E C O N O M I C S B U L L E T I N

A Note on Cost−Reducing Alliances in Vertically Differentiated Oligopoly

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Abstract

In a vertically differentiated oligopoly, firms raise cost−reducing alliances before competing with each other. It is shown that heterogeneity in quality and in cost functions reduces individual incentives to form links. Furthermore, both differentiated Cournot and Bertrand competition create qualitatively similar incitations to form alliances.

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1 Introduction

A recent literature in industrial economics puts ahead strategic incentives for rival firms form R&D alliances. Yet, this literature misses to analyze the impact of firm heterogeneity on incentives to form links. To fill this lack is the aim of this note. We consider a vertically (and horizontally) differentiated oligopoly, in which firms form collaborative links before competition (Cournot and Bertrand) in order to reduce their production costs. In this context, we explore the role of heterogeneity in both marginal cost functions and qualities inside the industry on building-up of stable networks.

We first argue that these heterogeneities *reduce* individual incentives to link formation: two firms are incited to form a cost-reducing alliance if they are close enough with regard to some individual parameter, explicitly the quality index multiplied by the variation of marginal cost inherent to the formation of one link. Hence, an increase in the variance of this parameter induces less connected stable networks. Precisely, stable networks are *locally complete* with respect to the parameter mentioned just above. This means that, when ranking firms by increasing value of this parameter, whenever two firms both gain to form a link, this is also the case for any pair of firms ranked between them. The distribution of this parameter thus determines the network density. Secondly, we observe that differentiated Cournot and Bertrand oligopoly provide similar incentives of link formation. However, our study has several deficiencies. Firstly, our paper lacks a full analysis for large cost of link formation. Nevertheless, it is shown that under large costs of link formation, quality index and marginal cost function do not play a symmetric role in the shaping of stable networks, so that locally completeness vanishes. Secondly, we are unable to provide a satisfying social welfare analysis.

This note is related to the literature on the formation of R&D associations and is inspired by Bloch (1995) and Goyal and Joshi (2003). In both approaches firms form associations prior to competition in order to reduce their marginal costs. In the former they form coalition, in the latter they form bilateral agreements. With regard to Goyal and Joshi notably, we extend their benchmark case of homogenous oligopoly to differentiated (in product) oligopoly with heterogenous firms (in qualities and marginal costs). Our main related contribution is to see that only firms producing closely related products or facing similar production costs will form a collaborative link; hence, our context exhibits the possible emergence of stable networks containing *incomplete* components. The reason why is that heterogeneity implies asymmetric gains from link formation, in such a way that the hypothesis (SY1) presented by the authors does not hold (the condition states that two symmetric firms have always an incentive to form a link). Let us also note that the stable architecture described here is formally equivalent to the networks exhibited by

Johnson and Gilles (2000) in a context where agents are not rival.

The note is organized as follows. Section 2 presents the model and the specific architectures emphasized in further results. Section 3 is devoted to Cournot and section 4 to Bertrand competition.

2 The model

We consider an industry with $N \geq 2$ firms, each producing a differentiated good. Each product i is described by a quality index u_i. We denote by x_i the quantity of product i sold to each user. There are S identical consumers. Following Sutton (1997) and Symeonidis (2003), the utility function of each consumer is given by the following quality-augmented version of the standard quadratic utility function:

$$U = \sum_{i \in N} \left(x_i - \frac{x_i^2}{u_i^2} \right) - \sigma \sum_{i,j<i} \frac{x_i}{u_i} \frac{x_j}{u_j} + M$$

where $M = Y - \sum_i p_i x_i$ denotes expenditures on outside goods, and the parameter σ ($\in [0,2]$ for Cournot competition and $\in [0,2[$ for Bertrand competition) is an inverse measure of the degree of horizontal product differentiation. from this utility function we derive the following individual inverse demand function for variety i:

$$p_i = 1 - \frac{2x_i}{u_i^2} - \frac{\sigma}{u_i} \sum_{j \neq i} \frac{x_j}{u_j}$$

(in the region where prices and quantities are positive)
and the demand function for $\sigma \neq 2$:

$$\frac{x_i}{u_i} = \frac{(2 + \sigma(N-2))u_i(1-p_i) - \sigma \sum_{j \neq i} u_j(1-p_j)}{(2-\sigma)(2+\sigma(N-1))}$$

We suppose that firms can engage collaboration links prior to market competition, in order to decrease their marginal cost. We represent by a non directed graph the structure of links between the firms. A *graph* g is a pair (N, L) where N is a set of firms and L is a set of pairs of firms in N. We denote by G the set of all non directed graphs with N nodes. We shall abuse the notation by writing that some link $ij \in g$, which means that a link exists between the two agents in the graph g. Symbol $g - ij$ (resp. $g + ij$ denotes the graph g less (resp. augmented by) the link ij. A *path* in the graph g is a sequence of nodes $\{a_0 a_p\}$ such that $a_i a_{i+1} \in g$ for all $i \in \{0, \ldots, p-1\}$. A *component* of a graph g is a subgraph such that there is a path between any pair of agent in the component, and there is no link between any agent inside the component and any agent outside the component. Given a network g, we denote by $\pi_i(g)$ the profit made by firm i in this network.

The marginal cost of a firm in a network is decreasing with regard to the number of collaborative ties the firm is involved in. Linearity of marginal cost functions is crucial for uniqueness. We say that marginal costs are *firm-specific linear* when marginal cost functions are linear and cost variations (*w.r.t.* the creation of a new link) are heterogenous. Hence,

$$\forall i \in N, \forall g \in G, c_i = c_0 + \Delta_i \cdot \eta_i(g),\ \Delta_i < 0$$

We use the usual pairwise stability as defined in Goyal and Joshi (2003). A *stable* network g satisfies two conditions:

(i) for $ij \in g$, $\pi_i(g) > \pi_i(g - ij)$ and $\pi_j(g) > \pi_j(g - ij)$
(ii) for $ij \notin g$, if $\pi_i(g + ij) > \pi_i(g)$, then $\pi_j(g + ij) \leq \pi_j(g)$

Finally, we identify the network structures emphasized in this note. The *empty network* is such that no pair of nodes forms a link. The *complete network* is the graph such that all pairs of nodes form a link. A *complete component* is a component such that all pairs of nodes in the component form a link.
A τ*-vicinity network* is built as follows. Suppose that each node i is associated with a positive real number $r(i)$. Consider a nonnegative real number τ. Then build the network g such that a link $ij \in g$ if and only if $min(\frac{r(i)}{r(j)}, \frac{r(j)}{r(i)}) > \tau$. Hence, if $\tau > 1$, the network is the empty graph. If $\tau = 0$, the network is the complete graph. Between these two extreme situations, that is for $\tau \in]0, 1]$, all networks can be generated according to the distribution of numbers. The network structure described here is *locally complete*, as defined in Johnson and Gilles (2000). That is, labelling without loss the firms by increasing number of index $r(i)$, if firms i and j form a link, with say $i < j$, then the link kl is formed whatever k, l such that $i \leq k < l \leq j$.

3 Cournot competition

The following proposition applies:

Proposition 3.1 *In this differentiated oligopoly à la Cournot with firm-specific linear marginal costs, there is a unique stable network. Particularly, two firms i and j form a link if and only if*

$$min\left(\frac{u_i \cdot \Delta_i}{u_j \cdot \Delta_j}, \frac{u_j \cdot \Delta_j}{u_i \cdot \Delta_i}\right) > \xi^C$$

with $\xi^C = \frac{\sigma}{4+\sigma(N-2)} \in]0, 1[$. *Hence, the unique stable network is the* ξ^C*-vicinity network with* $r(i) = u_i \cdot |\Delta_i|$.

Proof. Given the shape of the inverse demand function, standard derivations implicate the following equilibrium quantities on any graph g:

$$\frac{x_i^*}{u_i} = \frac{u_i(1-c_i(\eta_i(g)))}{4-\sigma} + \sigma \cdot \frac{\sum_{k=1}^{N} u_k(c_k(\eta_k(g))-1)}{(4-\sigma)(4+\sigma(N-1))}$$

Let us consider a graph g and agents i and j such that $ij \notin g$. We compute the net benefit for agent i from forming a link with agent j. Straightforward calculation entails (Symbol $\backsim$ means proportional to):

$$x_i(g+ij) - x_i(g) \backsim$$

$$\frac{-u_i}{4-\sigma} \cdot \Big(c_i(\eta_i(g)+1) - c_i(\eta_i(g)) \Big) + \sigma \cdot \frac{u_i \Big(c_i(\eta_i(g)+1) - c_i(\eta_i(g)) \Big) + u_j \Big(c_j(\eta_j(g)+1) - c_j(\eta_j(g)) \Big)}{(4-\sigma)(4+\sigma(N-1))}$$

Since quantities are positive, the formation of the link is profitable for agent i if and only if

$$\frac{u_i \cdot \big(c_i(\eta_i(g)) - c_i(\eta_i(g)+1)\big)}{u_j \cdot \big(c_j(\eta_j(g)) - c_j(\eta_j(g)+1)\big)} > \frac{\sigma}{4+\sigma(N-2)}$$

Then, if marginal costs are firm-specific linear, the condition becomes

$$\frac{u_i \cdot \Delta_i}{u_j \cdot \Delta_j} > \frac{\sigma}{4+\sigma(N-2)}$$

which is independent of the network structure. This ensures existence and uniqueness of the stable networks. **Q.E.D.**

Remarks:
1. Since $\xi^C \in]0,1[$, the complete network is not uniquely stable in general. and heterogeneity may lead to the emergence of stable *incomplete* components.
2. When integrating large costs of link formation, a major difference is that quality index and marginal cost functions do not play a symmetric role when integrating costs of link formation. Let us examine the homogenous case ($\sigma = 2$). Denoting by $f > 0$ the cost of link formation, straightforward computation indicates that the link ij is profitable to agent i in the graph g iff

$$u_i^2 \cdot \Gamma_{ij} \left[(N+1)u_i(1-c_i(g)) + \frac{\Gamma_{ij}}{2} - \sum_{p=1,\cdots,N} u_p(1-c_p(g)) \right] > (N+1)f$$

with $\Gamma_{ij} = Nu_i|\Delta_i| - u_j|\Delta_j|$. If $f = 0$, the sole condition $\Gamma_{ij} > 0$ is sufficient, and when ordering agents by increasing value of parameter $u|\Delta|$, if $u_i|\Delta_i| \leq u_k|\Delta_k| < u_l|\Delta_l| \leq u_j|\Delta_j|$, then $\Gamma_{ij} > 0$ implies $\Gamma_{kl} > 0$, which induces the emergence of locally complete networks. If $f > 0$, the above condition includes individual characteristics that can impede locally completeness.

3. The positivity of equilibrium quantities write for each firm i:

$$\frac{u_i(1-c_i(\eta_i(g)))}{\sum_{j\neq i} u_j(1-c_j(\eta_j(g)))} \geq \frac{\sigma}{4+\sigma(N-2)}$$

We note that $p_i^* > 0$ if and only if $y_i^* > \frac{-c_i(\eta_i)u_i}{2}$. Then, if the equilibrium quantity is positive, the equilibrium price is also positive. Note first that when $\sigma = 0$, the condition is automatically satisfied. Basically, the more differentiated the products, the less constraining the condition of positivity. Second, when $\sigma > 0$, the positivity of equilibrium quantities does not imply the condition of profitability of link formation, nor the converse. The worst situation with respect to incentive of link formation is attained for $\sigma = 2$. In this case, the right hand side of the above equation equals $\frac{1}{N-1}$; hence a N-firms network is viable iff for all pair of firms (i, j), $u_i(1 - c_i(\eta_i(g))) = u_j(1 - c_j(\eta_j(g)))$. For instance, when firms are homogenous and $\sigma = 2$, then the positivity is true if and only if the graph is symmetric, *i.e.* such that all firms have the same number of partners.

4 Bertrand competition

Bertrand competition entails the same stable network structures.

Proposition 4.1 *In this differentiated oligopoly à la Bertrand with firm-specific linear marginal costs, there is a unique stable network. Particularly, two firms i and j form a link if and only if*

$$min\left(\frac{u_i\cdot\Delta_i}{u_j\cdot\Delta_j}, \frac{u_j\cdot\Delta_j}{u_i\cdot\Delta_i}\right) > \xi^B$$

$$with\ \xi^B = \frac{\sigma(2+\sigma(N-2))}{(4+\sigma(N-3))(4+\sigma(2N-3))-(4+\sigma(N-2))(2+\sigma(N-2))}$$

Hence, the unique stable network is the ξ-vicinity network with $r(i) = u_i \cdot |\Delta_i|$.

Proof. The profit expression of firm i on graph g is:

$$\pi_i(g) = \frac{Su_i\big(p_i - c_i(\eta_i(g))\big)\Big((2+\sigma(N-2))u_i(1-p_i) - \sigma\sum_{j\neq i} u_j(1-p_j)\Big)}{(2-\sigma)(2+\sigma(N-1))}$$

Standard derivations enable to determine the price equilibrium:
$p_i^*(g)\cdot u_i(4+\sigma(2N-3))(4+\sigma(N-3)) = (4+\sigma(N-3))(2+\sigma(N-1))u_i$
$-\sigma(2+\sigma(N-2))\sum_{k=1}^N u_k$
$+(2+\sigma(N-2))\big[(2+\sigma(N-3))u_ic_i(\eta_i(g)) + \sigma\sum_{k=1}^N u_kc_k(\eta_k(g))\big]$
Thus, for graph g and link $ij \notin g$, and noting that $\pi_i \backsim (p_i - c_i)^2$:
$\pi_i^*(g+ij) - \pi_i^*(g) \backsim$

$\frac{(2+\sigma(N-2))}{u_i(4+\sigma(N-3))(4+\sigma(2N-3))} \cdot \Bigg[(4+\sigma(N-2))u_i\Bigg(c_i(\eta_i(g)+1)-c_i(\eta_i(g))\Bigg)+\sigma u_j\Bigg(c_j(\eta_j(g)+1)-c_j(\eta_j(g))\Bigg)\Bigg] - \Bigg(c_i(\eta_i(g)+1)-c_i(\eta_i(g))\Bigg)$

Considering firm-specific linear marginal costs, the result on uniqueness follows. To finish, a little calculus indicates that $\xi^B > 0$ as soon as $N > 0$. Indeed, the denominator is a polynomial expression of order 2 in N. When $\sigma > \frac{2}{5}$, there is no value of N that makes the expression null (then it is always positive). When $\sigma \leq \frac{2}{5}$, the polynomial is negative between two values of N, the greatest of which being negative whatever value of $\sigma \in [0, \frac{2}{5}]$.
Q.E.D.

Remarks:
1. Bertrand competition creates less incentive to link formation than Cournot competition since $\xi^B > \xi^C$ for all $N > 0$.
2. Under firms heterogeneity, the condition $(SY1)$ as expressed in Yi (1998) or Goyal and Joshi (2003) is not valid.
3. Goyal and Joshi (2003) show that Cournot competition entails drastically different incentives of link formation than Bertrand competition. Our study suggests that firms heterogeneity may qualify these differences in the context of differentiated goods.
4. The positivity conditions write for each agent i:

$$\frac{u_i(1-c_i(\eta_i(g)))}{\sum_{j\neq i} u_j(1-c_j(\eta_j(g)))} \geq \frac{\sigma(2+\sigma(N-2))}{(4+\sigma(N-3))(2+\sigma(N-1))-\sigma(2+\sigma(N-2))}$$

A similar discussion than that of the Cournot case applies. We note that when σ tends to 2 the value of the right hand side of the above equation tends to $\frac{2}{2N-3}$, which is less than $\frac{1}{N-1}$. This means that the system is not viable for nearly homogenous Bertrand oligopoly (there exists at least one agent such that the condition does not hold). Hence, 'almost' homogenous Bertrand oligopoly is incompatible with bilateral agreements.

References

Bloch, F., 1995, Endogenous structures of association in oligopolies, *Rand Journal of Economics*, **26**, 537-556.

Goyal, S. and S. Joshi, 2003, Networks of collaboration in oligopoly, forthcoming in *Games and Economic Behavior.*

Johnson, C. and R. Gilles, 2000, Spatial social networks, *Review of Economic Design*, **5**, 273-299.

Sutton, J., 1997, One smart agent, *Rand Journal of Economics*, **28**, 605-628.

Symeonidis, G., 2003, Quality heterogeneity and welfare, *Economics Letters*, **78**, 1-7.

Yi, S., 1998, Endogenous formation of joint ventures with efficiency gains, *Rand Journal of Economics*, **29**,610-631.