E C O N O M I C S   B U L L E T I N

Output persistence in human capital−based growth models

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Abstract

This note explores the persistence properties of a class of models proposed by Jones, Manuelli and Siu (2000) where growth stems from purposeful human capital accumulation. In doing so, we adopt Cogley and Nason's (1995) definition of output persistence. The propagation mechanism exhibited by this class of models appears unable to solve the output persistence puzzle.

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1 Introduction

In their famous contribution Cogley and Nason (1995) (CN hereafter) brought to light the inability of standard Real Business Cycle (RBC) models to reproduce some crucial dynamic properties of output. Using several econometric methods CN have defined a set of properties that characterize the persistence of output in postwar US data. Confronted with this set of properties, RBC models were shown to inherently lack propagation properties. These puzzling output dynamics have fostered a growing literature devoted to augmenting standard RBC models with powerful persistence mechanisms capable of overcoming CN's critique. Among others, Jones, Manuelli and Siu (2000) (JMS hereafter) have put forward human-capital-based endogenous growth as a way of generating strong persistence of output. The aim of this note is to clarify the output persistence properties associated with this mechanism[1] within the empirical setup of CN. In doing so, we emphasize the importance of preserving the whole of CN's empirical setup.

The gap between JMS's results and CN's analysis lies in two points. First, both the theoretical and empirical analyses considered by JMS are conducted at annual frequency, in contrast with the standard practice in the RBC literature -and in CN's analysis- of considering quarterly data. Second, JMS consider only the first order autocorrelation of output growth as their measure of persistence. Though crucial, this statistic does not encompass the whole set of properties highlighted by CN.

Thus, to clarify the persistence properties of the human capital accumulation channel, we first propose to study JMS's model both at annual and at quarterly frequency. Second, following CN[2], we study the whole autocorrelation function (ACF hereafter) as well as the spectrum of output growth.

We obtain the following results. At annual and quarterly frequencies, the model succeeds in generating the first positive value of the ACF, but fails to reproduce the consecutive negative values, and, consequently, is unable to account for the well-known peak of the spectrum at the business cycle frequencies. These results are shown to be robust to alternative values of the intertemporal elasticity of substitution. Hence, despite their strong propagation mechanism, human-capital-based growth models do not solve the output persistence puzzle as defined by CN.

The remainder is as follows. The model is described in section 2. Results are presented in section 3. Section 4 briefly concludes.

[1] We do not discuss the other advantages of this mechanism. These, such as the amplification of the volatility of total hours worked, are thoroughly described by JMS.

[2] We exclude from our analysis their results drawn from structural vector autoregressions. Indeed, since the model fails on the overall behavior of output growth, it would be useless to adress the issue of the sources of fluctuations.

2 The Model

JMS propose a class of endogenous growth models featuring investment in both human and physical capital. The model considered in this paper is their benchmark. The social planner's program writes

$$\max \ \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right) \right\} \tag{1}$$

subject to

$$c_t + x_{h,t} + x_{k,t} = y_t \tag{2}$$

$$k_{t+1} = \left(1 - \delta\right) k_t + x_{k,t} \tag{3}$$

$$h_{t+1} = \left(1 - \delta\right) h_t + x_{h,t} \tag{4}$$

$$y_t = s_t A k_t^{\alpha} \left(n_t h_t\right)^{1-\alpha} \tag{5}$$

where c_t is consumption, n_t is labor supply, $x_{k,t}$ is investment in physical capital, $x_{h,t}$ is investment in human capital, y_t is output, k_t is the stock of physical capital, h_t is the stock of human capital, and s_t is the technological shock. We impose an equal depreciation rate δ for both human and physical capital stocks. This assumption corresponds to the benchmark model studied by JMS. A is a scale parameter, α is the elasticity of output with respect to physical capital, and β is the subjective discount factor. The shock evolves according to

$$\log\left(s_{t+1}\right) = \rho \log\left(s_t\right) + \varepsilon_{t+1}, \ \ \varepsilon_t \sim iid\left(0, \sigma_{\varepsilon}\right), \tag{6}$$

where ρ is the persistence coefficient. Finally, the instantaneous utility function $u\left(c_t, n_t\right)$ is specified as follows

$$u\left(c_t, n_t\right) = \begin{cases} \dfrac{1}{1-\sigma}\left[c_t\left(1-n_t\right)^{\psi}\right]^{1-\sigma} & \text{with } \sigma > 0 \text{ and } \sigma \neq 1 \\ \log\left(c_t\right) + \psi \log\left(1 - n_t\right) & \text{with } \sigma = 1 \end{cases} \tag{7}$$

Once the first order conditions are derived, the model is loglinearized in the neighborhood of its deterministic steady-state. To simulate the model, we consider two alternative calibrations which differ according to the sampling frequency (annual or quarterly). The calibration constraints are taken from JMS, except for the steady growth rate which is taken from our data sample (described below). We impose $g = 1.0210^{1/T}$, $\beta = 0.950^{1/T}$, $\delta = 1 - \left(1 - 0.075\right)^{1/T}$, $\rho = 0.950^{1/T}$, $\alpha = 0.360$, $n = 0.170$, and $\sigma = 1$, with $T = 1$ for the annual calibration and $T = 4$ for the quarterly calibration. The implied values for ψ are 8.707 for $T = 1$ and 9.028 for $T = 4$.

3 Results

Our data set covers the period 1965-2001 for the U.S. Economy. Following CN, we estimate a bivariate VAR of output growth and the consumption-output ratio. The VAR is estimated twice, depending on the selected sampling frequency. The optimal lag is chosen by minimization of the AIC criterion and is equal to 1 for $T = 1$ and 4 for $T = 4$. In both cases, the associated ACF and spectrum of output growth are computed from the estimated VAR coefficients. Finally, the variance of the theoretical shock σ_ε^2 is calibrated so as to reproduce the variance of output growth implied by the VAR.

We begin our results description with the annual periodicity (which is the one considered by JMS). Figure 1 reports the theoretical and empirical ACFs of output growth. To begin with, output growth is positively autocorrelated at first order in the data. This property is also present in the theoretical model, and constitutes the definition of output persistence retained by JMS. Thus, the model succeeds in reproducing the positive first point of the ACF of output growth. However, when we look at higher orders, the theoretical ACF remains positive while its empirical counterpart becomes negative. This result comes from the endogenous growth mechanism which induces permanent effects of transitory shocks. Following a positive productivity shock, output monotonically rises until reaching its new long-run level. Hence, the deviation of output growth with respect to its average level always remains positive after the shock, thus explaining the ACF pattern. On the contrary, as documented notably by CN, a hump-shaped response of output would have generated an ACF similar to that found in the data[3]. We now propose to investigate the implication of the discrepancy on ACFs in the frequency domain.

Figure 2 reports the theoretical and empirical spectra of output growth. The vertical lines insulate the business cycle frequency band (i.e. movements whose period of reproduction lies between 2 and 8 years). The empirical spectrum exhibits a peak near business cycle frequencies. This pattern has been put forward by CN (with quarterly data) as an element of the definition of output persistence. This pattern is completely absent in the model. Indeed, the theoretical spectrum is concentrated at very low frequencies, and underestimates the volatility of output growth within the business cycle frequency band.

JMS show that the degree of intertemporal substitution of consumption is a major determinant of the second moment properties implied by their model. In particular, the parameter σ deeply influences the first point of the output growth ACF. Hence, as the authors, we perform a sensitivity analysis by taking two alternative values for $\sigma = \{0.9, 3.0\}$. For each calibration, the variance of the shock is adjusted to reproduce output growth volatility. Figures 3 and 4 depict the associated ACFs

[3]Notice that this property does not depend on the presence of permanent effects of shocks.

and spectra of output growth. As JMS, we conclude that the higher σ, the lower the first points of the ACF of output growth. However, the global pattern remains the same: the ACF is always strictly positive and, consequently, the spectrum does not exhibit the required peak at business cycle frequencies.

Finally, we propose to redo the previous analysis at quarterly frequency. As one might argue, following CN, it seems more convenient to take a quarterly data set to study the question of output growth persistence. Our results for $T = 4$, are reported in figures 5 and 6. The empirical pattern of output growth dynamics is consistent with CN estimates. The ACF of output growth is strictly positive for the first four points and negative beyond. The spectrum of output growth exhibits the well-known peak at business cycle frequencies. On the theoretical side, the predictions of the quarterly version of the model are very close to those of the annual version. The model is unable to reproduce the negative values of the ACF for high orders. Hence, it delivers a counterfactual spectrum pattern where the main part of output growth volatility is concentrated at very low frequencies.

4 Conclusion

JMS showed that human-capital-based endogenous growth models outperform standard RBC models on several dimensions, especially when it comes to their ability to reproduce the autocorrelation of output growth. Our objective was to gauge the propagation properties of their models, when considered from the viewpoint of CN's analysis.

We obtained disappointing conclusions: despite its strong propagation mechanism, JMS's model does not satisfactorily reproduce the spectrum and the ACF of output growth. This fact should not hide that JMS's benchmark mechanism constitutes a strong improvement over standard exogenous growth models. However, it might prove useful to devote further research into the study of alternative specifications of growth within this class of models. At this prospect, the aim of this note was to emphasize the importance of preserving the whole CN's empirical setup.

References

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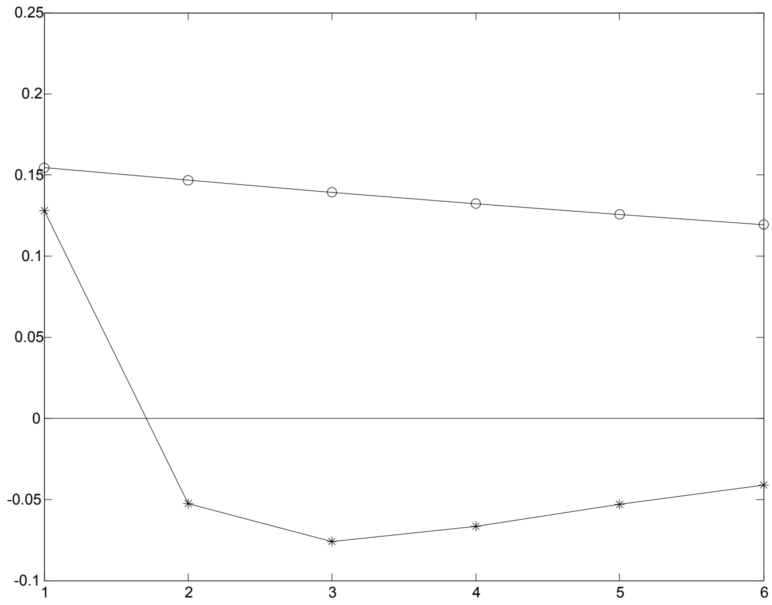

Figure 1: ACF of annual output growth in the model (line with circle) and in the data (line with star).

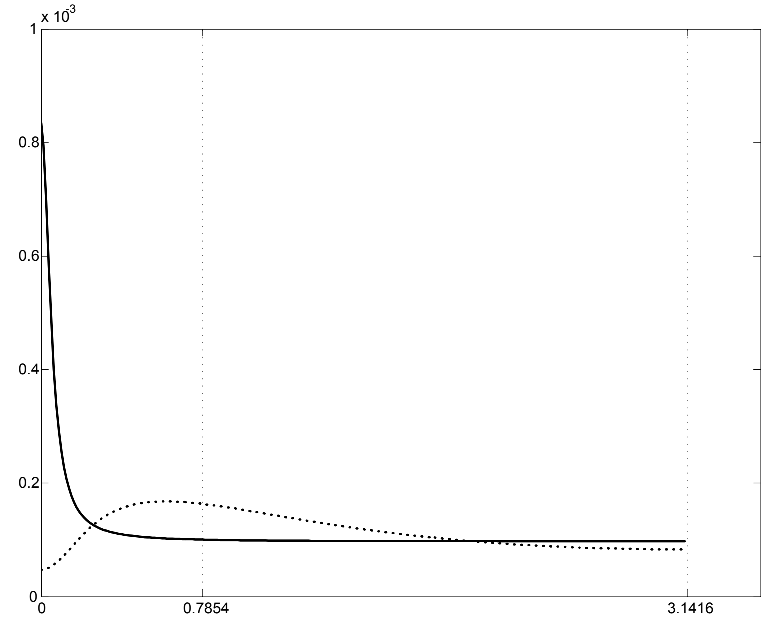

Figure 2: Spectrum of annual output growth in the model (solid line) and in the data (dotted line).

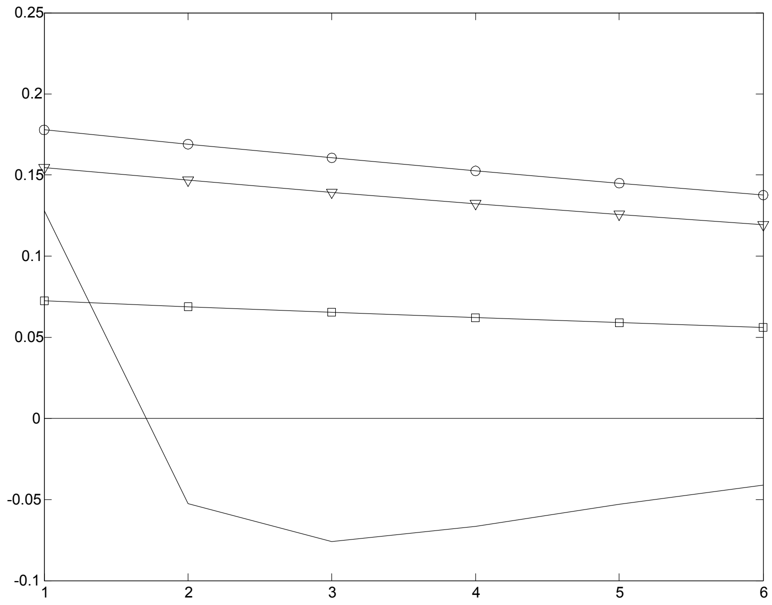

Figure 3: ACF of annual output growth in the model with $\sigma = 0.9$ (line with circle), with $\sigma = 1$ (line with triangle), with $\sigma = 3$ (line with square) and in the data (solid line).

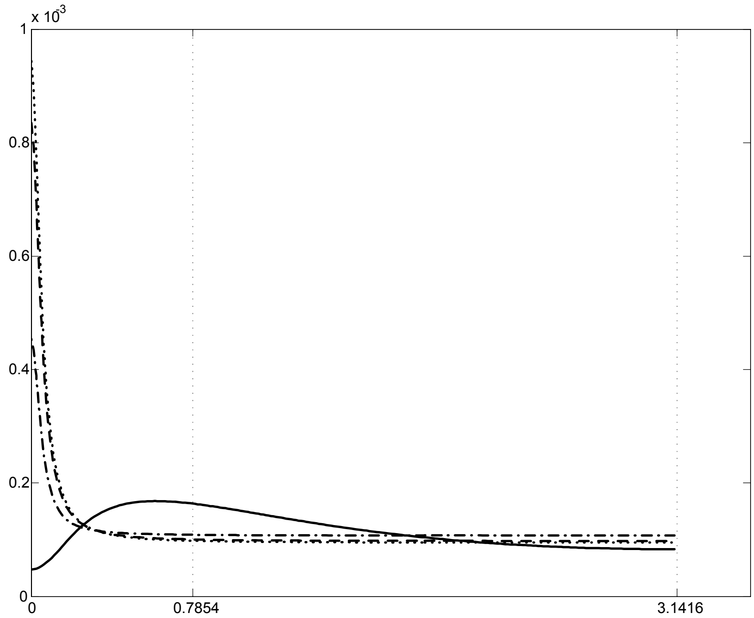

Figure 4: Spectrum of annual output growth in the model with $\sigma = 0.9$ (dotted line), with $\sigma = 1$ (dashed line), with $\sigma = 3$ (dash-dotted line) and in the data (solid line).

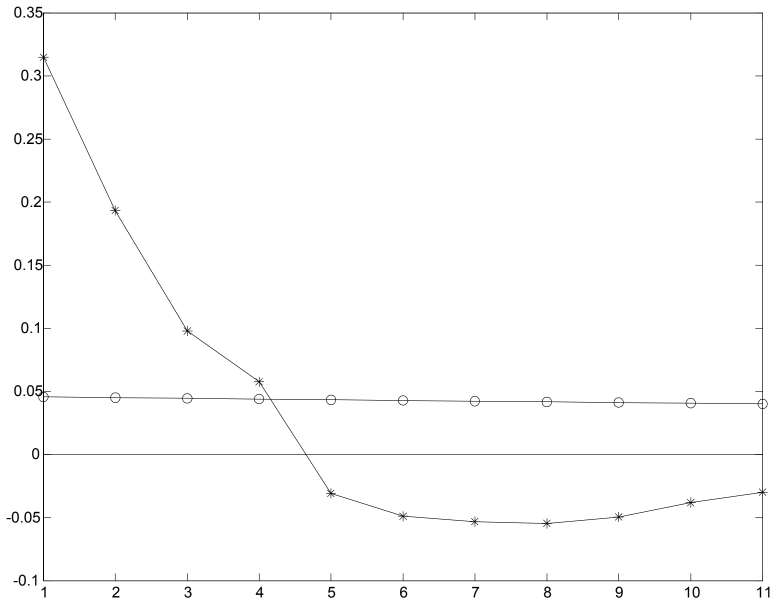

Figure 5: AFC of quarterly output growth in the model (line with circle) and in the data (line with star).

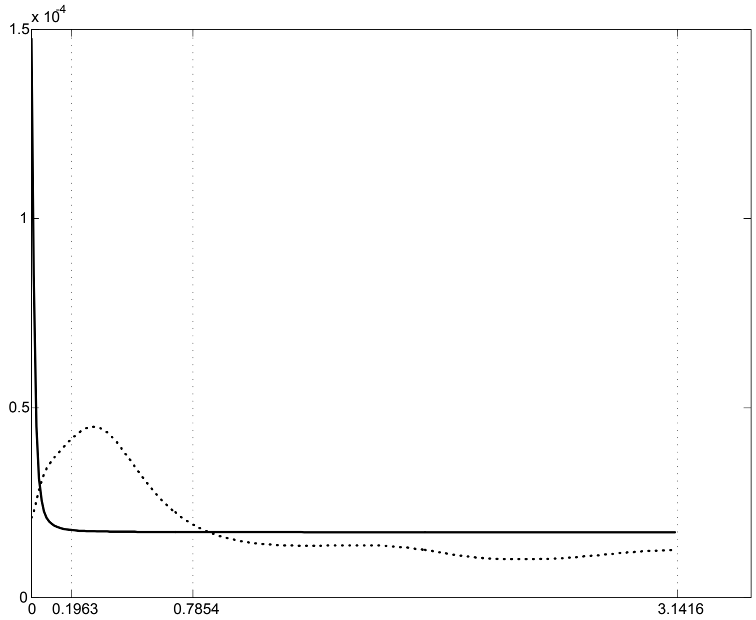

Figure 6: Spectrum of quarterly output growth in the model (solid line) and in the data (dotted line).