E C O N O M I C S B U L L E T I N

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Abstract

In this paper we develop new persistence change tests, similar in spirit to those of Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004). While the exisiting tests are based on the maximum over an appropriate sequence of ratios of sub−sample stationarity statistics, our proposed tests are based on the maximum of the sequence of the numerators of these ratios divided by the minimum of the sequence of the denominators of the ratios. The large sample properties of the tests are established, and both finite sample and asymptotic critical values are provided. Numerical evidence suggests that our proposed tests provide a useful complement to the extant tests.

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Some New Tests for a Change in Persistence*

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Abstract

In this paper we develop new persistence change tests, similar in spirit to those of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004). While the exisiting tests are based on the maximum over an appropriate sequence of ratios of sub-sample stationarity statistics, our proposed tests are based on the maximum of the sequence of the numerators of these ratios divided by the minimum of the sequence of the denominators of the ratios. The large sample properties of the tests are established, and both finite sample and asymptotic critical values are provided. Numerical evidence suggests that our proposed tests provide a useful complement to the extant persistence change tests.

Keywords: Persistence change; sub-sample stationarity tests; Brownian motion.

JEL Classification: C22.

1 Introduction

The ability to correctly decompose a time series into its separate difference stationary, $I(1)$, and trend stationary, $I(0)$, regimes, where they exist, has important implications for effective model building and forecasting in applied economics and finance. A number of recent testing procedures have been developed that aim to distinguish such behaviour. These include the ratio-based persistence change tests of, *inter alia*, Kim (2000), Kim *et al.* (2002), Busetti and Taylor (2004) (BT) and Leybourne and Taylor (2004) which test the null hypothesis of $I(0)$ throughout the sample against the alternative of a change from $I(0)$ to $I(1)$, or *vice versa*.

The test statistics developed by these authors are based on a sequence of ratios of sub-sample implementations of the Kwiatkowski *et al.* (1992) (KPSS) type stationarity test statistic. The sample is split in two at some point. The sub-sample KPSS test

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statistics are applied to each of the two sub-samples and then the ratio of the two statistics is taken. This is repeated over a range of possible changepoints and the test is then based on, for example, the maximum of the resulting sequence of ratios. In testing against $I(0)$ to $I(1)$ changes we divide the second sub-sample statistic by the first, and vice versa for tests against $I(0)$ to $I(1)$ changes. Now, it is well known that the KPSS statistic diverges with the sample size against $I(1)$ data, but is of $O_p(1)$ against $I(0)$ data; see, for example, KPSS pp.165-9. The ratio-based testing approach exploits these facts, since the largest ratio statistic will be of $O_p(1)$ against either constant $I(0)$ or constant $I(1)$ processes but will diverge where a persistence change occurs (due to the different orders in probability of the two sub-sample KPSS-type statistics).

In many ways it would seem more logical, rather than taking the largest of the sequence of ratios of sub-sample KPSS-type statistics, to take the ratio of the largest sub-sample statistic to the smallest sub-sample statistic. In testing against $I(0)$ to $I(1)$ changes the largest second sub-sample KPSS-type statistic would form the numerator and the smallest first sub-sample statistic the denominator, and vice versa for testing against $I(1)$ to $I(0)$ changes. In this way the numerator of the statistic will identify the sub-sample providing the most evidence against stationarity (for a given direction of change), while the denominator will identify the sub-sample which yields the least evidence. It seems, *a priori*, not unreasonable to expect that a test based on this statistic might be expected to be more powerful than that based on the maximal ratio of the sub-sample test statistics. Moreover, unlike existing tests, the sub-samples of data used in the numerator and denominator of the statistic will not necessarily span the whole of the data set.

The paper is organized as follows. Section 2 outlines the model of persistence change upon which we focus. In Section 3 we provide a brief review of the existing persistence change tests. In Section 4 we detail our new test statistics and derive their large sample properties. In Section 5, using Monte Carlo simulation, we provide critical values and compare the finite-sample size and power properties of the new tests with the corresponding tests of Section 3. Section 6 concludes.

2 The Persistence Change Model

As a model of persistence change, we follow BT and adopt the following data generating process (DGP):

$$\begin{aligned} y_t &= d_t + \varepsilon_t, \\ \varepsilon_t &= v_t + w_t, \qquad t = 1, ..., T, \end{aligned} \tag{2.1}$$

with either

$$w_t = w_{t-1} + \eta_t 1(t > \lfloor \tau^* T \rfloor) \tag{2.2}$$

or

$$w_t = w_{t-1} + \eta_t 1(t \leq \lfloor \tau^* T \rfloor) \tag{2.3}$$

with $\tau^* \in [0,1]$, and where $1(\cdot)$ denotes the indicator function and $\lfloor \cdot \rfloor$ denotes the integer part of its argument.

In (2.1), the deterministic kernel, $d_t = \mathbf{x}_t'\beta$, where $\mathbf{x}_t$ is a $(k+1)\times 1$ fixed sequence whose first element is fixed at unity throughout (so that (2.1) always contains an intercept term), with associated parameter vector β. The vector $\mathbf{x}_t$ is assumed to be a k-th order polynomial trend; that is, $\mathbf{x}_t = (1, t, ..., t^k)'$, within which the constant ($d_t = \beta_0$) and constant plus linear time trend ($d_t = \beta_0 + \beta_1 t$) are arguably the cases of most interest. The disturbances $\{v_t\}$ and $\{\eta_t\}$ are mutually independent mean zero processes satisfying the familiar α-mixing conditions of Phillips and Perron (1998,p.336), with strictly positive and bounded long-run variances $\omega^2 \equiv \lim_{T\to\infty} E\left(\sum_{t=1}^T v_t\right)^2$, and $\omega_\eta^2 \equiv \lim_{T\to\infty} E\left(\sum_{t=1}^T \eta_t\right)^2$, respectively.

Within this model, we consider four possibilities. The first of these is that y_t is $I(0)$ throughout the sample period. This is represented by (2.1) and (2.2) with $\tau^* = 1$ (or (2.3) with $\tau^* = 0$). We denote this H_0. Secondly, y_t may be $I(1)$ throughout; represented by (2.1) and (2.2) with $\tau^* = 0$ (or (2.3) with $\tau^* = 1$), denoted H_1. Thirdly, there may be a change from $I(0)$ to $I(1)$ at time $t = \lfloor \tau^* T \rfloor$; as given by (2.1) and (2.2) with the changepoint fraction $0 < \tau^* < 1$, denoted H_{01}. Finally, a change from $I(1)$ to $I(0)$ at time $t = \lfloor \tau^* T \rfloor$ is represented by (2.1) and (2.3) with $0 < \tau^* < 1$, denoted H_{10}.

3 Kim's Ratio-Based Tests

In order to test the null hypothesis that y_t is a constant $I(0)$ process against an $I(0)$-$I(1)$ shift at some unknown point in the sample, BT and Kim *et al.* (2002) have independently proposed the test which rejects for large values of the statistic

$$K = \max_{\tau\in\mathcal{T}} \frac{(T - \lfloor \tau T\rfloor)^{-2} \sum_{t=\lfloor \tau T\rfloor+1}^{T} (S_{t,2}(\tau))^2}{\lfloor \tau T\rfloor^{-2} \sum_{t=1}^{\lfloor \tau T\rfloor} (S_{t,1}(\tau))^2} \equiv \max_{\tau\in\mathcal{T}} \left\{ \frac{K_2(\tau)}{K_1(\tau)} \right\} \qquad (3.1)$$

where: $\mathcal{T} \equiv [\tau_l, \tau_u]$ is a given sub-interval of $[0,1]$; $S_{t,1}(\tau) \equiv \sum_{i=1}^t \hat{\epsilon}_{1,i}$ and $S_{t,2}(\tau) \equiv \sum_{i=\lfloor \tau T\rfloor+1}^t \hat{\epsilon}_{2,i}$, where $\{\hat{\epsilon}_{1,t}\}_{t=1}^{\lfloor \tau T\rfloor}$ and $\{\hat{\epsilon}_{2,t}\}_{t=\lfloor \tau T\rfloor+1}^{T}$ are the OLS residuals from the regressions of y_t on $\mathbf{x}_t$ for $t = 1, \ldots \lfloor \tau T\rfloor$ and for $t = \lfloor \tau T\rfloor + 1, ..., T$, respectively.

BT demonstrate that K is consistent at rate $O_p(T^2)$ under H_{01} but of $O_p(1)$ under H_{10}. They show, however, that the statistic $K' = \max_{\tau\in\mathcal{T}}\{K_1(\tau)/K_2(\tau)\}$ is of $O_p(T^2)$ under H_{10}, and of $O_p(1)$ under H_{01}. BT therefore also suggest a test based on the maximum of the these two statistics; that is, $K^* \equiv \max\{K, K'\}$, which they show to be of $O_p(T^2)$ under both H_{01} and H_{10}. All of these statistics are also shown to be of $O_p(1)$ under H_1.

Representations for the limiting null distributions of the foregoing statistics are given in BT. Notably, these do not depend on the long run variance ω^2, even though neither the numerator nor the denominator of K is scaled by a long run variance estimator. However, Leybourne and Taylor (2004) have shown that the finite sample size properties of the foregoing ratio-based tests against (constant parameter) weakly dependent $I(0)$ processes are not at all satisfactory.

Consequently, Leybourne and Taylor (2004) have proposed tests based on statistics where the denominator and numerator of (3.1) are scaled by appropriate sub-sample variance estimators; that is, they consider replacing $K_1(\tau)$ and $K_2(\tau)$ of (3.1), by $K_1(\tau, m) \equiv (\hat{\omega}^2_{1,\tau})^{-1}K_1(\tau)$ and $K_2(\tau, m) \equiv (\hat{\omega}^2_{2,\tau})^{-1}K_1(\tau)$, respectively, where following KPSS (Eq.(10), p.164)

$$\begin{aligned}
\hat{\omega}^2_{1,\tau} &= (\lfloor \tau T \rfloor)^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{\epsilon}^2_{1,t} + 2(\lfloor \tau T \rfloor)^{-1} \sum_{i=1}^{m} w(i,m) \sum_{t=i+1}^{\lfloor \tau T \rfloor} \hat{\epsilon}_{1,t}\hat{\epsilon}_{1,t-i} \\
\hat{\omega}^2_{2,\tau} &= (T - \lfloor \tau T \rfloor)^{-1} \sum_{t=\lfloor \tau T \rfloor+1}^{T} \hat{\epsilon}^2_{2,t} + 2(T - \lfloor \tau T \rfloor)^{-1} \sum_{i=1}^{m} w(i,m) \sum_{t=i+\lfloor \tau T \rfloor+1}^{T} \hat{\epsilon}_{2,t}\hat{\epsilon}_{2,t-i},
\end{aligned}$$

with $w(i, m) = 1 - i/(m + 1)$. With an obvious notation, we denote the resulting modified statistics by $K(m)$, $K'(m)$ and $K^*(m)$. Leybourne and Taylor (2004) report significant improvements in the finite sample size properties of the resulting tests. They find that setting $m = 0$ provides a useful pragmatic balance between re-dressing the size problems of the tests yet keeping power losses against persistence change processes relatively small. The results (including finite-sample critical values) presented in Section 5 all pertain to $m = 0$. Like Leybourne and Taylor (2004), we found that $m = 0$ appears to deliver the best size/power trade-off available. Results for other values of m are available on request.

4 Alternative Persistence Change Tests

Following the discussion in Section 1, our proposed new test against H_{01} rejects H_0 for large values of the statistic

$$L = \frac{\max_{\tau \in \mathcal{T}} K_2(\tau)}{\min_{\tau \in \mathcal{T}} K_1(\tau)} \tag{4.1}$$

where $K_1(\tau)$ and $K_2(\tau)$ are as defined in section 3. Similarly, to test H_0 against H_{10}, we also propose the test which rejects for large values of the statistic

$$L' = \frac{\max_{\tau \in \mathcal{T}} K_1(\tau)}{\min_{\tau \in \mathcal{T}} K_2(\tau)} \tag{4.2}$$

and, finally, in order to test H_0 against either H_{10} or H_{01}, we propose the test which rejects for large values of the statistic $L^* \equiv \max\{L, L'\}$.

As in Leybourne and Taylor (2004) studentised versions of these statistics can also be entertained simply by replacing $K_1(\tau)$ and $K_2(\tau)$ by $K_1(\tau, m)$ and $K_2(\tau, m)$

respectively in (4.1) and (4.2). Again with an obvious notation, we denote the resulting modified statistics by $L(m)$, $L'(m)$ and $L^*(m)$.

We now establish the limiting distributions of our proposed statistics under the null, H_0. BT demonstrate that for $0 < \tau < 1$,

$$\omega^{-1}T^{-1/2}(S_{\lfloor T\cdot\rfloor,2}(\cdot),\ S_{\lfloor T\cdot\rfloor,1}(\cdot)) \Rightarrow (N_2(\cdot,\cdot), N_1(\cdot,\cdot)) \tag{4.3}$$

where "⇒" denotes weak convergence and where

$$N_2(\tau,r) \equiv W(r) - W(\tau) - \int_\tau^1 \mathbf{x}(r)'dW(r)\left(\int_\tau^1 \mathbf{x}(r)\mathbf{x}(r)'dr\right)^{-1}\int_\tau^r \mathbf{x}(s)ds, \quad r \in (\tau,1]$$

$$N_1(\tau,r) \equiv W(r) - \int_0^\tau \mathbf{x}(r)'dW(r)\left(\int_0^\tau \mathbf{x}(r)\mathbf{x}(r)'dr\right)^{-1}\int_0^r \mathbf{x}(s)ds, \quad r \in [0,\tau],$$

where $\mathbf{x}(r) = (1, r, ..., r^k)'$, and $W(\cdot)$ is a standard Brownian motion process on $[0,1]$: here defined by $\omega^{-1}T^{-1/2}\sum_{t=1}^{\lfloor rT\rfloor} v_t \Rightarrow W(r)$, $r \in [0,1]$.

We then obtain immediately from (4.3) and applications of the CMT, that

$$\begin{aligned}
L &\Rightarrow \frac{\sup_{\tau\in[\tau_l,\tau_u]}(1-\tau)^{-2}\int_\tau^1 N_2(\tau,r)^2dr}{\inf_{\tau\in[\tau_l,\tau_u]}\tau^{-2}\int_0^\tau N_1(\tau,r)^2dr} \equiv B(\tau_l,\tau_u)\\
L' &\Rightarrow \frac{\sup_{\tau\in[\tau_l,\tau_u]}\tau^{-2}\int_0^\tau N_1(\tau,r)^2dr}{\inf_{\tau\in[\tau_l,\tau_u]}(1-\tau)^{-2}\int_\tau^1 N_2(\tau,r)^2dr} \equiv B'(\tau_l,\tau_u)\\
L^* &\Rightarrow \max\{B(\tau_l,\tau_u),\ B'(\tau_l,\tau_u)\}.
\end{aligned}$$

Again these limiting representations do not depend on the long-run variance, ω^2. The same limiting distributions also apply to the modified tests $L(m)$, $L'(m)$ and $L^*(m)$.

Under H_{01} it is trivial to show, using results given in the proof of Theorem 2.2 of BT, that both L and L^* will be of $O_p(T^2)$, while $L(m)$ and $L^*(m)$ will also be of $O_p(T^2)$ if m does not increase with T, but of $O_p(T/m)$ if $m \to \infty$ as $T \to \infty$, while L' and $L'(m)$ will be of $O_p(1)$. Similarly, under H_{10} it follows from results in the proof of Theorem 3.2 of BT that both L' and L^* will be of $O_p(T^2)$, while $L'(m)$ and $L^*(m)$ will also be of $O_p(T^2)$ if m does not increase with T, but of $O_p(T/m)$ if $m \to \infty$ as $T \to \infty$, while L and $L(m)$ will be of $O_p(1)$. Under H_1 all statistics are of $O_p(1)$.

5 Simulation Results

5.1 Critical Values

Table 1 reports both finite sample and asymptotic upper tail null critical values for the $L(0)$, $L'(0)$ and $L^*(0)$ persistence change tests of section 3. Precisely, the finite sample critical values of Table 1 were obtained by Monte Carlo simulation using pseudo-data generated according to the pure noise DGP:

$$y_t = \varepsilon_t \sim NIID(0,1), \quad t = 1, ..., T.$$

Results are reported for both de-meaned ($\mathbf{x}_t = 1$) and de-meaned and de-trended ($\mathbf{x}_t = (1,\ t)'$) data. In each case finite sample critical values are given for $T = 60, 120$ and 240, while the rows labelled '∞' give asymptotic critical values for the tests, obtained by direct simulation of the appropriate limiting functionals of section 4 using discrete approximations for $T = 1000$. For each test we used $\mathcal{T} = [0.2, 0.8]$, as is typical in this literature: this choice is applied throughout this section of the paper. All simulations in this paper were performed using 80,000 Monte Carlo replications and the RNDN function of Gauss 3.2.

Tables 1 – 3 about here

5.2 Size Properties

Table 3 reports, for $T = 60, 120$ and 240 and for both de-meaned and de-meaned and de-trended data, the empirical rejection frequencies of the $K(0)$, $K'(0)$ and $K^*(0)$ tests of section 3, relative to our new tests, $L(0)$, $L'(0)$ and $L^*(0)$, of section 4 when applied to data generated by the following stable and invertible $ARMA$ process:

$$y_t = \phi y_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1}, \quad t = -100, ..., T, \tag{5.1}$$

with $\varepsilon_t \sim NIID(0, 1)$, and the design parameters $\phi \in \{0.0, 0.50, 0.90\}$ and $\theta \in \{0.0, \pm 0.4\}$. Notice that in all cases y_t is an $I(0)$ process and, hence, H_0 holds. Notice that, due to invariance, we have set $d_t = 0$ with no loss of generality.

All tests in this and the following section were run at the nominal 5% level using exact (simulated) critical values for each test, and, in order to control for initial effects, the first 100 observations were discarded in each experiment.

In terms of a relative comparison there is really very little to choose overall between the finite sample size properties of the $K(0)$ and $L(0)$, $K'(0)$ and $L'(0)$, and $K^*(0)$ and $L^*(0)$ tests. For all of tests size distortions are, not surprisingly, worst for $\phi = 0.9$, where distortions are also worse, other things equal, for the de-meaned and de-trended case. Size distortions are ameliorated, other things equal, as the sample size is increased, as predicted by the limiting distribution theory. Although the tests display notable finite-sample size distortions when $\phi = 0.9$, a comparison with the results reported in KPSS, Table 3, p.171 shows that these distortions are overwhelmingly less serious than for the corresponding full-sample KPSS tests when the lag truncations $m = 0$ and $m = \text{int}[4(T/100)^{1/4}]$ are used in the full-sample long-run variance estimator, and roughly comparable where $m = \text{int}[12(T/100)^{1/4}]$ is used.

5.3 Empirical Power

Table 3 reports, for $T = 60, 120$ and 240 and for both de-meaned and de-meaned and de-trended data, the empirical rejection frequencies of the tests when the data are generated according to the $I(0) - I(1)$ switch DGP

$$y_t = \mu_t + \varepsilon_t, \quad t = -100, ..., T \tag{5.2}$$
$$\mu_t = \mu_{t-1} + 1(t > \lfloor \tau^* T \rfloor)\eta_t, \quad \eta_t \sim NIID(0, \omega_\eta^2). \tag{5.3}$$

with $\varepsilon_t \sim NIID(0,1)$, changepoint $\tau^* \in \{0.25, 0.50, 0.75\}$, and signal-to-noise ratio, $\omega_\eta^2 \in \{0.05, 0.10, 0.25\}$. Qualitatively similar conclusions are drawn for other values of τ^* and ω_η^2 and from size-adjusted results for other stable and invertible noise processes. Again we set $d_t = 0$ and discard the first 100 observations.

Summarizing the results in Table 3, there is not one case where $L(0)$ and $L^*(0)$ does not display higher power than $K(0)$ and $K^*(0)$, respectively.[1] Although the power improvements displayed by the new tests are not massive, they are statistically significant in the vast majority of cases.[2] For these tests it is also worth noting that, other things equal, power is not necessarily lower in the de-meaned and de-trended case than for the de-meaned case; this phenomenon is also apparent in the simulation results presented in, for example, Table 4 (p.172) of KPSS.

Although not reported, we also considered the corresponding $I(1)$-$I(0)$ switch DGP. These experiments yielded very similar results to those observed in Table 3, save that the relative behaviour of $K(0)$ and $K'(0)$ and $L(0)$ and $L'(0)$ are inter-changed, on switching τ^* for $(1 - \tau^*)$, noting that this model can also be viewed as a process with a switch from $I(0)$ to $I(1)$ at $(1 - \tau^*)$ when the data are taken in reverse order.

6 Conclusions

In this paper we have proposed a new set of tests for a change in persistence based on statistics formed from the ratio of the largest sub-sample KPSS-type statistic to the smallest sub-sample statistic. Asymptotic null distributions of the proposed statistics were derived and associated tables of critical values provided. The consistency of the proposed tests against persistence change processes was demonstrated. A Monte Carlo study comparing the finite sample size and power properties of the proposed tests with their existing counterparts formed from the maximal ratio of the sub-sample KPSS-type statistics. The results suggested that the new tests proposed in this paper provide a very useful complement to the existing maximal ratio tests, displaying comparable size properties coupled with somewhat superior power properties to the latter.

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[1] Recall that $L'(0)$ and $K'(0)$ are not consistent tests against the $I(0) - I(1)$ change model.

[2] To see this, denote each entry in the table (when divided by 100) as a, then observe that the standard error of a is approximately $\sqrt{a(1-a)/K}$, K the number of Monte Carlo replications used.

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Table 1. Critical values for tests of stationarity against a change in persistence.

Panel A. De-meaned Case									
		$L(0)$			$L'(0)$			$L^*(0)$	
T	10%	5%	1%	10%	5%	1%	10%	5%	1%
60	14.52	18.09	26.34	14.39	17.76	25.74	17.89	21.35	29.74
120	17.04	21.56	32.64	17.22	21.70	33.06	21.55	26.40	38.10
240	18.99	23.94	37.04	18.91	24.08	37.52	23.93	29.39	43.15
∞	21.32	27.26	43.30	21.32	27.26	43.30	27.17	33.82	51.05
Panel B. De-meaned and De-trended Case									
		$L(0)$			$L'(0)$			$L^*(0)$	
T	10%	5%	1%	10%	5%	1%	10%	5%	1%
60	6.60	7.68	10.34	6.55	7.73	10.29	7.69	8.83	11.48
120	7.81	9.34	12.97	7.89	9.49	13.21	9.39	10.97	14.63
240	8.73	10.57	15.14	8.76	10.60	15.01	10.55	12.40	17.06
∞	9.87	12.11	17.65	9.87	12.11	17.65	12.06	14.37	20.07

Table 2. Empirical rejection frequencies of nominal 5% tests against a change in persistence: DGP (5.1).

Panel A. De-meaned Case

		$T=60$						$T=120$						$T=240$					
ϕ	θ	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$
.0	.4	3.5	4.3	3.8	3.1	4.2	3.7	3.5	4.1	3.5	3.0	3.6	3.1	4.0	4.0	3.9	3.5	3.7	3.2
	-.4	4.3	5.2	4.6	3.9	5.2	4.5	4.3	5.1	4.4	4.1	4.7	4.5	4.7	4.9	4.8	4.6	4.8	4.5
.5	.0	5.0	6.4	5.9	4.7	6.5	5.9	4.7	5.5	5.1	4.6	5.3	5.0	4.8	4.9	4.9	4.6	4.8	4.6
	.4	5.2	6.1	5.9	5.0	6.4	6.0	5.2	5.9	5.7	5.4	6.1	6.0	5.2	5.3	5.6	5.3	5.7	5.7
	-.4	4.3	4.9	4.8	3.7	4.8	4.4	4.1	4.7	4.2	3.5	4.4	3.8	4.1	4.4	4.2	3.8	4.1	3.8
.9	.0	10.9	12.5	14.5	9.4	11.6	13.1	7.9	8.8	9.7	6.7	7.4	8.5	5.7	5.6	6.4	4.6	4.8	5.0
	.4	16.1	18.2	22.8	15.5	18.5	22.5	12.1	13.2	16.1	11.9	12.7	16.4	8.0	8.2	9.8	7.6	8.0	9.3
	-.4	8.3	9.8	10.7	6.7	8.4	8.9	6.7	7.0	7.5	5.2	5.6	6.0	5.1	4.9	5.5	4.0	4.1	4.3

Panel B. De-meaned and De-trended Case

		$T=60$						$T=120$						$T=240$					
ϕ	θ	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$
.0	.4	4.6	5.8	5.2	3.8	4.7	4.1	3.6	3.9	3.7	2.8	3.2	2.9	3.5	3.5	3.3	2.8	3.1	2.6
	-.4	4.0	5.0	4.3	4.0	5.1	4.3	4.5	4.8	4.6	4.5	5.0	4.6	4.5	4.7	4.6	4.6	4.9	4.7
.5	.0	6.6	8.0	7.7	7.4	8.6	8.5	5.8	6.1	6.2	6.2	6.8	6.7	4.7	5.1	4.8	5.1	5.5	5.3
	.4	5.2	6.3	6.1	5.6	7.0	6.5	5.4	6.1	6.0	5.7	6.8	6.5	5.0	5.5	5.4	5.4	6.2	6.0
	-.4	5.4	6.3	6.0	5.5	6.5	5.9	4.8	5.0	4.8	4.7	5.2	4.9	4.1	4.6	4.3	3.9	4.6	4.0
.9	.0	22.3	24.8	29.8	22.1	24.6	29.2	17.5	18.3	22.4	16.4	17.3	21.0	9.9	10.4	12.0	9.2	9.9	11.1
	.4	26.4	29.1	35.6	27.8	30.6	37.4	25.0	26.4	33.7	25.7	27.4	34.7	15.7	16.4	20.5	16.1	17.2	21.2
	-.4	17.3	20.0	22.8	16.1	18.7	20.9	13.7	14.4	17.1	12.3	13.2	15.2	8.4	8.9	10.0	7.5	8.1	8.9

Table 3. Empirical rejection frequencies of nominal 5% tests against a change in persistence: DGP (5.2)-(5.3).

Panel A. De-meaned Case

		$T=60$						$T=120$						$T=240$					
ω_η^2	τ^*	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$
.05	.25	6.8	6.0	6.6	7.1	6.5	7.5	13.2	9.7	13.5	14.5	10.2	14.9	34.5	21.8	37.9	36.6	22.1	39.2
	.50	6.7	4.6	5.6	7.0	4.9	6.2	12.8	4.8	10.5	13.9	5.1	11.4	32.6	7.3	27.8	34.9	7.6	29.9
	.75	5.9	4.7	5.2	6.0	4.8	5.5	8.0	4.2	6.6	8.2	4.2	6.7	17.2	2.8	13.0	18.3	3.1	13.8
.10	.25	12.7	9.2	12.6	14.0	10.5	14.9	32.7	21.3	36.0	35.0	21.7	37.9	63.4	34.9	66.7	66.3	33.9	68.3
	.50	12.5	4.5	9.8	13.8	5.3	11.6	31.5	7.3	26.8	34.2	7.5	29.1	62.0	14.9	57.8	65.1	13.9	60.1
	.75	8.2	4.0	6.4	8.3	4.4	7.0	16.7	2.8	12.6	17.9	2.9	13.6	39.4	1.3	32.2	41.3	1.4	33.9
.25	.25	37.8	23.4	40.5	39.8	24.1	43.3	66.0	33.6	68.3	69.2	32.4	70.4	87.4	35.8	87.0	90.3	33.1	89.8
	.50	37.1	8.0	31.5	40.7	8.5	35.6	67.8	15.3	63.5	71.3	14.3	66.4	89.3	18.3	86.6	91.6	16.8	89.1
	.75	21.5	2.1	15.5	22.8	2.3	17.4	46.0	1.0	38.8	48.3	1.0	40.8	73.8	0.2	67.9	76.0	0.2	70.0

Panel B. De-meaned and De-trended Case

		$T=60$						$T=120$						$T=240$					
ω_η^2	τ^*	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$	$K(0)$	$K'(0)$	$K^*(0)$	$L(0)$	$L'(0)$	$L^*(0)$
.05	.25	5.4	5.0	5.3	5.5	5.1	5.6	7.1	5.4	6.5	7.6	5.5	6.9	16.1	9.6	15.7	17.7	10.2	16.7
	.50	5.4	4.7	5.0	5.5	4.9	5.1	6.9	4.7	6.0	7.3	4.9	6.4	14.2	5.8	12.0	15.6	6.2	12.8
	.75	5.3	4.8	5.2	5.4	4.8	5.3	6.1	4.7	5.5	6.3	4.8	5.7	10.2	3.7	7.9	11.2	4.2	8.4
.10	.25	6.9	5.2	6.3	7.4	5.7	7.0	15.9	8.9	15.1	17.5	9.8	16.5	44.2	24.1	45.0	46.8	24.5	47.3
	.50	6.7	4.4	5.7	7.0	4.9	6.1	13.6	5.5	11.3	15.0	6.0	12.3	38.6	13.7	35.8	42.2	13.5	38.3
	.75	6.1	4.5	5.5	6.4	4.5	5.7	9.9	3.6	7.7	10.7	4.0	8.4	25.6	2.1	19.6	28.1	2.5	21.5
.25	.25	18.3	9.7	16.8	20.6	11.2	19.8	51.7	27.9	52.8	54.7	28.8	55.8	84.4	44.9	84.6	87.3	44.5	87.0
	.50	16.5	5.9	13.3	18.8	6.7	15.5	47.6	16.4	44.1	51.8	16.4	47.9	83.6	30.4	81.6	86.9	29.2	84.5
	.75	12.1	3.0	8.5	13.6	3.4	10.0	33.3	1.6	25.9	36.7	1.9	29.2	69.3	0.5	62.2	72.5	0.6	65.9