E C O N O M I C S B U L L E T I N

Testing of I(d) processes in the real output

Luis A. Gil−Alana
University of Navarre

Abstract

The real GDP series of sixteen European countries along with Japan, Canada and the US are examined in this paper by means of fractional integration techniques. The results crucially depend on how we specify the I(0) disturbances, as white noise or autoregressions. Thus, in the former case the orders of integration are higher than 1 in all cases, while using autoregressions the values are all strictly smaller than 1 implying mean reverting behaviour.

The author gratefully acknowledges financial support from the Government of Navarra ("Ayudas de Formación e Investigación y Desarrollo").
Citation: Gil−Alana, Luis A., (2004) "Testing of I(d) processes in the real output." *Economics Bulletin,* Vol. 3, No. 32 pp. 1−6
Submitted: July 6, 2004. **Accepted:** September 16, 2004.
URL: http://www.economicsbulletin.com/2004/volume3/EB−04C20027A.pdf

1. Introduction

For the purpose of the present paper, we define an I(0) process as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that a given process x_t, t = 0, ±1, … is I(d) if:

$$(1 - L)^d \, x_t \; = \; u_t, \quad t = 1, 2, ... \tag{1}$$

with I(0) u_t and where d can be any real number. If d > 0 in (1), x_t is said to be long memory, so-called because of the strong association between observations widely separated in time. These processes were introduced by Granger (1980) and Hosking (1981) and they were theoretically justified in terms of aggregation by Robinson (1978) and Granger (1980).[1] In this article, we use new statistical techniques developed by Robinson (1994) for testing I(d) processes in the real output. In particular, we use a parametric procedure that has several advantages with respect to other methods. Thus, it has standard null and local limit distributions, and this standard behaviour is unaffected by the inclusion of deterministic components or the type of disturbances used in the description of the short-run components underlying the series. The outline of the paper is as follows: Section 2 briefly describes the tests of Robinson (1994) and the economic implications of I(d) models on macroeconomic series. In Section 3, the tests are applied to the real GDP series of sixteen European countries along with Japan, Canada and the US. Section 4 concludes.

2. Testing of I(d) hypotheses

Following discussions of Bhargava (1986), Schmidt and Phillips (1992) and others of parameterization of unit-root models, we can consider the model,

$$y_t \; = \; \alpha \; + \; \beta t \; + \; x_t, \qquad t \; = \; 1,.2, ... \tag{2}$$

where y_t is the time series we observe, and x_t adopting the form as in (1) for a given real value d. We suppose that u_t in (1) has spectral density:

$$f(\lambda;\tau) \; = \; \frac{\sigma^2}{2\pi} \, g(\lambda;\tau),$$

with σ^2 and τ unknown but g of known form (e.g., g ≡ 1 if u_t is white noise). Robinson (1994) proposes a Lagrange Multiplier (LM) test of:

$$H_o: \; d \; = \; d_o, \tag{3}$$

in (1) and (2). Specifically, the test statistic is given by

[1] Similarly, Cioczek-Georges and Mandelbrot (1995), Taqqu et. al. (1997), Chambers (1998) and Lippi and Zaffaroni (1999) also use aggregation to motivate long memory processes, while Parke (1999) uses a closely related discrete time error duration model. Diebold and Inoue (2001) relate fractional integration with regime-switching models.

$$\hat{r} = \left(\frac{T}{\hat{b}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad (4)$$

where T is the sample size and

$$\hat{a} = \frac{2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\tau)^{-1}\, I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \quad \lambda_j = \frac{2\pi j}{T},$$

$$\hat{b} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)'\times\left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1}\times\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right); \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}).$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$, where

$$\hat{u}_t = (1-L)^{d_o} y_t - \hat{\beta}' w_t; \qquad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1}\sum_{t=1}^{T} w_t\,(1-L)^{d_o}\, y_t; \qquad w_t = (1-L)^{d_o}\begin{pmatrix}1\\ t\end{pmatrix},$$

and $\hat{\tau}$ in the above expressions is obtained by minimizing

$$\sigma^2(\tau) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\tau)^{-1} I(\lambda_j),$$

with $\hat{\sigma} = \sigma^2(\hat{\tau})$.

Based on H_o (3), Robinson (1994) showed that under certain regularity conditions,[2]

$$\hat{r} \rightarrow_d N(0,1) \qquad as \quad T \rightarrow \infty, \qquad (5)$$

and he also showed that the tests are efficient against alternatives of form H_a: $d = d_o + \delta T^{-1/2}$ for $\delta \neq 0$. They were applied to US historical annual macroeconomic data in Gil-Alana and Robinson (1997) and other versions of Robinson's (1994) tests based on monthly and quarterly data are respectively given in Gil-Alana (1999) and Gil-Alana and Robinson (2001).

The tests of Robinson (1994) thus are very useful if we want to investigate the order of integration of raw time series. This is crucial in macroeconomics. Thus, for example, if $d \in (0, 0.5)$, the series will be stationary and mean-reverting, while $d \in [0.5, 1)$ will imply nonstationarity but still mean reversion. On the other hand, $d \geq 1$ will imply nonstationarity and non mean-reverting, with the effects of the shocks persisting forever. The issue of mean reversion in I(d) models has also implications for economic planning. Thus, if a series is I(d) with $d \geq 1$, any shock to the economic system will have a permanent effect, so a policy action will be required to bring the variable back to its original form. On the other hand, if the series is I(d) with $d < 1$, there exist less need for policy action since the series will in any case return to its original form sometime in the future.

[2] These conditions are very mild regarding technical assumptions, which are satisfied by model (1) and (2).

3. An empirical application

We analyse the real GDP series in Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Holland, Ireland, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, the UK and the US. The data are annual and the starting date is 1945 for Ireland, Portugal and Greece; 1900 for Spain and Switzerland; 1885 for Japan; and 1870 for the remaining countries. All the series are at 1990 prices and end in 1997.

TABLE 1

Testing I(d) processes in the real output with white noise disturbances

Country / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
FINLAND	24.26	22.88	20.41	16.76	12.44	8.44	5.39	3.35	2.04	**1.15**	**0.49**
CANADA	24.91	24.51	23.55	21.65	18.44	14.02	9.18	5.01	2.02	**0.07**	**-1.17**
NORWAY	24.29	24.13	23.65	22.57	20.53	17.22	12.78	8.04	3.97	**1.02**	**-0.89**
JAPAN	23.08	23.05	22.62	21.44	19.06	15.24	10.46	5.85	2.30	**-0.04**	**-1.50**
IRELAND	10.58	10.30	9.65	8.62	7.26	5.72	4.16	2.68	**1.37**	**0.26**	**-0.64**
U.S.A.	23.40	22.32	20.38	17.34	13.33	9.08	5.45	2.80	**1.02**	**-0.15**	**-0.98**
SPAIN	21.53	21.08	19.84	17.47	14.00	10.01	6.33	3.45	**1.38**	**-0.05**	**-1.08**
GERMANY	23.39	22.21	20.04	16.68	12.44	8.14	4.57	2.01	**0.29**	**-0.83**	**-1.61**
SWEDEN	26.67	26.13	24.55	21.32	16.37	10.75	5.98	2.70	**0.65**	**-0.61**	**-1.46**
HOLLAND	24.45	23.62	21.95	19.12	15.09	10.46	6.16	2.82	**0.49**	**-1.05**	-2.10
ITALY	26.05	25.79	24.74	22.36	18.25	12.84	7.47	3.30	**0.55**	**-1.12**	-2.16
FRANCE	25.93	25.37	23.79	20.70	16.10	10.82	6.09	2.58	**0.26**	**-1.23**	-2.21
SWITZERLAND	21.75	20.16	16.98	12.40	7.71	4.07	1.70	**0.22**	**-0.72**	**-1.39**	-1.90
U.K.	23.99	22.34	19.24	14.92	10.33	6.42	3.53	**1.52**	**0.13**	**-0.86**	-1.62
DENMARK	24.66	23.84	22.06	18.85	14.18	8.93	4.38	**1.14**	**-0.91**	-2.20	-3.02
AUSTRIA	24.51	23.28	20.77	16.74	11.72	6.85	3.04	**0.44**	**-1.22**	-2.30	-3.03
BELGIUM	25.51	24.82	22.96	19.47	14.55	9.23	4.73	**1.50**	**-0.61**	-1.98	-2.90
PORTUGAL	11.61	9.93	7.07	4.05	1.85	**0.48**	**-0.37**	**-0.97**	**-1.43**	-1.81	-2.13
GREECE	12.73	11.80	9.45	6.06	3.08	**1.16**	**0.02**	**-0.72**	**-1.27**	-1.70	2.06

In bold, the non-rejection values at the 5% significance level.

Table 1 reports values of $\hat{r}$ in (4) in a model given by (1) and (2) with white noise u_t and $\alpha = \beta = 0$ a priori. Results based on $\alpha = 0$ a priori and β unknown (i.e., including an intercept) and both α and β unknown (including a linear time trend) were also obtained, the results being very similar to those reported in the table. We see that H_o (3) cannot be rejected for values of d_o constrained between 1 and 1.50. Thus, it cannot be rejected if $d_o = 1.40$ or 1.50 for Finland, Canada, Norway and Japan; if $d_o = 1.30$, 1.40 or 1.50 for Ireland, USA, Spain, Germany and Sweden; if $d_o = 1.30$ or 1.40 for Holland, Italy and France; if $d_o = 1.20$, 1.30 or 1.40 for Switzerland and the UK; For Denmark, Austria and Belgium, H_o (3) cannot be rejected if $d_o = 1.20$ or 1.30. while $d_o = 1$, 1.10, 1.20 and 1.30 are the non-rejection values for Portugal and Greece. As we can see from these results, imposing white noise disturbances, the orders of integration are in practically all cases higher than 1. However, a very different picture is obtained if we allow autocorrelations in the I(0) u_t.

Table 2 resumes the same statistic as in Table 1 but imposing AR(1) u_t. Higher order ARs were also performed and the results were consistent with those reported here. We see that, apart from Greece, for the remaining countries, all the non-rejection values are now constrained between 0.25 and 0.95, thus showing mean reversion. The series for Finland, Canada, Norway, Spain and Portugal clearly appear as nonstationary with orders of integration higher than 0.50. On the other hand, Austria, Denmark, Belgium and Greece seem to be the "most stationary" series with orders of integration smaller than 0.50 in all cases.

TABLE 2

Testing I(d) models in the real output with AR(1) disturbances

Country / d_o	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
FINLAND	4.49	4.53	3.97	2.88	1.98	**1.19**	**0.52**	**0.09**	**-0.08**	**-0.41**
CANADA	7.10	6.84	5.40	3.50	1.69	**0.28**	**-0.90**	-1.83	-2.29	-2.51
NORWAY	8.92	7.41	6.48	5.54	4.11	2.48	**0.86**	**-0.53**	**-1.57**	-2.03
JAPAN	9.86	9.31	6.73	3.34	**1.09**	**-0.38**	**-1.53**	-2.41	-2.84	-2.94
IRELAND	3.20	2.31	**1.58**	**1.37**	**0.92**	**0.32**	**-0.33**	**-0.97**	**-1.43**	-2.32
U.S.A.	4.56	3.73	3.19	2.24	**1.10**	**0.13**	**-0.65**	**-1.12**	-1.98	-2.10
SPAIN	8.71	8.24	6.46	4.36	2.77	**1.40**	**0.07**	**-1.06**	-1.94	-2.54
GERMANY	4.43	4.04	3.24	1.65	**0.49**	**-0.28**	**-0.73**	**-1.59**	-1.68	-1.97
SWEDEN	10.04	6.26	2.95	**1.27**	**0.38**	**-0.26**	**-0.86**	**-1.18**	**-1.62**	-2.51
HOLLAND	4.53	4.28	3.17	**1.43**	**-0.15**	**-1.42**	-2.48	-3.23	-3.86	-4.59
ITALY	9.87	8.08	4.02	**1.18**	**-0.33**	**-1.32**	-2.09	-2.56	-2.56	-3.40
FRANCE	9.50	6.72	4.29	1.70	**0.03**	**-1.12**	-2.03	-2.59	-3.15	-3.97
SWITZERLAND	3.74	1.80	**0.74**	**0.20**	**-0.05**	**-0.09**	**0.19**	**1.17**	2.28	2.47
U.K.	3.17	3.11	3.02	2.12	**1.23**	**0.50**	**-0.01**	**-0.18**	**-1.08**	**-1.17**
DENMARK	4.38	3.58	2.15	**0.33**	**-1.01**	-2.11	-3.04	-3.42	-4.22	-4.36
AUSTRIA	2.70	1.96	**0.75**	**-0.75**	-1.92	-2.85	-3.46	-3.66	-3.96	-3.98
BELGIUM	4.68	4.14	2.64	**0.57**	**-1.08**	-2.36	-3.42	-4.10	-4.79	-4.97
PORTUGAL	3.89	3.27	2.80	2.34	1.79	**1.12**	**0.31**	**-0.71**	**-1.55**	-1.79
GREECE	**0.17**	**-0.44**	**-1.02**	**-1.56**	-2.09	-2.5	-3.42	-3.48	-3.80	-3.90

In bold, the non-rejection values at the 5% significance level.

4. Conclusions

Testing the order of integration of the real GDP series with the tests of Robinson (1994), we see that the results substantially change depending on how we specify the I(0) disturbances. If they are white noise, the orders of integration are in all cases higher than 1, while imposing autocorrelations they become smaller than 1. Thus, it seems that there exists some kind of competition between the AR parameters and the fractional differencing parameter in describing the series. (Note that we use Yule-Walker estimates of the AR coefficient, which entail AR roots that are automatically less than one in absolute value, but can be arbitrarily close to one). Thus, a model selection criterion based on diagnostic checking on the residuals should perhaps be elaborated to correctly specify these and other macroeconomic series. In any case, it is reasonable to assume that some type of weak-autocorrelation structure is present in the data in order to describe the short-run dynamic behaviour of the series and, in that respect, the results presented in this paper are very conclusive against the existence of unit roots, finding strong evidence of mean reversion in all cases.

References

Bhargava, A. (1986) "On the theory of testing for unit roots in observed time series" *Review of Economic Studies* **53**, 369--384.

Chambers, M. (1998) "Long memory and aggregation in macroeconomic time series" *International Economic Review* **39**, 1053-1072.

Cioczek-Georges, R. and B.B. Mandelbrot (1995) "A class of micropulses and anti-persistent fractional Brownian motion" *Stochastic Processes and Their Applications* **60**, 1-18.

Gil-Alana, L.A. (1999) "Testing fractional integration with monthly data" *Economic Modelling* **16**, 613--629.

Gil-Alana, L.A. and P.M. Robinson (1997) "Testing of unit roots and other nonstationary hypotheses in macroeconomic time series" *Journal of Econometrics* **80**, 241--268.

Gil-Alana, L.A. and P.M. Robinson (2001) "Testing seasonal fractional integration in UK and Japanese consumption and income" *Journal of Applied Econometrics* **16**, 95-114.

Granger, C.W.J. (1980) "Long memory relationships and the aggregation of dynamic models" *Journal of Econometrics* **14**, 227--238.

Hosking, J.R.M. (1981) "Fractional differencing" *Biometrika* **68**, 168-176.

Lippi, M. and P. Zaffaroni (1999) "Contemporaneous aggregation of linear dynamic models in large economies" Manuscript, Research Department, Bank of Italy.

Parke, W.R. (1999) "What is fractional integration?" *The Review of Economics and Statistics* **81**, 632-638.

Robinson, P.M. (1978) "Statistical inference for a random coefficient autoregressive model" *Scandinavian Journal of Statistics* **5**, 163--168.

Robinson, P.M. (1994) "Efficient tests of nonstationary hypotheses" *Journal of the American Statistical Association* **89**, 1420--1437.

Schmidt, P. and P.C.B. Phillips (1992) "LM tests for a unit root in the presence of deterministic trends" *Oxford Bulletin of Economics and Statistics* **54**, 257--287.

Taqqu, M.S., W. Willinger and R. Sherman (1997) "Proof of a fundamental result in self-similar traffic modelling" *Computer Communication Review* **27**, 5-23.