E C O N O M I C S B U L L E T I N

Collusion and the elasticity of demand

David Collie
Cardiff Business School, Cardiff University

Abstract

The analysis of collusion in infinitely repeated Cournot oligopoly games has generally assumed that demand is linear, but this note uses constant−elasticity demand functions to investigate how the elasticity of demand affects the sustainability of collusion.

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1. Introduction

Most analyses of the sustainability of collusion in infinitely-repeated Cournot oligopoly games either prove general propositions about the sustainability of collusion as a subgame perfect equilibrium for a sufficiently large discount factor, as in Friedman (1971), or they assume that demand is linear so that they can explicitly solve for the discount factor and relate it to the parameters of the model. For example, various authors have used linear demand functions to analyse how product differentiation, the number of firms, cost asymmetries, or the type of competition (Bertrand or Cournot) affects the sustainability of collusion.[1] Although such analyses are useful in the exploration of the factors that affect the sustainability of collusion, they are somewhat limited by being restricted to a particular functional form i.e. linear demand. One important factor that may be thought to affect the sustainability of collusion is the elasticity of demand but, obviously, the effect of this parameter on the sustainability of collusion cannot be addressed using linear demand.[2] It would be natural to address this question using a constant-elasticity demand function. However, the constant-elasticity demand function does not allow explicit analytical solutions to be obtained when a firm deviates from collusion so it is not possible to obtain an explicit solution for the critical discount factor required to sustain collusion. Therefore, the only way to assess the effect of the elasticity of demand on the sustainability of collusion in an infinitely repeated Cournot oligopoly is to resort to numerical solutions, and that is what will be done in this note. As there are only two significant parameters in the model (the elasticity of demand and the number of firms) it straightforward to plot the critical discount factor as a function of these parameters. Despite using numerical solutions, the results obtained are unambiguous and seem to be quite general. It is shown that the larger is the elasticity of demand then the easier it is for firms to sustain collusion at the monopoly price.[3]

2. The Model

Consider an infinitely-repeated Cournot oligopoly game with J firms producing a homogeneous product and facing a constant-elasticity demand function, $P(Q) = Q^{-1/\eta}$, where P is the market price, Q is total industry output, and η is the elasticity of demand, which is assumed to be greater than one. All firms have identical and constant marginal cost c, and they maximise the present discounted value of future profits using a common discount factor, δ. In each stage of the infinitely repeated game, the output of the *j*th firm is q_j and its profits are

[1] See, *inter alia*, Deneckere (1983,1984), Majerus (1988), Wernerfelt (1989), Albæk and Lambertini (1998), and Rothschild (1999). Exceptions to the assumption of linear demand are Lambertini (1996) and Tyagi (1999) both of whom allow non-linearity by using a particular functional form that allows demand to be concave or convex, but which does not include constant elasticity demand as a special case.

[2] Jacquemin and Slade (1989, p. 421) note that 'The elasticity of the individual-firms demand curve is an important factor' affecting the incentive to cheat by cutting price and increasing sales.

[3] This note only considers collusion at the monopoly price (full or perfect collusion) and does not deal with the possibility of partial collusion. For an analysis of partial collusion in an infinitely repeated Cournot duopoly see Verboven (1997).

$\pi_j = (P-c)q_j$ while total industry output is $Q = \sum_{j=1}^{J} q_j$ and total industry profits are $\Pi = (P-c)Q$.

In such an infinitely-repeated game, Friedman (1971) has shown that collusion at the monopoly price can be sustained as a subgame-perfect equilibrium using Nash-reversion trigger strategies, where firms revert to the Cournot-Nash equilibrium if any firm deviates from collusion, provided that the discount factor is sufficiently large.[4] Collusion at the monopoly price is sustainable if the present discounted value of profits from collusion, π^M, exceed the profits from deviating from collusion, π^D, followed by the Cournot-Nash equilibrium profits, π^N, forever thereafter. Thus, collusion at the monopoly price is sustainable if the discount factor exceeds the critical value, δ^*, defined as follows:

$$\frac{\pi^M}{1-\delta} > \pi^D + \frac{\delta \pi^N}{1-\delta} \qquad \Rightarrow \qquad \delta > \delta^* \equiv \frac{\pi^D - \pi^M}{\pi^D - \pi^N} \tag{1}$$

When the firms collude, they are assumed to behave as a monopolist and maximise total industry profits. Straightforward calculations yield the monopoly price, output and profits:

$$P^M = \frac{\eta c}{\eta - 1} \qquad Q^M = \left(\frac{\eta-1}{\eta c}\right)^{\eta} \qquad \Pi^M = \frac{1}{\eta}\left(\frac{\eta-1}{\eta c}\right)^{\eta-1} \tag{2}$$

Given that all firms are identical, it is natural to assume that all firms produce the same output when they collude so each firm produces $q^M = Q^M / J$ and earns profits equal to:

$$\pi^M = \frac{\Pi^M}{J} = \frac{1}{J\eta}\left(\frac{\eta-1}{\eta c}\right)^{\eta-1} \tag{3}$$

In the Cournot-Nash equilibrium, each firm maximises its profits given the outputs of the other firms and this leads to the following first-order conditions:

$$\frac{\partial \pi_j}{\partial q_j} = P + q_j P' - c = Q^{\frac{-1}{\eta}} - q_j \frac{1}{\eta} Q^{\frac{-\eta-1}{\eta}} - c = 0 \qquad j = 1, \ldots J \tag{4}$$

Summing over the J first-order conditions yields an equation that can be solved for total industry output and the market price. Thus, the market price, total industry output, and total industry profits in the Cournot-Nash equilibrium are:

$$P^N = \frac{J\eta c}{J\eta - 1} \qquad Q^N = \left(\frac{J\eta-1}{J\eta c}\right)^{\eta} \qquad \Pi^N = \frac{1}{J\eta}\left(\frac{J\eta-1}{J\eta c}\right)^{\eta-1} \tag{5}$$

Since all the firms have the identical and constant marginal cost, the Cournot-Nash equilibrium is symmetric with each firm producing $q^N = Q^N / J$ and earning profits equal to:

[4] For simplicity, only Nash-reversion trigger strategies will be considered in this note. The optimal penal code for sustaining collusion has been derived by Abreu (1986) and it would be interesting to analyse such strategies with constant elasticity demand functions.

$$\pi^N = \frac{\Pi^N}{J} = \frac{1}{J^2\eta}\left(\frac{J\eta-1}{J\eta c}\right)^{\eta-1} \tag{6}$$

For later reference, it is useful to calculate the ratio of Cournot-Nash equilibrium profits and the profits from collusion at the monopoly price:

$$\Omega = \frac{\pi^N}{\pi^M} = \frac{\Pi^N}{\Pi^M} = \frac{1}{J^\eta}\left(\frac{J\eta-1}{\eta-1}\right)^{\eta-1} \tag{7}$$

Note that this profit ratio does not depend upon the costs of the firms, but only on the number of firms and the elasticity of demand. Differentiating this profit ratio with respect to the elasticity of demand yields:

$$\frac{\partial\Omega}{\partial\eta} = \frac{-1}{J^\eta}\left(\frac{J\eta-1}{\eta-1}\right)^{\eta-1}\left[\frac{J-1}{J\eta-1} + \ln\left(1-\frac{J-1}{J\eta-1}\right)\right] > 0 \tag{8}$$

This can be signed by noting that the term in square brackets is negative since $x+\ln(1-x)<0$ for $0<x<1$. Therefore, as the elasticity of demand increases Cournot-Nash equilibrium profits increase relative to the profits from collusion at the monopoly price. Intriguingly, note that the limit of this profit ratio as the elasticity of demand tends to infinity is equal to $\exp(1-1/J)/J<1$, which implies that there are always gains from collusion (and losses from deviation) whatever the elasticity of demand.

If a firm deviates from collusion in any stage of the game then it maximises its profits in that stage given that all the other firms produce the collusive output, q^M. Therefore the first-order condition for profit maximisation by the deviating firm is:

$$\frac{\partial\pi_j}{\partial q_j} = P + q_jP' - c = \left[q_j + (J-1)q^M\right]^{\frac{-1}{\eta}} - q_j\frac{1}{\eta}\left[q_j + (J-1)q^M\right]^{\frac{-\eta-1}{\eta}} - c = 0 \tag{9}$$

The solution to this equation gives the output of the deviating firm, $q_j = q^D$, which can be used to calculate total industry output, market price, and the profits of the deviating firm, π^D. Unfortunately, this equation cannot be solved analytically as the situation is no longer symmetric, but it can be solved numerically using *Mathematica*, see Wolfram (1999). The marginal cost of the firms can be normalised to unity, $c=1$, without any loss of generality since the profit ratios: π^N/π^M, see (7), and π^D/π^M are independent of marginal cost so the critical discount factor (1) will be independent of costs.[5] Equation (9) can be solved for a given number of firms and a given value for elasticity of demand. The solution can then be used to solve for the profits of the deviating firm and this together with the profits from collusion (3) and the Cournot-Nash equilibrium profits (6) can be used to calculate the critical discount factor(1). Thus, as shown in figure 1, the critical discount factor can be plotted as a function of the elasticity of demand for given numbers of firms ($J = 2, 5, 20$). Figure 2 shows the critical discount factor as a

[5] Numerical solutions can be used to show that the second profit ratio is independent of costs.

function of the number of firms for given values of the elasticity of demand ($\eta = 1 \cdot 1, 5$). In both figures, it can be seen that the critical discount factor is decreasing in the elasticity of demand and increasing in the number of firms. Therefore, the larger is the elasticity of demand then the easier it is to sustain collusion at the monopoly price.

The explanation for this result can be understood if one realises that the elasticity of demand has three effects on the sustainability of collusion. Firstly, the larger is the elasticity of demand then the lower will be the price-cost margin when the firms collude at the monopoly price. This effect reduces the incentive for the firms to deviate from collusion as the elasticity of demand increases. Secondly, the larger is the elasticity of demand then the smaller will be the reduction in price if a firm deviates from collusion and increases its output. This effect increases the incentive to deviate from collusion as the elasticity of demand increases. Thirdly, as shown in (8) , the larger is the elasticity of demand then the larger will be the Cournot-Nash equilibrium profits relative to the profits from collusion at the monopoly price. This effect reduces the losses from reversion to the Cournot-Nash equilibrium following any deviation from collusion, and thereby increases the incentive to deviate from collusion as the elasticity of demand increases. It turns out that the first effect dominates the second and third effects so that the incentive to deviate from collusion decreases as the elasticity of demand increases. Therefore, the larger is the elasticity of demand then the easier it is to sustain collusion at the monopoly price.

3. Conclusions

This note has investigated the relationship between the elasticity of demand and the sustainability of collusion at the monopoly price. It has been shown that the larger is the elasticity of demand then the easier it is for firms to sustain collusion at the monopoly price. Despite using numerical solutions, the results obtained are unambiguous and seem to be quite general. Since firms are colluding at the monopoly price, the dominant effect that drives this result is that the larger is the elasticity of demand then the lower is the price-cost margin and the lower is the incentive to deviate from collusion. If one considered partial collusion where firms colluded at the same price whatever the elasticity of demand then the result will probably be reversed so that the larger is the elasticity of demand then the harder it is to sustain collusion.

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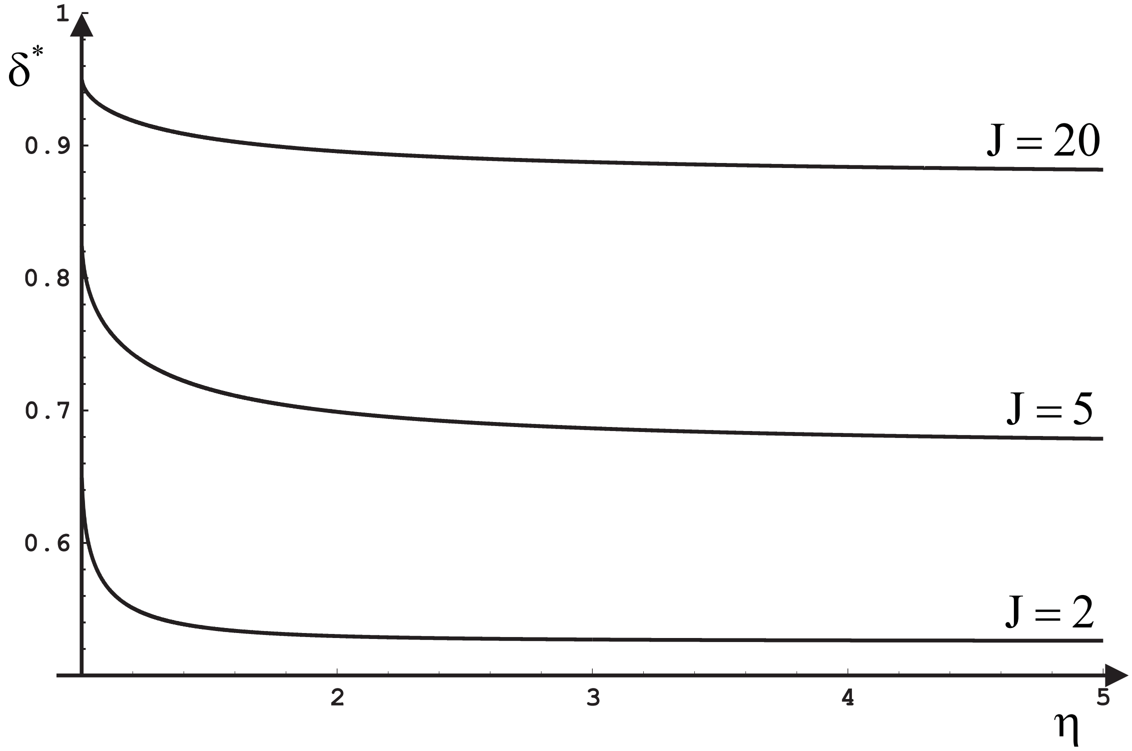

Figure 1: Critical discount factor and the elasticity of demand

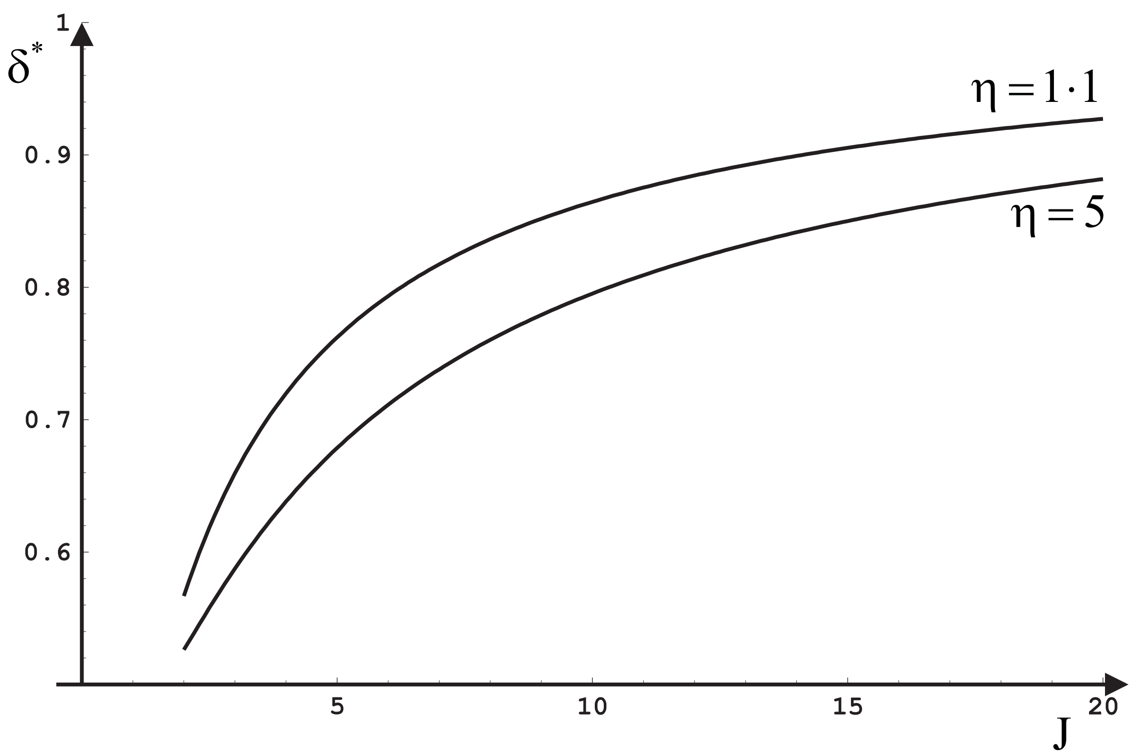

Figure 2: Critical discount factor and the number of firms