E C O N O M I C S   B U L L E T I N

Risk neutral valuation and uncovered interest rate parity in a stochastic two−country−economy with two goods

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Citation: Costa, Vincenzo, (2004) "Risk neutral valuation and uncovered interest rate parity in a stochastic two−country−economy with two goods." *Economics Bulletin*, Vol. 3, No. 43 pp. 1−10
Submitted: July 31, 2004. **Accepted:** November 4, 2004.
URL: http://www.economicsbulletin.com/2004/volume3/EB−04C00007A.pdf

RISK NEUTRAL VALUATION AND UNCOVERED INTEREST RATE PARITY IN A STOCHASTIC TWO-COUNTRY-ECONOMY WITH TWO GOODS

Vincenzo Costa*

Abstract. In this paper we consider a model of a stochastic two-country economy and we use the martingale properties, the change of the numéraire technique and the risk neutral evaluation for achieving some important relations between interest rate of two markets, in particular the so-called uncovered interest rate parity.

keywords : change of the numéraire, equivalent martingale measure, interest rate parity

AMS subject classification: 91B28, 91B70, 60G46, 60H10, 60H30
JEL subject classification: C65, C69, D51, E43, G12

1. Introduction.

In this paper we consider a two-country-economy with two consumption goods in a such way that each of them constitues the local numéraire, for respective country.

In the two markets, which are analysed along a finite interval of time $[0, T]$, there are two non-risky assets (the saving accounts), two zero coupon bonds and two productive assets. In particular, the possession of a part of unique share for each of two productive assets gives the right to receive a stream of dividends, expressed in unit of goods.

The possibility to value the foreign financial and productive assets is made possible by using the stochastic exchange rate which is the key quantity for our set-up.

Besides, the presence of the financial assets (risky and non-risky) makes both of the two markets complete so that it is possible to define a risk-premium for each market and to construct an equivalent martingale measure (EMM) in each country (which is the risk neutral one).

Making a change of a probability, which is a change of numéraire for a financial point of view, it is possible to obtain some relation between the interest (non-risky)-rates of the two country, in particular the most important one we want to achieve: the uncovered interest rate parity (UIRP).

We can show also a relation for judging the usefulness to invest or not in the own foreign country, under a risk neutral valuation.

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We point out that the central aim of this paper is to give the basis and some important relations between interest rates, for a maximization and an equilibrium problem (making use of Karatzas et al.'s technique, see Karatzas et al, 1990) for two agents, the domestic one and the foreign one, in a model of the kind of that considered here.

The paper is organized as follows: in section 2 we give our model and we construct the equivalent martingale measure with relative risk-neutral probability space, for each country. In section 3, we obtain the UIRP while in section 4 we show the relation mentioned before.

2. The model

Our model represents a stochastic two-country-economy, in the absence of arbitrage (NFL i.e. no-free-lunch), in which there are two agents - Domestic (D) and foreign (F) - who can consume two different goods (D and F). We have to point out that these goods are not storable (and so they are perishable). Besides, the agents can buy and sell, import and export them. In our context, there are two productive assets (D and F) whose, resp., dividends (D and F) give our agents the goods they consume along the finite horizon of our analysis. In the market, we can find also two saving accounts (D and F) (with the concerning risk-less interest rates) and two zero coupon bonds (D and F). In the end, there is a variable, which is crucial for us: the exchange rate. We specify that we consider a moneyless model. In each country, the numéraire is the local consumption good and we suppose existence and uniqueness of solution of all stochastic differential equations we will write.

As we said before, the analysis of agents' behaviour is made in a finite interval of time $[0, T]$ with $T < +\infty$. The economy lives on a probability space $(\Omega, \mathcal{F}, P)$ which is complete and we assume that an exogenous standard 5-dimensional Wiener process $\{W_t := (W_1, W_2, W_3, W_4, W_5)^\top\}_{t\in[0,T]}$ is defined on it. The space has a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ which is generated by W and which is augmented and completed. Doing so, it satisfies the so-called "usual hypothesis" (see Protter, 1986) for more details). In this context, the brownian motion represents the (only) source of randomness (and then of risk) of our model.

More in details, we give now the equations for our variables, expressed in units of own consumption good and written for $t \in [0, T]$.

The flow of "**dividends**" generated by domestic (resp. foreign) productive assets satisfies $(i = D, F)$

$$d\delta_{i,t} = \delta_{i,t}[\mu_{\delta(i),t}dt + \sigma_{\delta(i),t}dW_t] \quad \text{with} \quad \delta_{i,0} = \bar{\delta}_{i,0} \ , \tag{2.1}$$

with relative "**productive assets**" price per share ex-dividend $(i = D, F)$

$$dA_{i,t} + \delta_{i,t}dt = A_{i,t}[\mu_{A(i),t}dt + \sigma_{A(i),t}dW_t] \quad \text{with} \quad A_{D,T} = 0 \ , \tag{2.2}$$

Regarding (2.2), we observe that it is a Backward Stochastic Differential Equation ($BSDE$ - see Antonelli, 1993 and El Karoui et al, 1990 for their theory and applications in economy and finance). The meaning of its final condition is that productive assets have paid out all their dividends in T and so they have no further value.

The "**saving accounts**" (which are no-risky asset in own country), instead, satisfies ($i = D, F$)

$$df_{i,t} = r_{i,t} f_{i,t} dt \quad \text{with} \quad f_{i,0} = 1_i \ . \tag{2.3}$$

where r_i is the "**risk-less interest rate**" of country i while 1_i means 1 unit of consumption goods in the country i.

Besides, the "**zero coupon bonds**" (ZCB), with maturity T follows another $BSDE$

$$dB_{i,t,T} = B_{i,t,T}[\mu_{B(i),t,T} dt + \sigma_{B(i),t,T} dW_t] \quad \text{with} \quad B_{i,T,T} = 1_i \ . \tag{2.4}$$

Finally, we have the equation of "**exchange rate (foreign vs domestic)**":

$$dq_{D,t} = q_{D,t}[\mu_{q(D),t} dt + \sigma_{q(D),t} dW_t] \quad \text{with} \quad q_{D,0} = \bar{q}_{D,0} \ , \tag{2.5}$$

where we have the equivalence

$q_{D,t}$ units of domestic consumption good

= 1 unit of foreign consumption good at t

For the case domestic vs foreign, the exchange rates follows an equivalent equation

$$dq_{F,t} = q_{F,t}[\mu_{q(F),t} dt + \sigma_{q(F),t} dW_t] \quad \text{with} \quad q_{F,0} = \bar{q}_{F,0} \ . \tag{2.6}$$

Remark 2.1: *We observe that our hypothesis of absence of arbitrage in the domestic market is legitimate by the number of risky-assets (which includes the risk-less asset of the foreign country) while the completeness is guaranteed by this number plus the presence of the non-risky-asset. The same is true for foreign market.*

Lemma 2.2: *Given the equations (2.5) and (2.6), we have* $\forall t \quad$ *a.s.:*

$$q_{D,t} \cdot q_{F,t} = 1 \tag{2.7}$$

$$\sigma_{q(D),t} + \sigma_{q(F),t} = 0 \tag{2.8}$$

$$\mu_{q(D),t} + \mu_{q(F),t} = \|\sigma_{q(D),t}\|^2 \tag{2.9}$$

Proof: The (2.7) is an immediate consequence of financial considerations.

If we apply Itô's lemma, with function $f(x, y) := xy$, to $x := q_{D,t}$ and $y := q_{F,t}$, we obtain

$$\begin{aligned} d(q_D q_F)_t &= q_{D,t} dq_{F,t} + q_{F,t} dq_{D,t} + d < q_D, q_F >_t \\ &= (q_{D,t} q_{F,t})\{[\mu_{q(D),t} + \mu_{q(F),t} + \sigma_{q(D),t} \sigma^\top_{q(F),t}]dt \\ &+ [\sigma_{q(D),t} + \sigma_{q(F),t}]dW_t\} \ . \end{aligned}$$

From that and since (2.7) implies

$$d(q_D q_F)_t = 0 \ , \tag{2.10}$$

we have

$$\sigma_{q(D),t} + \sigma_{q(F),t} = 0$$

i.e. the (2.8). This relation gives $\sigma_{q(F),t} = -\sigma_{q(D),t}$ and then using (2.10) too, we can write

$$\begin{aligned} 0 &= \mu_{q(D),t} + \mu_{q(F),t} + \sigma_{q(D),t}\sigma^{\top}_{q(F),t} \\ &= \mu_{q(D),t} + \mu_{q(F),t} + \sigma_{q(D),t}[-\sigma^{\top}_{q(D),t}] \\ &= \mu_{q(D),t} + \mu_{q(F),t} - \|\sigma_{q(D),t}\|^2 . \end{aligned}$$

We can conclude achieving the (2.10) thanks to this chain of equalities. $\square$

Lemma 2.3: *It is possible to rewrite the equations of A_F, f_F, δ_F, B_F in terms of units of domestic consumption good:*

$$\begin{aligned} d(q_D A_F)_t &= (q_{D,t}A_{F,t})\{[\mu_{q(D),t} + \mu_{A(F),t} + \sigma_{q(D),t}\sigma^{\top}_{A(F),t}]dt \qquad (2.11) \\ &\qquad + [\sigma_{q(D),t} + \sigma_{A(F),t}]dW_t\} \ , \\ d(q_D f_F)_t &= (q_{D,t}f_{F,t})\{[\mu_{q(D),t} + r_{F,t}]dt + \sigma_{q(D),t}dW_t\} \ , \qquad (2.12) \\ d(q_D \delta_F)_t &= (q_{D,t}\delta_{F,t})\{[\mu_{q(D),t} + \mu_{\delta(F),t} + \sigma_{q(D),t}\sigma^{\top}_{\delta(F),t}]dt \qquad (2.13) \\ &\qquad + [\sigma_{q(D),t} + \sigma_{\delta(F),t}]dW_t\} \ , \\ d(q_D B_F)_{t,T} &= (q_{D,t}B_{F,t,T})\{[\mu_{q(D),t} + \mu_{B(F),t,T} + \sigma_{q(D),t}\sigma^{\top}_{B(F),t,T}]dt \qquad (2.14) \\ &\qquad + [\sigma_{q(D),t} + \sigma_{B(F),t,T}]dW_t\} \ . \end{aligned}$$

All equations are taken with the correspondent initial or final conditions.

Proof: It is sufficient to use the exchange rate and Itô 's rule, with the same function and in a equivalent manner as in previous lemma. Besides, it needs to remember that $\sigma_{f(F),t} = 0$ and $\mu_{f(F),t} = r_{F,t}$, for writing (2.12). $\square$

Let us introduce, now, the 5×5 "**volatility matrixes**" (in order to simplify our notation, we will omit dependence on T):

Definition 2.4:

$$\Sigma_{D,t} := \begin{pmatrix} \sigma_{A(D),t} \\ \sigma_{A(F),t} + \sigma_{q(D),t} \\ \sigma_{q(D),t} \\ \sigma_{B(D),t} \\ \sigma_{B(F),t} + \sigma_{q(D),t} \end{pmatrix} \quad \textit{and} \quad \Sigma_{F,t} := \begin{pmatrix} \sigma_{A(F),t} \\ \sigma_{A(D),t} + \sigma_{q(F),t} \\ \sigma_{q(F),t} \\ \sigma_{B(F),t} \\ \sigma_{B(D),t} + \sigma_{q(F),t} \end{pmatrix} . \qquad (2.15)$$

We are ready to make our hypotheses on the model

Hypothesis 2.5: *a) For $i = D, F \quad \exists \epsilon_i > 0$ s.t. for $(t,\omega) \in [0,T] \times \Omega$:*

$$\Sigma_{i,t}\Sigma^{\top}_{i,t} \geq \epsilon_i \mathbf{I}_5 \qquad (2.16)$$

where $\top$ means transposition operator and $\mathbf{I}_5$ means identity matrix of order 5;

b) for $i = D, F$: $\quad r_{i,t}$, $\mu_{B(i),t}$, $\mu_{\delta(i),t}$, $\mu_{A(i),t}$, $\mu_{q(i),t}$, $\mu_{Q(i),t}$, one-dimensional drifts, $\Sigma_{i,t}$ matrixes 5×5, $\sigma_{Q(i),t}$ vectors 1×5 are processes bounded uniformly and $\{\mathcal{F}_t\}_{t \geq 0}$-adapted;

c) For $i = D, F$:

$$\int_0^T [\nu^2_{A(i),t} + \|\sigma_{A(i),t}\|^2] dt < +\infty \qquad \text{a.e.} \tag{2.17}$$

d) for $i = D, F$: $\Sigma_{i,t}$, $r_{i,t}$, $\mu_{q(i),t}$ are <u>endogenous</u> ;
e) for $i = D, F$: $\mu_{\delta(i),t}$, $\sigma_{\delta(i),t}$ are <u>exogenous</u> .

Remark 2.6: *By hypothesis 2.5-a, we have that $\Sigma_{D,t}$ and $\Sigma_{F,t}$ are a.e. invertible.*
Besides, it is possible to show (see Karatzas et al, 1986) that $\forall \xi \in \mathcal{R}^5$, $\forall (t,\omega) \in [0,T] \times \Omega$

$$\|[\Sigma^\top_{i,t}]^{-1}\xi\| \le \frac{1}{\sqrt{\epsilon_i}}\|\xi\| \qquad\qquad \|[\Sigma_{i,t}]^{-1}\xi\| \le \frac{1}{\sqrt{\epsilon_i}}\|\xi\| \; .$$

Now, let us introduce

Definition 2.7: *The 5×1 "**appreciation rate vectors**" is defined by:*
a) in the domestic market

$$b_{D,t} := \begin{pmatrix} \mu_{A(D),t} \\ \mu_{A(F),t} + \mu_{q(D),t} + \sigma_{A(F),t}\sigma^\top_{q(D),t} \\ r_{F,t} + \mu_{q(D),t} \\ \mu_{B(D),t} \\ \mu_{B(F),t} + \mu_{q(D),t} + \sigma_{B(F),t}\sigma^\top_{q(D),t} \end{pmatrix} \tag{2.18-a}$$

b) in the foreign market

$$b_{F,t} := \begin{pmatrix} \mu_{A(F),t} \\ \mu_{A(D),t} + \mu_{q(F),t} + \sigma_{A(D),t}\sigma^\top_{q(F),t} \\ r_{D,t} + \mu_{q(F),t} \\ \mu_{B(F),t} \\ \mu_{B(D),t} + \mu_{q(F),t} + \sigma_{B(D),t}\sigma^\top_{q(F),t} \end{pmatrix} . \tag{2.18-b}$$

So far, we can show the existence of "**risk-premium**" or, equivalently, "**market price of risk**".

Proposition 2.8: *For $i = D, F$: in the market i, there exists a unique 5×1-vector $\theta_{i,t}$ which is the solution of the following equation*

$$\Sigma_{i,t}\theta_{i,t} = b_{i,t} - r_{i,t}\mathbf{1}_5, \tag{2.19}$$

where $\mathbf{1}_5$ indicates the vector of dimension 5×1 with 1 on each component.

PROOF: Thanks to invertibility of $\Sigma_{i,t}$, we can define $\theta_{i,t}$ as

$$\theta_{i,t} := (\Sigma_{i,t})^{-1}[b_{i,t} - r_{i,t}\mathbf{1}_5] \qquad \square \tag{2.20}$$

Remark 2.9: *For $i = D, F$: in the market i, always since $\Sigma_{i,t}$ is invertible and, besides, for the boundeness of coefficients we use in definition of $\theta_{i,t}$, we can say that $\|\theta_{i,t}\|$ is bounded uniformly in $(t, \omega) \in [0, T] \times \Omega$. Then, by Novikov's condition, the exponential process*

$$Z_{i,t} := exp[-\int_0^t \theta_{i,s} dW_s - \frac{1}{2}\int_0^t \|\theta_{i,s}\|^2 ds] \qquad t \in [0, T] \tag{2.21}$$

is a martingale. Therefore, if we put

$$P_i(A) := E[Z_{i,T}\mathbf{1}_A] \quad \text{with } A \in \mathcal{F}_T \tag{2.22}$$

*(where $\mathbf{1}_A$ is indicator function of (measurable) set A), we get a probability measure equivalent to P - a so-called "**equivalent martingale measure of P**" (EMM) -. In particular, we can specify that this probability is unique since the historical market is complete and so it is "**the risk-neutral probability**". Besides, P_D has a Radon-Nikodym derivative w.r. to P given by*

$$\frac{dP_i}{dP} \restriction_{\mathcal{F}_t} = Z_{i,t} \qquad t \in [0, T].$$

As consequence (by Girsanov's theorem), the process

$$W_{i,t} := W_t + \int_0^T \theta_{i,s} ds \qquad t \in [0, T] \tag{2.23}$$

*is the standard brownian motion under probability P_i. The relative market, besides, is "**standard**" in Karatzas's sense. In the end, we have also the "**deflator**" (which can be defined thanks to completeness of the market i):*

$$\zeta_{i,t} := Z_{i,t} \cdot exp[-\int_0^t r_{i,s} ds] \ . \tag{2.24}$$

3. The uncovered interest rate parity

We can proof some results about the interest rates, which connect them to risk-premia.

Theorem 3.1: *$\theta_{D,t}$ and $\theta_{F,t}$ satisfy, resp.,*

$$\mu_{q(D),t} = r_{D,t} - r_{F,t} + \sigma_{q(D),t}\theta_{D,t} \tag{3.1}$$
$$\mu_{q(F),t} = r_{F,t} - r_{D,t} + \sigma_{q(F),t}\theta_{F,t} \quad . \tag{3.2}$$

Proof: Let us consider 3-td row of equation which defines $\theta_{D,t}$:

$$\sigma_{q(D),t}\theta_{D,t} = r_{F,t} + \mu_{q(D),t} - r_{D,t} \ .$$

This relation implies obviously (3.1) . We can proceed in a similar way for (3.2). $\square$

Corollary 3.2: *Under risk-neutral valuation, we have the "***uncovered interest rates parity***"*

$$r_{D,t} = r_{F,t} + \mu_{q(D),t} \qquad \textit{in the domestic market (i.e. under } P_D\textit{);} \tag{3.3}$$
$$r_{F,t} = r_{D,t} + \mu_{q(F),t} \qquad \textit{in the domestic market (i.e. under } P_F\textit{).} \tag{3.4}$$

Proof: It suffices to recall relations (3.1) and (3.2) and to remark that, under the risk-neutral probability, the (corrispondent) risk-premium becomes zero. □

Remark 3.3: *Relation (3.3) implies that the risk-less domestic interest rate should be higher (resp. lower) than the risk-less foreign interest rate, by an amount equal to depreciation (resp. appreciation) of domestic currency (which is, in our case, the unit of domestic consumption good). See Copeland, 1994, regarding this comment. Mutatis mutandis, the same is true for (3.4).*

4. Other important relations between interest rates

Now, we can arrive to the relevant

Theorem 4.1: *Under P_D, we have*

$$\sigma_{q(D),t}\theta_{F,t} = -\sigma_{q(D),t}\sigma^{\top}_{q(D),t} \quad , \tag{4.1}$$
$$Pr_{\sigma_{q(D),t}}(\theta_{F,t}) = -\sigma^{\top}_{q(D),t} \quad , \tag{4.2}$$
$$Pr_{\sigma_{q(D),t}}(\theta_{F,t}) = \sigma^{\top}_{q(F),t} \quad , \tag{4.3}$$

(i.e. the orthogonal projection of $\theta_{F,t}$ on $\sigma_{q(D),t}$ is $\sigma^{\top}_{q(F),t}$) .

Proof: - By virtue of (3.1) and (3.2), we get, resp.,

$$r_{D,t} - r_{F,t} = \mu_{q(D),t} - \sigma_{q(D),t}\theta_{D,t} \tag{4.4}$$

$$r_{D,t} - r_{F,t} = -\mu_{q(F),t} + \sigma_{q(F),t}\theta_{F,t} \ . \tag{4.5}$$

If we make equal the right-hand sides of (4.4) and (4.5), we can say that

$$\mu_{q(D),t} - \sigma_{q(D),t}\theta_{D,t} = -\mu_{q(F),t} + \sigma_{q(F),t}\theta_{F,t} \ .$$

This relation gives us

$$\sigma_{q(D),t}\theta_{D,t} + \sigma_{q(F),t}\theta_{F,t} = \mu_{q(D),t} + \mu_{q(F),t} \ . \tag{4.6}$$

If we recall (2.8) and (2.9), we can claim that

$$\sigma_{q(F),t} = -\sigma_{q(D),t} \tag{4.7}$$

and

$$\mu_{q(D),t} + \mu_{q(F),t} = \sigma_{q(D),t}\sigma^{\top}_{q(D),t} \ . \tag{4.8}$$

Now, if we substitute (4.7) and (4.8) in (4.6), we obtain

$$\sigma_{q(D),t}\theta_{D,t} - \sigma_{q(D),t}\theta_{F,t} = \sigma_{q(D),t}\sigma_{q(D),t}^{\top} \ .$$

Moreover,

$$\sigma_{q(D),t}(\theta_{F,t} - \theta_{D,t}) = -\sigma_{q(D),t}\sigma_{q(D),t}^{\top} \ .$$

Under P_D, $\theta_{D,t} = 0$, then the above equality implies (4.1) . Besides, the orthogonal projection of $\theta_{F,t}$ on $\sigma_{q(D),t}$ is

$$Pr_{\sigma_{q(D),t}}(\theta_{F,t}) = \frac{\sigma_{q(D),t} \cdot \theta_{F,t}}{\sigma_{q(D),t} \cdot \sigma_{q(D),t}^{\top}}\sigma_{q(D),t}^{\top}$$

and so, from (4.1), just proved, we arrive to (4.2) after a simple substitution. In the end, we can deduce (4.3) from (4.2) and (4.7) . □

Remark 4.2: *Financial interpretation of relation (4.3) is very interesting.*

In order that the agent may judge remunerative (i.e. acceptable w.r. to his/her aim: to maximize his/her expected utility) to invest into domestic market by a valuation under domestic risk-neutral environment, he/she must receive a foreign risk-premium (i.e. on own market) equal, by projection, to the volatility of q_F, the exchange (domestic vs foreign) rate. There is a lot of sense in this behaviour, because he/she invests (also) into domestic market but, in the end, he/she values everything in own market. In other words, he/she has to be able hedging himself/herself from exchange rate risk.

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