_{ss} (%)	λ_{trans} (%)	λ (%)
$\mu^* = 0.00\%$, $\varepsilon = 0.0$	0.1985	−0.1884	0.0101
$\mu^* = 10.0\%$, $\varepsilon = 0.0$	0.6814	−0.5915	0.0899
$\mu^* = 100\%$, $\varepsilon = 0.0$	5.7471	−3.2225	2.5246
$\mu^* = 10.0\%$, $\varepsilon = 0.2$	−0.2093	0.3117	0.1024
$\mu^* = 10.0\%$, $\varepsilon = 0.4$	−1.0961	1.2140	0.1179

movements substantially diminish the costs of inflation. For the 0% and 10% experiments, λ is still higher than Gomme found, but quite negligible.

These results can be interpreted as follows. Our model is able to produce the negative long-run relationship between inflation and growth (i.e. the above mentioned growth effect). As explained by Wu and Zhang (1998), an increase in money growth rate increases the opportunity cost of holding money rather than physical capital. The stock of capital increases, which reduces the marginal product of capital. Due to factor complementarity, households decrease their work effort and their human capital stock. This in turn lowers n_k and n_h in the long-run. An increase in the money growth rate then has two opposite effects[5], summarized in equation (18): a negative effect on the growth rate of human capital (the higher μ, the lower g) and a positive effect on leisure (the higher μ, the higher $1 - n_k - n_h$). Consequently, the growth rate is higher in the optimal context rather than in the suboptimal one (*growth effect*). This growth effect explains the high value of λ_{ss} (in this case the leisure effect is flattened out). Nevertheless, such an argument is only valid in the long-run. Indeed, the growth rate experiences high values for the first years of the transition after a policy change (*transitional effect*). This lowers the costs of leaving the optimal context, and in turn explains the low value of λ relative to λ_{ss}.

With $\varepsilon = 0.2$ or 0.4, we only consider the $\mu^* = 10\%$ experiment, as in Wu and Zhang (1998). In this case, we find *negative* values for λ_{ss}. This would mean that households would rather immediately live in the suboptimal steady-state than in the optimal one. This obviously makes no sense and shows that ignoring the transition induces heavily distorted conclusions.

[5] The effect on c/h in equation (18) is negligible.

Notice that even though the leisure effect grows with ε, the growth effect is still present. However, for $\varepsilon > 0$, the former dwarves the latter. This case illustrates clearly the inconsistency of measuring only the long-run WCOI (λ_{ss}). This spurious result disappears when transitional dynamics are correctly taken into account: λ is positive. Moreover, λ is almost insensitive to ε for the range of values considered here.

5 Conclusion

This paper has examined the welfare costs of inflation in a simple endogenous growth model. We have shown that transitional dynamics play a crucial role in the assessment of these costs. We report costs of approximately 0.1% for a 10% money growth, much smaller than in the literature (even without endogenous growth, e.g. Cooley and Hansen, 1989), Gomme (1993) excepted. Further research should extend this procedure to a larger set of monetary models designed to reproduce some potentially important facts in terms of welfare, such as the well-known liquidity and output effects.

References

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