E C O N O M I C S   B U L L E T I N

Equilibrium selection in coordination games: Why do dominated strategies matter?

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Abstract

In this paper I illustrate by an example that strictly dominated strategies may affect the process of the equilibrium selection in coordination games. The strategy profile that gets selected may be both Pareto and risk dominated. This distinguishes it from the examples provided in Ellison (2000) and Maruta (1997).

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1 Introduction

Coordination games characterize economic interactions in a large number of settings. The defining feature of such games is the existence of the multiple strict Pareto ranked Nash equilibria. The tools for equilibrium selection in such games are provided by the evolutionary game theory. The pioneering papers in this area are Foster and Young (1990), Kandori, Mailath, and Rob (1993) (henceforth, KMR), and Young (1993), which applied such models to 2×2 coordination games and showed that in the medium run players can coordinate on any strict Nash equilibrium, while in the long-run the risk-dominant outcome is selected as the unique stochastically stable solution.

In this paper I investigate the sensitivity of the equilibrium selection results to the inclusion of strictly dominated strategies. Such an investigation is important, because the games we study in economics are usually stylised descriptions of real life strategic interaction, which leave out a lot of details. Therefore, the strategy set available to the players is sensitive to the modelling decisions.

2 An Example

In this Section I consider a simple coordination game and show that the presence of a strictly dominated strategy can affect the long-run outcome. Consider coordination game represented on Figure 1:

	A	B	C
A	*2,2*	*0,0*	*c,0*
B	*0,0*	*3,3*	*0,0*
C	*0,c*	*0,0*	*0,4*

Figure 1.

Here $c>4$. This game has two pure strategy Nash equilibria *(A,A)* and *(B,B)*, and a mixed strategy equilibrium *(0.6A+0.4B, 0.6A+0.4B)*. Strategy *C*, on the other hand, is strictly dominated (for example, by strategy *0.9A+0.1B*) and is weakly dominated by *A*.

If one believes that strictly dominated strategies should not affect the outcome of the equilibrium selection process, the long-run prediction for this game should be the same as for the following *2X2* coordination game:

	A	B
A	*2,2*	*0,0*

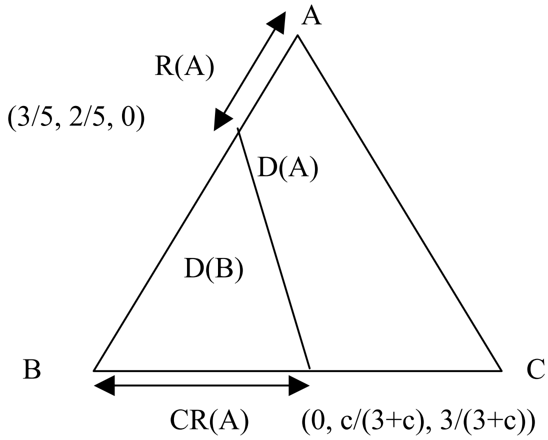
B	*0,0*	*3,3*

Figure 2.

However, as I will show shortly, the standard KMR dynamics selects different outcomes for these games, provided $c>4.5$.

To describe the standard KMR dynamics assume that the population consists on N players and at period $t=0$ each player is characterized by a particular strategy she plays. The strategy choice of a player in the next period is the best response to the current population strategy profile with probability $1-\varepsilon$, but with probability ε the player suffers from noise, in which case the strategy is selected at random and all strategies are selected with positive probabilities. Noise occurs independently across both players and periods. If $\varepsilon>0$ the model described about possesses a unique steady state distribution. The limit of this distribution as ε goes to zero is known as stochastically stable equilibrium.

It is a well-known result that for the game depicted at Figure 2 the stochastically stable equilibrium is *(B, B)*. The stochastically stable outcome of the game depicted at Figure 1, however, depend on the value of *c*. To see this let us denote by *D(A) (D(B))* the basin of attraction of pure strategy *A (B)*, i. e. the set of all mixed strategies to which *A (B)* is a best reply. These sets are illustrated on Figure 3. The vertices represent monomorphic populations playing particular strategies. It is easy to check that strategies *A* and *B* earn the same payoffs against strategy profiles *(3/5, 2/5, 0)* and *(0, c/(3+c), 3/(3+c))*, where numbers in brackets represent the fractions of *A, B,* and *C*-strategists respectively.



Following Ellison (2000), define *R(A)*, the radius of *D(A),* as the minimal distance from *A* to *D(B)* and *CR(A)*, the coradius of *D(A),* as the minimal distance from B to *D(A)*. Then from Figure 3

$$R(A)=2/5,\ CR(A)=3/(3+c).$$

As the radius exceeds the coradius *A* is the unique stochastically stable equilibrium. This happens for $c>9/2$. Therefore, if an *A*-strategist fairs against a *C*-strategist sufficiently better than a *B*-strategist, *(A, A)* rather then *(B, B)* will become the stochastically stable equilibrium. Therefore, the long-run outcome of a strategic interaction can be affected by the presence of a strictly dominated strategy.

3 Conclusions

In this note I demonstrated by an example that the long-run equilibrium of a game may be sensitive to the presence of strictly dominated strategies. Indeed, players may coordinate on a strategy that is both Pareto and risk dominated, provided it fairs well against a strictly dominated strategy. The result is rather disturbing because it can be interpreted as the sensitivity to the modelling assumptions. Indeed, assume that players are firms. Let us interpret different strategies as R&D programmes, and suppose there is complementarity between the programmes of different firms. Since the number of different R&D programmes can be numerous and had to model explicitly, an economist conducting a study of the industry might be willing to concentrate only on viable alternatives, leaving others out. One definition of a viable alternative is that it is not strictly dominated. As we have seen, however, leaving out alternatives that are judged non-viable can affect the long-run prediction for the behaviour of the industry.

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