

Threshold stationary real exchange rates: a nonlinear, multivariate approach

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Abstract

Purchasing power parity implies that real exchange rates are stationary. However, the finding of nonstationary real exchange rates has been difficult to dismiss. Using a nonlinear, three-regime structural bivariate threshold model, this paper finds evidence of threshold stationary real exchange rates which is consistent with purchasing power parity adjusted for market frictions such as transaction costs.

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1. Introduction

Tests for stationary versus nonstationary real exchange rates are equivalent to tests of purchasing power parity, a fundamental arbitrage condition in international finance which maintains that real exchange rates are stationary. Although few, if any, economists believe that real exchange rates are nonstationary, the finding of nonstationary real exchange rates has been difficult to dismiss. For example, see Adler and Lehmann (1983), Cumby and Obstfeld (1984), Edison (1987), Frenkel (1981), Krugman (1978) and Roll (1979) who apply regression (based) tests, Abuaf and Jorion (1990) and Meese and Rogoff (1988) who apply the Dickey - Fuller unit-root test, and Corbae and Ouliaris (1988), Mark (1990), Patel (1990) and Taylor (1988) who apply cointegration tests.

As econometric tests have become more powerful and data spanning longer time periods are used, some recent studies have found evidence that real exchange rates are consistent with a very long-run stationary process. For example, see Abuaf and Jorion (1990) and Lothian and Taylor (1996) who apply the Dickey - Fuller unit-root test, Cheung and Lai (1993a) and Steigerwald (1996) who apply cointegration tests, Cheung and Lai (1993b) who apply fractional integration techniques, and Glen (1992) and Grilli and Kaminsky (1991) who apply the variance ratio test.

Econometric tests of stationary versus nonstationary real exchange rates are generally univariate tests of temporary versus permanent innovations. It is possible to determine the size of independent temporary and permanent real-exchange-rate innovations using multivariate, structural models which exploit the joint behavior of real exchange rates and additional, related time series. For example, Clarida and Gali (1994) and Evans and Lothian (1993) estimate trivariate structural models which find important short-run temporary real-exchange-rate innovations, but that permanent innovations dominate real exchange rates. Lastrapes (1992) estimates a bivariate structural model and finds that permanent innovations dominate real exchange rates across all horizons.

There is a growing consensus that real exchange rates follow a nonlinear process. Theoretically, a nonlinear process for real exchange rates is supported by models that include

market frictions such as the model in Dumas (1992) which includes transaction costs. Empirically, a nonlinear process for real exchange rates is supported by recent studies such as Bleaney and Mizen (1996) who test for threshold nonlinearity, Coakley and Fuertes (2001) who test for nonparametric cointegration, Serletis and Gogas (2000) who test for nonlinear chaotic dynamics, and Chen and Wu (2000), Michael, Nobay and Peel (1997), Sarantis (1999), and Sarno (2000) who test for nonlinearity using various forms of smooth transition autoregressive models.

This paper takes the nonlinearity and (near) nonstationarity of real exchange rates as its starting point. It estimates a nonlinear, structural bivariate threshold model in order to determine the size of independent temporary and permanent real-exchange-rate innovations in three real-exchange-rate regimes. The idea is that there is a middle regime where real exchange rates are nonstationary due to market frictions such as transaction costs, and two outer regimes where real exchange rates are stationary. This nonlinear, threshold stationary process for real exchange rates is consistent with purchasing power parity adjusted for market frictions such as transaction costs.

The model exploits the joint behavior of real exchange rates and real interest-rate differentials, which are related through uncovered interest parity adjusted for differential inflation rates. The model identifies independent temporary and permanent real-exchange-rate innovations in each regime using covariance restrictions and the Blanchard and Quah (1989) restriction on the long-run multiplier of one structural innovation. This nonlinear, three-regime structural bivariate threshold model finds that permanent innovations dominate real exchange rates in the middle regime, but that temporary innovations have a large impact on real exchange rates in the two outer regimes. This evidence of threshold stationary real exchange rates is consistent with purchasing power parity adjusted for market frictions such as transaction costs.

2. The Model

Let s_t be the natural logarithm of the domestic price of a unit of foreign currency and let p_t be the natural logarithm of a price index. Then the natural logarithm of the time

t real exchange rate is $q_t = s_t + p_t^* - p_t$, where an asterisk denotes a variable from the foreign country. Let $\pi_t = p_t - p_{t-1}$ be the inflation rate and let i_t be a nominal interest rate, then the real interest rate is $\rho_t = i_t - \pi_t$ and the time t real-interest-rate differential is $\hat{\rho}_t = (\rho_t - \rho_t^*)$. Let $y_t = (\Delta q_t, \hat{\rho}_t)'$ where $\Delta q_t = q_t - q_{t-1}$ is the percent change (first difference) in the real exchange rate. Then the bivariate threshold vector autoregression (TVAR) under consideration is

$$y_t = c_j + \sum_{i=1}^{p_j} \phi_i^{(j)} y_{t-i} + \epsilon_t^{(j)} \quad \text{for } \tau_{j-1} < q_t \leq \tau_j \quad (1)$$

where $j = \{1, 2, 3\}$ indexes the three real-exchange-rate regimes, c_j is a constant vector, $p_j > 0$, $\phi_i^{(j)}$ is a (2×2) parameter matrix, and $\epsilon_t^{(j)} = (\epsilon_t^{\Delta q_j}, \epsilon_t^{\hat{\rho}_j})'$ is a mean zero vector of innovations with covariance structure Σ_j . The threshold variable used to partition the three real-exchange-rate regimes is q_t , and the threshold values are $-\infty = \tau_0 < \tau_1 < \tau_2 < \tau_3 = \infty$ where τ_1 and τ_2 are the two nontrivial threshold value which partition the three real-exchange-rate regimes.

Threshold models are linear within a regime but nonlinear across regimes. They can be estimated in each regime using least squares conditional on the threshold variable, the number of regimes, and the lag lengths. Based on a nonrestrictive set of assumptions, Tsay (1998, p.1194-5) shows that least squares estimates are strongly consistent as the regime sample sizes go to infinity.

The TVAR model in Equation (1) can be rewritten as

$$\Phi_j(L)y_t = \epsilon_t^{(j)} \quad \text{for } \tau_{j-1} < q_t \leq \tau_j \quad (2)$$

where $\Phi_j(L)$ has the appropriate lags p_j and the constant vector c_j is suppressed. The model assumes that the percent real-exchange-rate changes and the real interest-rate differentials are stationary. Therefore Equation (2) can be inverted and represented by the infinite-order moving-average model

$$y_t = C_j(L)\epsilon_t^{(j)} \quad \text{for } \tau_{j-1} < q_t \leq \tau_j \quad (3)$$

where $C_j(L) = \Phi_j(L)^{-1}$ and the contemporaneous effect of $\epsilon_t^{(j)}$ on y_t is the identity matrix.

The structural, bivariate threshold model which identifies the independent temporary and permanent real-exchange-rate innovations in each regime is

$$y_t = A_j(L)\eta_t^{(j)} \quad \text{for } \tau_{j-1} < q_t \leq \tau_j \quad (4)$$

where $\eta_t^{(j)} = (\eta_t^{T_j}, \eta_t^{P_j})'$ is a mean zero vector of orthogonalized innovations with a covariance structure normalized to the identity matrix. The structural model is identified in each regime by comparing Equations (3) and (4) and observing that $\epsilon_t^{(j)} = A_j(0)\eta_t^{(j)}$ and $A_j(k) = C_j(k)A_j(0)$, where $A_j(0)$ is the contemporaneous effect of $\eta_t^{(j)}$ on y_t in regime j . Therefore the four elements of $A_j(0)$ just identify the structural model in each regime. Covariance restrictions establish three of the four restrictions necessary to identify $A_j(0)$ since $\Sigma_j = A_j(0)A_j(0)'$. The fourth restriction is the Blanchard and Quah (1989) identifying restriction that the long-run dynamic response of the *level* of the real exchange rate to the structural innovation $\eta_t^{T_j}$ is zero, or $\sum_{k=1}^{\infty} a_{1,1}^{(j)}(k) = 0$. These four restrictions just identify the four elements of $A_j(0)$ in each regime. The last restriction identifies $\eta_t^{T_j}$ as the temporary real-exchange-rate innovation and $\eta_t^{P_j}$ as the permanent real-exchange-rate innovation.

3. Empirical Results

Nominal exchange rates (line ae) for Germany, Japan and the United Kingdom (U.K.) relative to the United States (U.S.) as the domestic currency; wholesale price indexes (line 63) for Germany, Japan, the U.K. and the U.S.; and money-market interest rates (line 60b) for Germany, Japan, the U.K. and the U.S. are from the International Monetary Fund's *International Financial Statistics*. The data are monthly during the floating-exchange-rate period from April 1973 through December 2000 except for Germany which ends in December 1998 when the German mark was linked to the Euro.

The TVAR model assumes that the percent real-exchange-rate changes and the real interest-rate differentials are stationary. The Dickey - Fuller unit-root t test finds that all of the percent real-exchange-rate changes and real interest-rate differentials decisively reject the unit-root null hypothesis at the 0.01 level in favor of the mean-stationary alternative.

The TVAR model is estimated using 4 VAR lags in each regime, a choice supported by a likelihood ratio test which finds that the reduced form VAR residuals are highly consistent with white noise. Tables I, II and III present real-exchange-rate forecast-error variance decompositions along with standard errors which are bootstrapped using 1,000 repetitions of the model. The forecast-error variance decompositions are normalized such that the variances of the two structural innovations sum to 100%. Therefore the tables only list the percentage of the variance due to temporary real-exchange-rate innovations.¹

The two threshold values τ_1 and τ_2 are selected in a grid search across the empirical range of each real exchange rate such that the two outer regimes have either 25, 35, 45 or 55 observations. Empirical results are presented for the two threshold values which have the largest percentage of temporary real-exchange-rate innovations in the two outer regimes. The Germany - U.S. real exchange rate selects threshold values which place 25 observations in the lower regime and 45 observations in the upper regime. The Japan - U.S. real exchange rate selects threshold values which place 35 observations in both outer regimes. And the U.K. - U.S. real exchange rate selects threshold values which place 55 observations in both outer regimes.

Summary of Results: When the regimes are *not* taken into account, then the forecast-error variance decompositions in Tables I, II and III show that permanent innovations dominate all three real exchange rates. This is consistent with previous empirical evidence. When the regimes are taken into account, then the forecast-error variance decompositions in Tables I, II and III show that permanent innovations dominate each real exchange rate in the middle regime, but that temporary innovations have a large impact on all three real exchange rates in the two outer regimes.

Germany - United States Real Exchange Rate in Table I: In the lower regime with 25 observations, temporary innovations have a large impact on the Germany - U.S. real exchange rate accounting for 70% of the initial forecast-error variance. In the middle regime with 234 observations, permanent innovations dominate the Germany - U.S. real

¹Forecast-error variance decompositions are not presented for the real interest-rate differentials since their purpose here is to identify independent temporary and permanent real-exchange-rate innovations.

exchange rate accounting for over 98% of the forecast-error variance at all forecast horizons. In the upper regime with 45 observations, temporary innovations have an extremely large and persistent impact on the Germany - U.S. real exchange rate accounting for over 88% of the forecast-error variance through the 8-month forecast horizon and over 50% of the forecast-error variance through the 41-month forecast horizon.

Japan - United States Real Exchange Rate in Table II: In the lower regime with 35 observations, temporary innovations have an extremely large impact on the Japan - U.S. real exchange rate accounting for 82% of the initial variance. In the middle regime with 258 observations, permanent innovations dominate the Japan - U.S. real exchange rate accounting for over 88% of the forecast-error variance at all forecast horizons. In the upper regime with 35 observations, temporary innovations have an extremely large and persistent impact on the Japan - U.S. real exchange rate accounting for 99% of the initial variance and over 50% of the forecast-error variance through the 29-month forecast horizon.

United Kingdom - United States Real Exchange Rate in Table III: In the lower regime with 55 observations, temporary innovations have a large impact on the U.K. - U.S. real exchange rate accounting for 64% of the initial forecast-error variance. In the middle regime with 218 observations, permanent innovations dominate the U.K. - U.S. real exchange rate accounting for over 97% of the forecast-error variance at all forecast horizons. In the upper regime with 55 observations, temporary innovations have a large and persistent impact on the U.K. - U.S. real exchange rate accounting for 68% of the initial variance.

4. Conclusion

This paper estimates a nonlinear, three-regime structural bivariate threshold model using Germany - U.S., Japan - U.S. and U.K. - U.S. data, and finds evidence of threshold stationary real exchange rates. This evidence is consistent with purchasing power parity adjusted for market frictions such as transaction costs.

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Table I
Forecast-Error Variance Decompositions:
Germany - United States Real Exchange Rate

Forecast Horizon (Months)	No Regimes (304 obs)	Lower Regime (25 obs)	Middle Regime (234 obs)	Upper Regime (45 obs)
Percentage of the Variance Due to Temporary Innovations:				
1	4.7 (12.2)	70.2 (29.7)	1.1 (10.3)	89.8 (31.7)
2	5.0 (12.0)	54.2 (26.9)	1.3 (10.2)	88.6 (31.0)
3	5.2 (11.9)	47.4 (25.5)	1.4 (10.0)	90.4 (30.6)
4	4.8 (11.4)	35.5 (22.4)	1.2 (9.6)	90.3 (30.0)
5	4.4 (10.9)	25.0 (18.9)	1.1 (9.1)	90.0 (29.3)
6	4.2 (10.4)	18.0 (16.0)	0.9 (8.6)	90.0 (28.6)
9	3.5 (9.0)	9.2 (11.9)	0.8 (7.2)	87.7 (26.5)
12	3.0 (7.8)	6.7 (10.7)	0.6 (6.1)	85.0 (24.6)
24	1.9 (4.8)	3.6 (9.2)	0.3 (3.5)	71.9 (18.2)
36	1.3 (3.3)	2.4 (8.4)	0.2 (2.4)	57.0 (13.5)
48	1.0 (2.4)	1.8 (7.7)	0.2 (1.7)	43.2 (10.0)
60	0.8 (1.9)	1.4 (7.2)	0.1 (1.4)	31.7 (7.5)
72	0.6 (1.5)	1.2 (6.8)	0.1 (1.1)	23.0 (5.7)
84	0.6 (1.3)	1.0 (6.5)	0.1 (0.9)	16.6 (4.4)
96	0.5 (1.1)	0.9 (6.4)	0.1 (0.8)	11.9 (3.6)
108	0.4 (0.9)	0.8 (6.2)	0.1 (0.7)	8.7 (3.0)
120	0.4 (0.8)	0.7 (6.2)	0.1 (0.6)	6.3 (2.6)

Standard errors in parentheses are bootstrapped using 1,000 repetitions of the model. The data are monthly from April 1973 through December 1998.

Table II
Forecast-Error Variance Decompositions:
Japan - United States Real Exchange Rate

Forecast Horizon (Months)	No Regimes (328 obs)	Lower Regime (35 obs)	Middle Regime (258 obs)	Upper Regime (35 obs)
Percentage of the Variance Due to Temporary Innovations:				
1	17.4 (14.6)	82.3 (30.5)	9.8 (12.8)	99.4 (29.7)
2	17.4 (14.5)	68.0 (28.5)	10.5 (13.0)	98.8 (29.8)
3	17.9 (14.5)	64.6 (27.4)	11.3 (13.2)	98.2 (30.3)
4	17.5 (14.2)	54.3 (23.4)	11.7 (13.2)	92.8 (29.3)
5	16.8 (13.7)	44.9 (20.5)	11.6 (12.9)	90.6 (29.2)
6	16.1 (13.1)	37.9 (18.3)	11.4 (12.5)	87.1 (28.5)
9	13.5 (11.1)	24.4 (13.8)	10.1 (11.0)	81.8 (28.0)
12	11.3 (9.3)	17.6 (11.1)	8.7 (9.4)	78.3 (28.0)
24	6.0 (4.9)	8.4 (6.9)	4.9 (5.3)	58.7 (24.6)
36	3.9 (3.1)	5.5 (5.3)	3.2 (3.3)	41.9 (20.6)
48	2.8 (2.2)	4.1 (4.3)	2.3 (2.3)	29.6 (17.0)
60	2.2 (1.6)	3.3 (3.7)	1.8 (1.7)	21.0 (14.0)
72	1.8 (1.3)	2.7 (3.3)	1.5 (1.4)	15.1 (11.4)
84	1.5 (1.1)	2.3 (3.0)	1.3 (1.1)	11.1 (9.1)
96	1.3 (0.9)	2.0 (2.8)	1.1 (1.0)	8.3 (7.3)
108	1.2 (0.8)	1.8 (2.6)	1.0 (0.8)	6.4 (6.0)
120	1.0 (0.7)	1.6 (2.5)	0.9 (0.8)	5.0 (5.1)

Standard errors in parentheses are bootstrapped using 1,000 repetitions of the model. The data are monthly from April 1973 through December 2000.

Table III
Forecast-Error Variance Decompositions:
United Kingdom - United States Real Exchange Rate

Forecast Horizon (Months)	No Regimes (328 obs)	Lower Regime (55 obs)	Middle Regime (218 obs)	Upper Regime (55 obs)
Percentage of the Variance Due to Temporary Innovations:				
1	21.6 (16.3)	64.9 (28.8)	0.0 (9.5)	68.6 (28.3)
2	18.8 (15.1)	54.5 (28.2)	0.3 (9.5)	64.1 (26.8)
3	15.7 (13.6)	38.1 (23.3)	0.8 (9.6)	51.5 (22.7)
4	13.4 (12.4)	34.7 (22.3)	1.7 (9.7)	51.4 (22.3)
5	11.9 (11.4)	28.9 (19.4)	1.9 (9.6)	48.6 (21.4)
6	10.7 (10.5)	23.4 (16.1)	2.0 (9.2)	47.5 (20.9)
9	8.2 (8.4)	15.4 (11.7)	1.9 (8.0)	41.8 (19.0)
12	6.5 (6.8)	11.4 (9.3)	1.7 (6.8)	36.9 (17.3)
24	3.3 (3.5)	5.6 (5.3)	1.0 (3.9)	22.5 (12.5)
36	2.2 (2.2)	3.7 (3.7)	0.7 (2.6)	14.3 (9.6)
48	1.6 (1.5)	2.8 (2.8)	0.5 (1.9)	9.5 (7.7)
60	1.3 (1.2)	2.2 (2.3)	0.4 (1.5)	6.5 (6.4)
72	1.0 (1.0)	1.9 (1.9)	0.3 (1.3)	4.7 (5.4)
84	0.9 (0.8)	1.6 (1.6)	0.3 (1.1)	3.4 (4.6)
96	0.8 (0.7)	1.4 (1.5)	0.3 (0.9)	2.6 (4.0)
108	0.7 (0.6)	1.2 (1.3)	0.2 (0.8)	2.0 (3.5)
120	0.6 (0.5)	1.1 (1.2)	0.2 (0.7)	1.6 (3.0)

Standard errors in parentheses are bootstrapped using 1,000 repetitions of the model. The data are monthly from April 1973 through December 2000.