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Equilibrium dynamics with different types of pay−as−you−go pension schemes

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Abstract

We analyse the steady−state equilibrium dynamics of an OLG economy with a pay−as−you−go (PAYG) pension scheme that relates old−age pensions to previous earnings. Contrary to an economy where PAYG pensions depend on the earnings of those currently working, such an economy may experience complex equilibrium dynamics with endogenous cycles and bifurcations.

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1. Introduction

The defining property of pay-as-you-go (PAYG) pension systems, namely that total pension payouts in any period equal the sum of contributions raised in that period, is consistent with a large variety of schemes. In this paper we distinguish two benefit rules or, which is the same here, two policies to keep the PAYG budget intact when fluctuations in revenues or payouts of the scheme occur. In one setting, pensioners receive old-age incomes as a given fraction (called *replacement ratio*) of their incomes during working age; contributions are then adjusted to make means meet ends. In the other setting, contributions are levied as a constant percentage of the income of the currently working; pension benefits emerge endogenously from the revenues thus raised.

Consider an economy of two overlapping generations where life cycles consist of one period of work and one period of retirement. Assume that generations are equally populous. Denoting by b_t and p_{t-1}, respectively, period-t contributions to the PAYG scheme per capita of workers, and period-t pension payments to the retirees of generation $t-1$, a PAYG scheme is defined by $b_t = p_{t-1}$. We analyze the following specifications:

- Generation $t-1$'s pension p_{t-1} depends on that generation's previous earnings w_{t-1}. In a linear specification, $p_{t-1} = \alpha \cdot w_{t-1}$ where α is the replacement ratio. Contributions levied on those working emerge residually as $b_t = \alpha w_{t-1}$; the implicit contribution rate, thus, amounts to $b_t/w_t = \alpha w_{t-1}/w_t$.

- Alternatively, contributions in t are raised as a fixed percentage (the *contribution rate*) β of workers' income: $b_t = \beta \cdot w_t$. Generation $t-1$'s pensions then residually result as $p_{t-1} = \beta \cdot w_t$; the implicit replacement ratio is $p_{t-1}/w_{t-1} = \beta \cdot w_t/w_{t-1}$.

We call the first policy a *fixed replacement* (FR) policy, and the second a *fixed contributions* (FC) policy. The distinction between these policies is akin to the defined-benefit/defined-contribution dichotomy in the context of private, funded pensions. In stochastic settings, FR- and FC-policies perform differently with respect to intergenerational risk-sharing (Thøgersen, 1998; Wagener, 2003). The FC/FR distinction also bears linkages to the degree of intragenerational redistribution through the pension scheme: In Bismarckian systems pensions are indexed to previous earnings (as with a FR policy) while they are unrelated to previous earnings in Beveridgean schemes (as under a FC regime).

Prima facie, FR- and FC-policies look equivalent in models with identical agents if the wage is time-invariant; then $\alpha = \beta$ represents both the contribution and the replacement rate. (With population growth at rate $n > -1$, set $\alpha = \beta(1+n)$.) It therefore seems perfectly justified to focus on one of the two policies, especially when one is interested in steady state economies. Indeed, most contributions in the literature only analyse FC schemes and do not even mention FR policies.[1]

Here we argue that the distinction between FR and FC PAYG policies might indeed matter. In particular, equilibrium dynamics and stability properties of steady states largely differ under the two regimes. Technically, the equilibrium of an OLG economy with a FR scheme is characterized by a difference equation of order two. This is due to the fact that today's pension contributions to the pension scheme depend on past rather than on current wages. In contrast, with a FC policy the equilibrium is characterized by a first-order difference equation. Hence, while equilibrium dynamics with FC schemes do not exhibit any surprising features, FR equilibrium dynamics are more colourful. Changes in FR policies may expose the economy to periodic or unstable dynamics. Moreover, we show that such peculiarities can occur under circumstances (namely, for high elasticities of substitution in production) that would, without wage-related pension liabilities, not give rise to bifurcations or endogenous cycles. The differences between FC and FR equilibria are due to the endogeneity of the contribution rate under a FR scheme; this works to magnify the effects of changes in the capital-labour ratio on the rent-wage ratio.

Section 2 presents a standard 2-OLG economy with FR PAYG pensions and Section 3 analyzes and illustrates its equilibrium dynamics. Section 4 relates our observations to the literature and compares them to a FC policy. Section 5 concludes.

2. Model and Equilibrium

We consider a Diamond-type 2-OLG model with capital accumulation (see Ihori, 1996, for a survey). Generation t ($t = 0, 1, \ldots$) consists of N_t identical individuals each of whom lives for the two periods t (youth) and $t+1$ (old age). From t to $t+1$ the population grows by (the possibly negative) rate $n_t := N_t/N_{t-1} - 1 > -1$. We denote the life-cycle utility function by U and assume that it is additively time-separable with instantaneous utility functions u and $\hat{u}$: $U_t = u(c_{1,t}) + \hat{u}(c_{2,t})$. By $c_{1,t}$ and $c_{2,t}$ we denote consumption of a member of generation t when young and old, respectively. Sub-utility functions are well-behaved with $\nu'(c) > 0 > \nu''(c)$ for all $c > 0$ and $\nu'(0) = \infty$ for $\nu = u, \hat{u}$. To obtain that savings increase in their rate of return we require that the marginal utility of old-age

[1] See, e.g., Burbidge (1983), Ihori (1996) or Jäger (1994). Even more widespread are, however, lump-sum (rather than earnings-related) pension schemes. See, e.g., Blanchard and Fischer (1989).

consumption decreases less than proportionally with consumption:

$$\hat{u}'(c) + c\hat{u}''(c) > 0 \quad \text{for all} \quad c > 0. \tag{1}$$

Each individual inelastically supplies one unit of labour during working age and is fully retired during old-age. Hence, total labour supply in period t amounts to N_t.

The economy has available a time-invariant, aggregate neoclassical production technology that employs inputs capital and labour with constant returns to scale. Per-worker output in t then amounts to $f(k_t)$ with k_t as the capital stock in period t divided by N_t. We suppose that the production function f possesses the standard Inada properties: $f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$ for all $k > 0$ and $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$.

We denote by s_t, b_t, p_t, w_t, R_t, respectively, generation t's per-capita savings, per-capita social security taxes, per-capita old-age pensions, the net wage rate and the interest factor that prevail in period t. Budget constraints for the two periods in the life of generation t are then given by $c_{1,t} = w_t - b_t - s_t$ and $c_{2,t} = R_{t+1} \cdot s_t + p_t$. We concentrate on PAYG schemes (i.e., $(1 + n_t)b_t = p_{t-1}$) with a FR policy (for FC policies see Section 4.2). I.e., $p_t = \alpha \cdot w_t$ and $b_t = \alpha w_{t-1}/(1 + n_t)$.

Capital market equilibrium in period t requires total saving of the previous period to be equal to capital demand in t:

$$s_{t-1} = (1 + n_t) \cdot k_t. \tag{2}$$

Competitive profit maximization requires firms to hire labour and capital such as to equate marginal factor productivities and factor prices: $f'(k_t) = R_t$ and $f(k_t) - f'(k_t) \cdot k_t = w_t$. Individuals take factor prices and the parameters of the PAYG schemes as given. Utility maximization entails equalizing the discounted periodwise marginal utilities of consumption: $u'(c_{1,t}) = R_{t+1}\hat{u}(c_{2,t})$. This implicitly defines generation t's saving as a function of the own wage rate, the interest factor, the replacement rate, and the wage rate of the previous generation which enters via pension contributions: $s_t = s_t(w_t, w_{t-1}, R_{t+1}, \alpha)$.

- s_{w_t} is ambiguous in sign due to a positive first-period income effect and a negative second-period income effect;
- $s_{w_{t-1}} < 0$: savings are lower the higher are past wages – which imply a higher pension liability for current workers;
- $s_R > 0$: savings increase in their rate of return, provided that (1) holds;
- $s_\alpha < 0$: savings decrease in the replacement rate α due to smaller disposable incomes in working age and higher retirement incomes.

Plugging the (appropriately lagged) factor price relations and the savings function into the equilibrium condition (2) for the capital market, we can summarize equilibrium dynamics under a FR regime in the following second-order difference equation:

$$(1+n)k_{t+1} - s\left(f(k_t) - k_t f'(k_t), f(k_{t-1}) - k_{t-1} f'(k_{t-1}), f'(k_{t+1}), \alpha\right) = 0. \qquad (3)$$

Existence of equilibrium (i.e., a solution for (3)) can be established under fairly general assumptions (see Geanakoplos and Polemarchakis, 1991), which are met here. A *stationary state* for the dynamics of (3) is a level $\bar{k}$ such that:

$$(1+n)\cdot\bar{k} = s\left(f(\bar{k}) - \bar{k}\cdot f'(\bar{k}), f(\bar{k}) - \bar{k}\cdot f'(\bar{k}), f'(\bar{k}), \alpha\right). \qquad (4)$$

One – uninteresting – fixed point of (4) is given by $\bar{k} = 0$. In what follows we will discuss positive steady states with $\bar{k} > 0$ only.

3. Stability Analysis

3.1 General Observations

Eq. (3) being of order two, the stability analysis proves a bit tricky. We use the "geometrical" method developed in Azariadis (1993) and Grandmont et al. (1998).[2] Along with the substitution $k_t =: x_{t-1}$ we rewrite (3) as a dynamic system $x_t = G(k_{t-1}, x_{t-1})$ and $k_t = x_{t-1}$ with Jacobian matrix J of partial derivatives:

$$J = \begin{pmatrix} G_k & G_x \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial k_{t-1}} \\ 1 & 0 \end{pmatrix}. \qquad (5)$$

The elements in the first line of J can be calculated by implicit differentiation of (3):

$$\frac{\partial k_{t+1}}{\partial k_t} = -\frac{s_{w_t}\cdot f''(k_t)\cdot k_t}{1+n-s_R\cdot f''(k_{t+1})} \quad \text{and} \quad \frac{\partial k_{t+1}}{\partial k_{t-1}} = -\frac{s_{w_{t-1}}\cdot f''(k_{t-1})\cdot k_{t-1}}{1+n-s_R\cdot f''(k_{t+1})} < 0. \qquad (6)$$

The sign of the second term comes from $s_R > 0 > s_{w_{t-1}}$, the sign of the first one is unclear. With $\det J = -G_x > 0$ and $\mathrm{tr} J = G_k$, the characteristic polynomial of J is

$$\phi(\lambda) = \lambda^2 - \lambda\frac{\partial k_{t+1}}{\partial k_t} - \frac{\partial k_{t+1}}{\partial k_{t-1}},$$

where all the derivatives have to be evaluated at $\bar{k}$. The discriminant of ϕ, $D = G_k^2 + 4G_x$, can take any sign. Hence, J may have complex eigenvalues; equilibria may thus be periodic. Moreover, the positions of the eigenvalues of J relative to the unit circle are unclear,

[2]For recent applications see Koskela et al. (2000), Aloi et al. (2000) or Coimbra et al. (2000).

too. Hence, equilibria may be stable or not. Furthermore, upon changes of the replacement rate α (possibly also of other parameters of the model) there may be bifurcations of different types, including period-doubling Hopf bifurcations . I.e., by slightly changing a parameter of the model, *qualitative* changes in the dynamic properties of the equilibrium trajectories nearby the steady state may occur. We illustrate this in a numerical example.

3.2 An Example

Let utility be Cobb-Douglas and production technology be CES:

$$U = \log c_{1,t} + \log c_{2,t} \quad \text{and} \quad f(k_t) = A[1 + a \cdot k_t^{\eta}]^{1/\eta}$$

with parameters $A > 0$, $a \in (0,1)$, $\eta \leq 1$ and $\eta \neq 0$. The elasticity of substitution in production is $1/(1-\eta)$. For this specification we obtain a saving function

$$s(w_t, w_{t-1}, R_{t+1}; \alpha) = \frac{1}{2} \cdot \left[w_t \left(1 - \frac{\alpha}{R_{t+1}} \right) - \frac{\alpha}{1+n} w_{t-1} \right].$$

It is plausible to assume $\alpha \leq R_{t+1}$; hence, $\partial s/\partial w_t \geq 0$. Furthermore, $\partial s/\partial w_{t-1} < 0 < \partial s/\partial R_{t+1}$ for $\alpha > 0$, as expected. By an appropriate choice of A, we adapt units of measurement such that $\bar{k} = 1$ in the steady state. Then,

$$\bar{R} = (1+a)^{1/\eta - 1} \quad \text{and} \quad \bar{w} = \frac{1}{a} \cdot (1+a)^{1/\eta - 1}.$$

We index the economy by the replacement rate and write all expressions as functions of α. The top-row elements of the Jacobian $J(\alpha)$ can be calculated as:

$$G_k(\alpha) = \frac{1}{2} \cdot \frac{(1+a)^{1/\eta-1} - \alpha}{\frac{1+n}{(1-\eta)\cdot(1+a)} + \frac{a\alpha}{2}} > 0 \quad \text{and} \quad G_x(\alpha) = -\frac{1}{2} \cdot \frac{\frac{\alpha}{1+n} \cdot (1+a)^{1/\eta-1}}{\frac{1+n}{(1-\eta)\cdot(1+\eta)} + \frac{a\alpha}{2}} < 0,$$

which has characteristic polynomial $\phi(\lambda, \alpha) = \lambda^2 + \lambda G_k(\alpha) - G_x(\alpha)$ and discriminant $D(\alpha) = G_k^2(\alpha) + 4G_x(\alpha)$. We set $n = 0$. By appropriately choosing the free parameters we can generate a large variety of stability patterns: fixed and periodic points both with contracting and expanding directions. We illustrate this for $\eta = 0.25$ and $a = 0.4$.

Figure 1 goes here.

The dotted arrow in Fig. 1 shows what happens when α varies from 0 to closely below 1:[3]

[3]The precise values of the boundary points for the numerical specification chosen are: $\alpha_1 = 0.198$, $\alpha_2 = 0.259$, and $\alpha_3 = 0.621$. Calculations have been made in a MATHEMATICA notebook that is available from the author.

- $0 < \alpha < \alpha_1$: Here we have $D(\alpha) > 0$ (real eigenvalues) and $\phi(1, \alpha) < 0$ (eigenvalues on different sides of $+1$); furthermore $g(\alpha) > 0$. Hence, the steady state is a *saddle*.

- $\alpha_1 < \alpha < \alpha_2$: Eigenvalues are still real (since $D(\alpha) > 0$), but now are both positive and fall on the same side of both $+1$ and -1. Hence, the steady state is *asymptotically stable*.

- $\alpha_2 < \alpha < \alpha_3$: Eigenvalues are complex and come as a conjugate pair. As $\det J(\alpha) < 1$, the oscillatory orbits in the neighbourhood of the steady state are *stable spirals*.

- $\alpha_3 < \alpha < 1$: Eigenvalues are complex. Since $\det J(\alpha) > 1$, we obtain *unstable spirals* near to the steady state.

For $\alpha = \alpha_1$, a saddle-node bifurcation occurs (there is one eigenvalue of modulus one and $|\det J| < 1$), and for $\alpha = \alpha_3$ we detect a Hopf bifurcation (an invariant closed curve in the neighbourhood of the steady state).[4] Summing up, we get

Result 1 *In an economy with FR pensions equilibria may be periodic and unstable. Small changes in the replacement rate may lead to drastic changes in the system's orbit structure and dynamic behaviour.*

4. Discussion and Comparison

4.1 Bifurcations and the Elasticity of Substitution

While saddle-node bifurcations are quite common in planar dynamical models with some sort of debt (see Azariadis (1993) for several examples), Hopf bifurcations are rarer. They can occur in many-good OLG models (Grandmont, 1985) and are not new in one-good, Diamond-type OLG models either; see, e.g., Farmer (1986), Reichlin (1986), or Azariadis (1993). Typically, however, they emerge at *low* values of the elasticity of substitution in production, in particular for values well below unity (i.e., for $\eta < 0$ in CES functions):

[4] Application of the Hopf bifurcation theorem requires several conditions to be met at once (Azariadis, 1993, Theorem 8.5): The dynamic mappings that give rise to the Jacobian J must be C^k for some $k \geq 6$. This is satisfied here, since in the relevant range all functions are smooth. The steady state must be a non-hyperbolic equilibrium at α_3, and there must be a conjugate pair of complex eigenvalues $\lambda(\alpha_3)$ of modulus 1. This is exactly what $\det J(\alpha_3) = 1$ means. Furthermore $\lambda^k(\alpha) \neq 1$ for $k = 1, 2, 3, 4$, which is also satisfied. Finally, $d|\lambda(\alpha_3)|/d\alpha \neq 0$, which can be visually verified by plotting the graph of $g(\alpha)$.

Reichlin (1986) discusses the Leontief case, and in Farmer (1986)'s example for the CES-case Hopf bifurcations occur only when technologies exhibit lower factor substitutability than Cobb-Douglas functions.

In the example presented above, we find Hopf bifurcations for elasticities of substitution well above unity, which may be regarded as the empirically more relevant case. They originate from opposing effects of higher wages in different periods on saving. In standard models, higher wages will stimulate saving which, in a growing economy, will promote capital accumulation, further raise the wages and lower the interest rate. With FR pensions there is, however, a negative effect on saving which runs counter to this process: High wages today mean high pension liabilities for the subsequent generation — which will depress their saving. Eventually, the dynamic growth process might reverse itself.

Result 2 *In an economy with a FR PAYG pension scheme, endogenous cycles and bifurcations may occur even when the elasticity of substitution in production is above unity.*

4.2 FR- versus FC-policies

Contrast these observations with the case of FC pensions. The FC analogue to eq. (3) is

$$(1+n)k_{t+1} - \hat{s}\left(f(k_t) - k_t f'(k_t), f(k_{t+1}) - k_{t+1} f'(k_{t+1}), f'(k_{t+1}), \beta\right) = 0, \tag{7}$$

where $\hat{s} = \hat{s}(w_t, w_{t+1}, R_{t+1}, \beta)$ emerges from utility maximization, $\max_{s_t}[u(w_t(1-\beta) - s_t) + \hat{u}(R_{t+1}s_t + (1+n_{t+1})\beta w_{t+1})]$.

Result 3 *In an economy with FC pensions equilibrium dynamics are monotonic. Furthermore, the steady state is stable if capital income increases with the capital stock:*

$$f'(k) + k \cdot f''(k) \geq 0 \quad \textit{for all} \quad k > 0. \tag{8}$$

Proof: The properties of a FC steady state are determined by

$$\frac{\mathrm{d}k_{t+1}}{\mathrm{d}k_t} = -\frac{\hat{s}_{w_t} k_t f''(k_t)}{1 + n + f''(k_{t+1}) \cdot \left[\hat{s}_{w_{t+1}} k_{t+1} - \hat{s}_R\right]}$$

(see Galor and Ryder, 1989; Azariadis, 1993). Uniqueness of the steady state and monotonicity of dynamics will hold when $\mathrm{d}k_{t+1}/\mathrm{d}k_t > 0$. This provided, stability of the steady state will prevail whenever $\mathrm{d}k_{t+1}/\mathrm{d}k_t < 1$.[5] It can easily be shown that $\hat{s}_{w_t} > 0$, $\hat{s}_{w_{t+1}} < 0$,

[5] Precisely, stability will prevail if $-1 < \mathrm{d}k_{t+1}/\mathrm{d}k_t < 1$. The case $-1 < \mathrm{d}k_{t+1}/\mathrm{d}k_t < 0$ (which cannot occur here) is problematic; see Jäger (1994) for a discussion.

$\hat{s}_R > 0$, and $\hat{s}_{w_{t+1}} k_{t+1} - \hat{s}_R < 0$. Hence, $\mathrm{d}k_{t+1}/\mathrm{d}k_t > 0$. Now we evaluate $\mathrm{d}k_{t+1}/\mathrm{d}k_t$ in the steady state. Invoking these intermediate results one gets that $\mathrm{d}k_{t+1}/\mathrm{d}k_t < 1$ for $k_{t+1} = k_t = \bar{k}$ is equivalent to

$$\left((1+n) - f''\bar{k}(1-\beta)\right) u'' < -\hat{u}''(1+n)f'\left[f' + (1-\beta)f''\bar{k}\right] - f''\hat{u}'.$$

This always holds as the LHS is negative while the RHS is positive, given (8). ■

The dynamics of the economy with a FC PAYG scheme are, thus, well-behaved, compared to those under a FR scheme: They are monotonic and never exhibit endogenous cycles or bifurcations.[6] The difference between FC and FR schemes traces back to eqs. (3) and (7): The former difference equation is of order two, the latter of order one.

4.3 Increasing the Replacement Rate

The higher the contribution rate in a FC scheme, the lower is the steady-state capital stock; see, e.g., Burbidge (1983) and Azariadis (1993, ch. 18). What does happen in a FR scheme when we increase the replacement rate? From (4),

$$\frac{\mathrm{d}\bar{k}}{\mathrm{d}\alpha} = \frac{s_\alpha}{1 + n + f''(\bar{k})\left(-s_R + s_{w_t}\bar{k} + s_{w_{t-1}}\bar{k}\right)} \tag{9}$$

with $s_\alpha < 0$. Without further restrictions the sign of the denominator in (9) is unclear. It is therefore conceivable that increasing α raises the capital stock. However, we show

Result 4 *If the steady state is stable for $\alpha = 0$, then introducing a small FR scheme depresses the capital stock:* $\mathrm{d}\bar{k}/\mathrm{d}\alpha < 0$.

Proof: For $\alpha = 0$ we are in a standard laissez-faire OLG economy without pensions. Galor and Ryder (1989) show that the conditions for a unique and globally stable steady state (characterized by $\bar{k}^0$) include $\mathrm{d}k_{t+1}/\mathrm{d}k_t < 1$. This is equivalent to $1 + n + f''(\bar{k}^0) \cdot (-s_R + s_{w_t}\bar{k}^0) > 0$ here. Taking into account that $s_{w_{t-1}} < 0$ for $\alpha > 0$ while $s_{w_{t-1}} = 0$ for $\alpha = 0$, the denominator in (9) is positive for small positive values of α. ■

[6] This only holds for perfect-foresight equilibria (where individuals correctly anticipate future interest and wage rates). With myopic foresight, equilibria might be oscillatory and exhibit deterministic stable cycles (Michel and de la Croix, 2000).

5. Conclusion

We analyzed economic dynamics in an OLG model with capital accumulation and PAYG social security schemes. Unlike the literature, which concentrates on pension policies with constant contribution rates, our focus was on pension policies with a fixed replacement ratio. Our results are the following: (i) Equilibrium dynamics differ considerably between economies with FC and FR pensions. (ii) With FR pensions, equilibria may be periodic and unstable, prompting endogenous cycles and bifurcations. (iii) With FR pensions, bifurcations might even occur when the elasticity of substitution in production is high.
The distinction between FR- and FC-policies currently figures prominently on the pension reform agenda. While PAYG schemes throughout the world traditionally are of the FR-type, several countries recently switched to a FC scheme (e.g., Sweden) or at least moved into that direction (e.g., Italy, Germany). Latvia and Poland chose the FC option when newly designing their pension system in the late 1990s. While (to our knowledge) so far no empirical comparison of the dynamic stability of the two PAYG options is available, our analysis indicates that addressing that issue in pension studies might be worthwile.

References

Aloi, M., H. Dixon, and T. Lloyd-Braga (2000) "Endogenous fluctuations in an open economy with increasing returns to scale" *Journal of Economic Dynamics and Control* 24, 97–125.

Azariadis, C. (1993) *Intertemporal Macroeconomics.* Blackwell: Oxford/Cambridge.

Blanchard, O. J. and S. Fischer (1989) *Lectures on Macroeconomics.* The MIT Press: Cambridge.

Burbidge, J. B. (1983) "Social security and saving plans in overlapping-generations models" *Journal of Public Economics* 21, 73–92.

Coimbra, R., T. Lloyd-Braga, and L. Modesto (2000) "Unions, increasing returns and endogenous fluctuations" IZA Discussion Paper No. 229, Bonn.

Farmer, R. E. (1986) "Deficits and cycles" *Journal of Economic Theory* 40, 77–86.

Galor, O., and H. Ryder (1989) "Existence, uniqueness and stability in overlapping generations models with productive capital" *Journal of Economic Theory* 49, 360–375.

Geanakoplos, J. and H. M. Polemarchakis (1991) "Overlapping generations" in *Handbook of Mathematical Economics*, Vol. IV, by W. Hildenbrand and H. Sonnenschein, Eds., North-Holland: Amsterdam etc., pp. 1899–1960.

Grandmont, J.-M. (1985) "On endogenous business cycles" *Econometrica* 53, 995–1045.

Grandmont, J.-M., P. Pintus, and R. de Vilder (1998) "Capital-labor substitution and competitive nonlinear endogenous business cycles" *Journal of Economic Theory* 80, 14–59.

Ihori, T. (1996) *Public Finance in an Overlapping Generations Economy*. St. Martin's Press: New York.

Jäger, K. (1994) "On the influence of pay-as-you-go financed pension systems on saving: The role of stability" *Scottish Journal of Political Economy* 41, 358–371.

Koskela, E., M. Ollikainen, and M. Puhakka (2000) "Saddles, indeterminacy and bifurcations in an overlapping generations economy with a renewable resource" CESifo Working Paper No. 298, CESifo, München.

Michel, P., and D. de la Croix (2000) "Myopic and perfect foresight in the OLG model" *Economics Letters* 67, 53–60.

Reichlin, P. (1986) "Equilibrium cycles in an overlapping generations economy with production" *Journal of Economic Theory 40*, 89–102.

Thøgersen, Ø. (1998) "A note on intergenerational risk sharing and the design of pay-as-you-go pension programs" *Journal of Population Economics* 11, 373–378.

Wagener, A. (2003) "On intergenerational risk sharing within social security schemes" Forthcoming, *European Journal of Political Economy*.

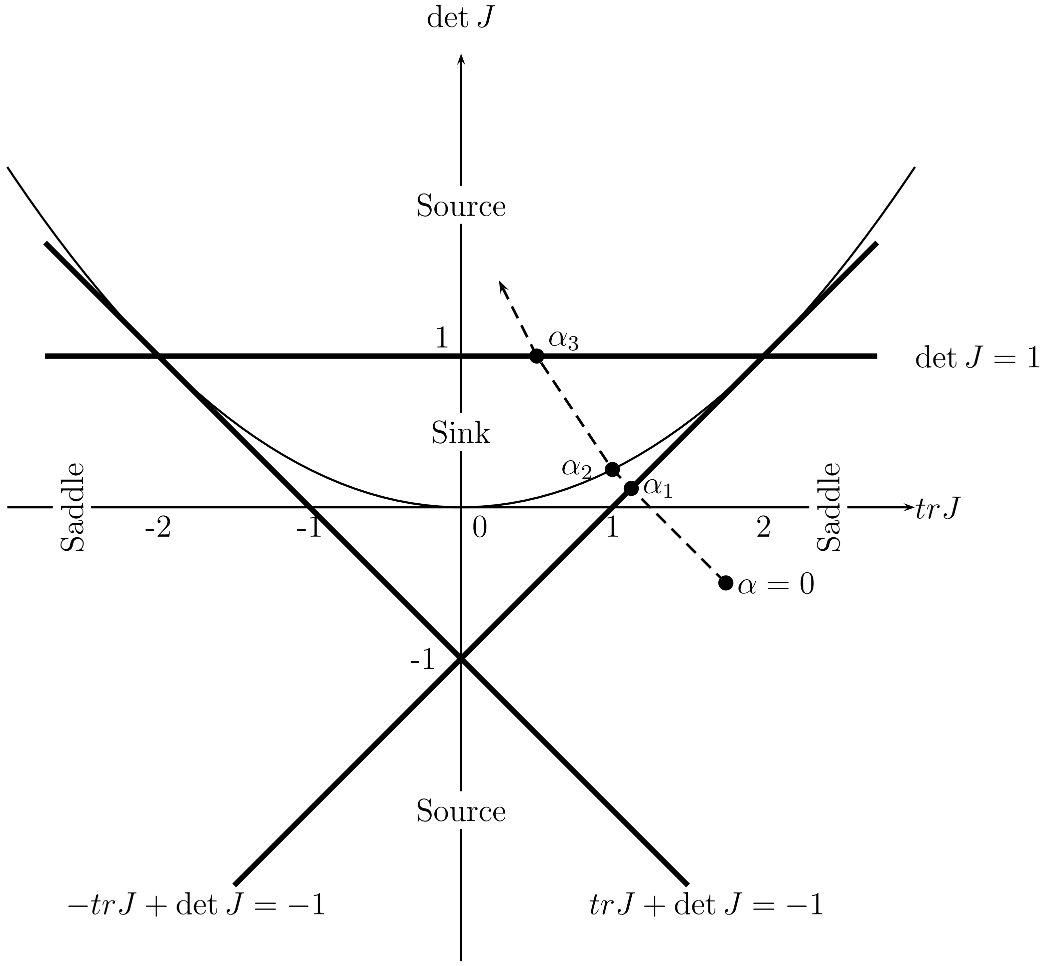

Figure 1: Characteristics of stability ($n = 0$; $\eta = 0.25$; $a = 0.4$)