E C O N O M I C S   B U L L E T I N

The effects of additive outliers on stationarity tests: a monte carlo study

Olivier Darné
LAMETA−CNRS, Faculty of Economics, University of Montpellier I

Abstract

Monte Carlo simulations are used to study the size and power properties of two stationarity tests developed by Kwiatkowski et al. (1992) [KPSS] and Leybourne and McCabe (1994) [LMC] when the data contain additive outliers. We show that the KPSS tests are very robust to additive outliers whereas the LMC test exhibits size distorsions and loss of power.

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1 Introduction

Since Nelson and Plosser (1982), substantial literature has developed on the nature of the trend (deterministic or stochastic) in the economic times series from the unit root tests. These tests test the null hypothesis of a difference stationary (or I(1)) process against the alternative of a level stationary (or I(0)) process.

In practice, time-series data often contain aberrant observations referred to as outliers, in particular the additive outliers (AO's). Such AO's affect observations in isolation due to some nonrepetitive events and may occur as a results of measurement errors or economic, political and financial events such as oil shocks, wars, financial crashes and changes in policy regimes. In recent studies, Franses and Haldrup (1994) and Shin *et al.* (1996) established that the presence of additive outliers in a univariate time series disturbs the properties of Dickey-Fuller (1979) unit root tests[1]. Indeed, the presence of additive outliers induces in the errors a negative moving-average component which causes the unit root tests to exhibit substantial size distorsions towards rejecting the null hypothesis too often (Vogelsang, 1999).

Recently, there has been increasing interest in tests of the null hypothesis of level stationary (or I(0)) process against the alternative of a difference stationary (or I(1)) process. These tests are widely used in empirical applications, notably as complements to tests of the unit root hypothesis (Caner and Kilian, 2001) in order to obtain more robust results.
Therefore, we study the effects of additive outliers on the two most widely used stationarity tests developed by Kwiatkowski *et al.* (1992) and Leybourne and McCabe (1994) from simulation experiments.

2 Stationarity Tests

The two most widely used tests of the I(0) null hypothesis are due to Kwiatkowski *et al.* (1992) [KPSS] and to Leybourne and McCabe (1994) [LMC]. These two tests differ in how they account for serial correlation under the null. Whereas the KPSS test employs a nonparametric correction similar to the Phillips-Perron test, the LMC test allows for additional autoregressive lags similar to the augmented Dickey-Fuller test.

2.1 Leybourne-McCabe test

Leybourne and McCabe (1994) consider the generalized local-level model

1. Shin *et al.* (1996) also studied the innovational outliers and showed that they did not affect the Dickey-Fuller unit root tests

$$\Phi(L)y_t = \alpha_t + \beta t + \varepsilon_t \quad (1)$$
$$\alpha_t = \alpha_{t-1} + \nu_t, \qquad \alpha_0 = \alpha, \qquad t = 1,\ldots,T \quad (2)$$

where $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ is a pth-order autoregressive polynomial in the lag operator L with roots outside the unit circle, $\varepsilon_t \sim$ iid $(0,\sigma_\varepsilon^2)$ and $\nu_t \sim$ iid $(0,\sigma_\nu^2)$, and ε_t and ν_t being mutually independent. This model can be shown to be second-order equivalent in moments to the ARIMA(p,0,0) process

$$\Phi(L)(1-L)y_t = \beta + (1-\theta)\xi_t \qquad 0 < \theta \leq 1 \quad (3)$$

with $\xi_t \sim$ iid $(0,\sigma_\xi^2)$, $\sigma_\xi^2 = \sigma_\varepsilon^2\theta^{-1}$, and θ related to σ_ν^2 according to $\theta = (r+2-(r^2+4r)^{1/2})/2$, r being the signal-to-noise ratio $r = \sigma_\nu^2/\sigma_\varepsilon^2$. Here, $\sigma_\nu^2 = 0$ implies $\theta = 1$ and $\sigma_\nu^2 > 0$ implies $0 < \theta < 1$. Thus, stationarity test is concerned with the null hypothesis $H_0 : \sigma_\nu^2 = 0$, i.e. y_t follows a trend-stationary ARIMA(p,0,0) process, and the alternative $H_1 : \sigma_\nu^2 > 0$, i.e. y_t is I(1) and follows an ARIMA(p,1,1) process.

To implement the LMC test we construct the series

$$y_t^* = y_t - \sum_{i=1}^{p} \phi_i^* y_{t-i} \quad (4)$$

where the θ_i^* are the maximum likelihood estimates of θ_i from the fitted ARIMA(p,1,1) model

$$\Delta y_t = \beta + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \xi_t - \theta\xi_{t-1} \quad (5)$$

and then calculate the residuals, noted $\hat{\epsilon}_t$, from the least-squares regression of y_t^* on a constant and an deterministic time trend. The test statistic is defined by

$$\text{LMC} = \frac{\hat{\epsilon}' V \hat{\epsilon}}{\hat{\sigma}_\epsilon^2 T^2} \quad (6)$$

where $\hat{\sigma}_\epsilon^2 = \hat{\epsilon}'\hat{\epsilon}/T$ is a consistent estimator and V is a $(T \times T)$ matrix with ijth element equal to the minimum of i and j.

2.2 Kwiatkowski-Phillips-Schmidt-Shin test

The KPSS test of stationarity is based on the same model as the LMC test and has the same general structure. The KPSS test statistic for the model with time trend is computed as

$$\text{KPSS } = \frac{\hat{\epsilon}' V \hat{\epsilon}}{\hat{\sigma}_{\epsilon}^2 T^2} \tag{7}$$

where $\hat{\epsilon}_t$ is the least-squares residual from a regression of y_t^* on a constant and an deterministic time trend. The difference to the LMC test is that the KPSS test relies on a nonparametric estimator of the long-run variance of ϵ_t

$$\hat{\sigma}_{\epsilon}^2 = \hat{\epsilon}'\hat{\epsilon}/T + 2\sum_{i=1}^{l} w(i,l)\hat{\epsilon}'\hat{\epsilon_{t-i}}/T \tag{8}$$

where $w(i,l)1 - i/(l+1)$ is the Bartlett kernel. This estimator is consistent if the truncation lag l increases with the sample size. As parameter l is to be determined, we set $l = int[8(T/100)^{1/4}]$ and $l = int[12(T/100)^{1/4}]$, where $int[.]$ denotes the integer part, as there is no consensus concerning the choice of it. The asymptotic critical values for the LMC and KPSS statistics are identical and provided in Kwiatkowski *et al.* (1992).

3 Monte Carlo Study

In order to assess the performance of the LMC and KPSS test statistics in the presence of additive outliers, a Monte Carlo study is performed. The results are reported in Tables 1 and 2. The data-generating-process is given by $y_t = \alpha y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. The sample size is $T = 100$ and 200, and all experiments are based on 5000 replications. We consider the following three additive outlier situations: a single AO at $k = T/2$, two AOs at $k = 2T/5$ and $4T/5$, and three AOs at $k = 2T/5$, $T/2$ and $4T/5$[2]. Results are reported for outliers of magnitude (ω) 0, 5 and 10. For size simulations $\alpha = 0.8$ and for power simulations $\alpha = 1$.

For the KPSS test, we choose two values of l such that $l = int[8(T/100)^{1/4}]$ and $int[12(T/100)^{1/4}]$, noted KPPS(8) and KPSS(12), respectively. These choices tended to produce the most accurate test results in previous studies. For the LMC test we set $p = 1$, noted LMC(1), since the test is not sensitive to the lag order used. We focus on the nominal 5% test.

Table 1 displays the empirical size of stationarity test. The KPPS tests have an excellent size when there is one or several outliers. Therefore, the size of KPSS

2. We do not add more outliers in order the amount of outliers is in accordance with the view that outliers are *rare* events.

tests seems to be not affected by the presence of outliers, whatever the sample size. On the other hand, the LMC test is inflated size when the magnitude and amount of outliers increase. For example, with $T = 200$ and when the data contain three additive outliers, the size increases from 0.075 to 0.301 when the outlier magnitude increases from 0 to 10.

Power of the tests are given in Table 2. From this table we see that the power of the KPSS tests is good in all cases. If the outlier magnitude is low ($\omega = 5$), the power of LMC test is closed enough to that with no outlier. However, there is evidence of serious loss of power for the LMC statistic when the outlier magnitude is high ($\omega = 10$).

Overall, the simulation results suggest that the KPSS tests are very robust to additive outliers and the LMC test exhibits size distorsions and loss of power in the presence of outliers.

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TAB. 1 – *Empirical size of test statistics in the presence of additive outliers*

		Single outlier			Two outliers			Three outliers		
T	ω	KPSS(8)	KPSS(12)	LMC(1)	KPSS(8)	KPSS(12)	LMC(1)	KPSS(8)	KPSS(12)	LMC(1)
100	0	0.158	0.096	0.070	0.168	0.103	0.074	0.166	0.104	0.072
	5	0.157	0.096	0.114	0.156	0.099	0.152	0.164	0.098	0.196
	10	0.151	0.094	0.212	0.131	0.083	0.248	0.156	0.097	0.287
200	0	0.157	0.093	0.069	0.162	0.101	0.075	0.173	0.106	0.075
	5	0.158	0.094	0.116	0.156	0.098	0.159	0.165	0.102	0.206
	10	0.154	0.092	0.219	0.126	0.080	0.245	0.159	0.097	0.301

Notes: Probability of rejection at a nominal 5% level. The data-generating-process is given by $y_t = 0.8y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. T is the sample size and ω the outlier magnitude. Results are based on 5000 replications. KPPS(8) and KPSS(12) denote the KPSS test statistics with $l = int[8(T/100)^{1/4}]$ and $int[12(T/100)^{1/4}]$, respectively, and LMC(1) the LMC test statistics with $p = 1$.

TAB. 2 – *Power of test statistics in the presence of additive outliers*

		Single outlier			Two outliers			Three outliers		
T	ω	KPSS(8)	KPSS(12)	LMC(1)	KPSS(8)	KPSS(12)	LMC(1)	KPSS(8)	KPSS(12)	LMC(1)
100	0	0.588	0.415	0.328	0.569	0.413	0.318	0.572	0.409	0.309
	5	0.586	0.416	0.311	0.563	0.406	0.325	0.567	0.407	0.388
	10	0.577	0.415	0.448	0.547	0.395	0.597	0.554	0.396	0.683
200	0	0.581	0.426	0.334	0.570	0.413	0.312	0.568	0.420	0.312
	5	0.579	0.426	0.318	0.560	0.407	0.327	0.571	0.417	0.379
	10	0.574	0.422	0.455	0.542	0.394	0.597	0.552	0.407	0.694

Notes: Empirical probabilities of rejection at a nominal 5% level. The data-generating-process is given by $y_t = y_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$. T is the sample size and ω the outlier magnitude. Results are based on 5000 replications. KPPS(8) and KPSS(12) denote the KPSS test statistics with $l = int[8(T/100)^{1/4}]$ and $int[12(T/100)^{1/4}]$, respectively, and LMC(1) the LMC test statistics with $p = 1$.