E C O N O M I C S   B U L L E T I N

Costly participation in voting and equilibrium abstention: a uniqueness result

Takanori Adachi
Department of Economics, University of Pennsylvania

Abstract

This note shows that a unique mixed Nash equilibrium obtains when there are three voters in Palfrey and Rosenthal's (1983) costly voting game under complete information. Experimental investigation of this result might be interesting.

I am grateful to Richard McLean for invaluable comments, which greatly improved the exposition of the note. I also thank Andy Postlewaite for giving me an opportunity to write this note, and Yoichi Hizen and Yasutora Watanabe, the discussion with whom stimulated my interests in this subject. The usual disclaimer applies.
Citation: Adachi, Takanori, (2004) "Costly participation in voting and equilibrium abstention: a uniqueness result." *Economics Bulletin*, Vol. 4, No. 2 pp. 1−5
Submitted: November 9, 2003. **Accepted:** January 17, 2004.
URL: http://www.economicsbulletin.com/2004/volume4/EB−03D70012A.pdf

1. Introduction

In this note, I investigate endogenous abstention by considering a positive cost of participating in a meeting for decision-making.[1] I choose a model with the number of members being finite, actually three. In this sense, if that is literally taken, the target of this paper is not for so called *The Paradox of Voting*, as Börgers (2003) emphasizes.[2] We may also interpret this situation as when there are three populated groups (say, senior creditors, junior creditors and share holders in corporate reorganization processes) and each representative has one vote.

I confine my attention to the case where there are two alternatives to avoid such a problem as Condorcet paradox. I call one of the two alternatives a *majority* opinion in a society if the number of supporters for that alternative is larger than that of supporters for the other alternative, which is called a *minority* opinion. By assuming that each member has to pay the same participation cost to go to a meeting place, I analyze how each member in the majority and in minority behave differently in equilibrium.

In an influential paper, Palfrey and Rosenthal (1983) show that there are typically two types of equilibria in this type of game: one is where all voters choose mixed strategies (which are different depending on whether he is in majority or in minority) over whether he abstains or goes to vote, and the other includes some voters who definitely abstain and those who definitely go to vote. Their main focus is on the *asymptotic* properties of these equilibria: when the number of voters is very large, the probability of going to vote tends to be zero for all the voters in the former type of equilibrium, which seems a natural consequence. But, the second type of equilibrium does not have such a property. On the contrary, this note shows that when there are *three* voters a unique mixed Nash equilibrium obtains in Palfrey and Rosenthal's (1983) costly voting game under complete information.[3] Palfrey and Rosenthal (1983) do not find or at least do not mention the uniqueness result when the number of voters is three. The intuitive reason for the uniqueness is that a voter in the minority group does not dare to abstain for sure because he is the only voter who can be pivotal.

In the next section, I introduce a three-player model of endogenous abstention. If there is no participation cost, then the only plausible (in the sense of trembling perfection) Nash equilibrium is such that all the voters participate in voting. After that, I show for a participation cost near to zero, the unique Nash equilibrium is such that the members in the majority participate in voting with probability near one, and the members in the minority participate in voting with probability near zero.

The strong imposition on the numbers of voters could be a good news for laboratory experiments; our uniqueness result might be checked by experiments with just three subjects.[4] The results in the model below depend on the ratio of relative benefit to participation cost, i.e. we can

[1]For endogenous abstention without participation cost, see Feddersen and Pesendorfer (1996, 1999). There the driving force of endogenous abstention is the assumption that there are two types of citizens all of whom has a common benefit: one is well-informed type and the other is uninformed type.

[2]For a recent analysis of a model with a continuum of members, see, e.g. Coate and Conlin (2002). Their behavioral assumption is that each voters maximize the aggregated expected utility of the group to which they belong.

[3]In a companion paper, Palfrey and Rosenthal (1985) consider a model with incomplete information on the costs and/or preferences of other members.

[4]For an experimental study of strategic voting models, see e.g. Fisher and Myatt (2002).

normalize the benefit to a fixed number. But, I do not do so because for experimental purposes we can have two controlling instruments rather than one.

2. The model

There is a social group which consists of three members. They are faced with a problem of choosing one of two alternatives, A and B. Suppose that the number of supporters for A is *two* (voters 1 and 2), and the number of supporters for B is *one* (voter 3). In this sense, alternative A (resp., B) represents a *majority* (resp., *minority*) opinion in this group. We assume that these alternatives are fixed.[5]

A supporter for alternative $x \in \{A, B\}$ gains $u > 0$ if alternative x wins, and gains nothing if alternative $y \in \{A, B\}$, $y \neq x$ wins. We assume that she has to pay a (physical, mental, opportunity, whatever) cost $c \in [0, u/2)$ to participate in the decision-making or abstain from it. After she decides to join the meeting, what she does at the meeting place is to write "A" or "B" on a ballot and cast a vote. The rule to decide a winning alternative is a majority rule.[6] Lastly, what has been described so far is common knowledge among the three members.

Now, member i's ($i = 1, 2, 3$) pure strategy s_i is an element of the following set:

$$S_i = \{(s_i^1, s_i^2) | s_i^1 \in \{P, N\}, s_i^2 : \{P, N\}^3 \to \{Write \text{ "}A\text{"}, Write \text{ "}B\text{"}\}\},$$

where P means "Participate" and N means "Not Participate (i.e. Abstain)."

However, notice here that after she decided to participate in voting (i.e. $s_i^1 = P$), her choice $s_i^2 =$ "Against her preferred alternative" is a weakly dominated strategy for any observation $s_{-i}^1 \in \{P, N\}^2$. Thus, supposing a weakly dominated strategy is not played, we will focus on endogenous determination of $\{s_i^1\}_i$, i.e. what will be a Nash equilibrium in the first stage (deciding whether to participate or not).

We can describe the normal form of the participation game by the following matrices where voter 1 chooses a row , 2 chooses a column and 3 chooses a matrix (P or N). The triples in each box indicate the payoffs to voters 1, 2 and 3 in that order.

P

	P	N
P	$u-c, u-c, -c$	$\frac{u}{2}-c, \frac{u}{2}, \frac{u}{2}-c$
N	$\frac{u}{2}, \frac{u}{2}-c, \frac{u}{2}-c$	$0, 0, u-c$

N

	P	N
P	$u-c, u-c, 0$	$u-c, u, 0$
N	$u, u-c, 0$	$\frac{u}{2}, \frac{u}{2}, \frac{u}{2}$

Let $\sigma_i \in [0, 1]$ be the probability of voter i's participating in voting. The reduced normal form for voters 1 and 2 associated with a mixed strategy σ_3 of voter 3 is given by

[5]Osborne, Rosenthal and Turner (2000) consider a model where alternatives are not fixed, but the final decision is a function (which they call a *compromise function*) of how many and what types of members have participated. Turner and Weninger (2001) is an empirical analysis based on the results of Osborne, Rosenthal and Turner (2000).

[6]When tied or nobody votes, the winner is decided by the fair lottery.

	P	N
P	$u-c, u-c$	$\sigma_3(\frac{u}{2}-c)+(1-\sigma_3)(u-c), \sigma_3\frac{u}{2}+(1-\sigma_3)u$
N	$\sigma_3\frac{u}{2}+(1-\sigma_3)u, \sigma_3(\frac{u}{2}-c)+(1-\sigma_3)(u-c)$	$(1-\sigma_3)\frac{u}{2}, (1-\sigma_3)\frac{u}{2}$

The first observation is that if there is no cost to participate in voting (i.e. $c = 0$), then there is a continuum of Nash equilibria: any σ_3 can consist of a Nash equilibrium as long as voters 1 and 2 participate in voting for sure.[7]

Proposition 1. If we assume that $c = 0$, then the set of Nash equilibria of the participation game is given by $\{(1, 1, \sigma_3)|0 \leq \sigma_3 \leq 1\}$.

Proof. If $c = 0$, then the normal forms defined above are given as follows.

P

	P	N
P	$u, u, 0$	$\frac{u}{2}, \frac{u}{2}, \frac{u}{2}$
N	$\frac{u}{2}, \frac{u}{2}, \frac{u}{2}$	$0, 0, u$

N

	P	N
P	$u, u, 0$	$u, u, 0$
N	$u, u, 0$	$\frac{u}{2}, \frac{u}{2}, \frac{u}{2}$

and

	P	N
P	u, u	$\sigma_3\frac{u}{2}+(1-\sigma_3)u, \sigma_3\frac{u}{2}+(1-\sigma_3)u$
N	$\sigma_3\frac{u}{2}+(1-\sigma_3)u, \sigma_3\frac{u}{2}+(1-\sigma_3)u$	$(1-\sigma_3)\frac{u}{2}, (1-\sigma_3)\frac{u}{2}$

If $0 < \sigma_3 \leq 1$, then $(\sigma_1, \sigma_2) = (1, 1)$ is the unique equilibrium of the reduced game since P then strictly dominates N. If $0 < \sigma_3 \leq 1$, it follows that the only candidates for equilibria in the actual three voter game belong to the set $\{(1, 1, \sigma_3)|0 < \sigma_3 \leq 1\}$. Clearly, these are all equilibria. If $\sigma_3 = 0$, then the equilibria of the reduced game are precisely the strategy pairs in the set $\{(1, \sigma_2)|0 \leq \sigma_2 \leq 1\} \cup \{(\sigma_1, 1)|0 \leq \sigma_1 \leq 1\}$. However, $(\sigma_1, \sigma_2) = (1, 1)$ is the only equilibrium strategy pair of the reduced game to which $\sigma_3 = 0$ is a best response by voter 3 in the actual three voter game. This completes the proof. *QED*

It is easy to see that the participation game with $c = 0$ is nongeneric and that trembling-hand perfection eliminates all the equilibria except $(\sigma_1, \sigma_2, \sigma_3) = (1, 1, 1)$ because $\sigma_3 = 1$ is voter 3's best response to any completely mixed pair of strategies of voters 1 and 2. In all the equilibria, the social welfare is maximized (in the case where $c = 0$) since the majority opinion wins for sure and the sum of utilities is $2u$.

3. Equilibrium under costly participation

[7] Since the game is a finite one, the existence of a Nash equilibrium is obvious.

In this section, we are going to find an equilibrium of the game with non zero participation cost. The following proposition is obtained.

Proposition 2. If $c \in (0, u/2)$, then the unique Nash equilibrium of the participation game is

$$(\sigma_1, \sigma_2, \sigma_3) = (\sqrt{1 - \frac{2c}{u}}, \sqrt{1 - \frac{2c}{u}}, 1 - \sqrt{1 - \frac{2c}{u}}).$$

Proof. First note that, in any equilibrium, it must be the case that $\sigma_3 \leq 2c/u$. To see this, notice that if $\sigma_3 > 2c/u$, then $(\sigma_1, \sigma_2) = (1, 1)$ is the unique equilibrium of the reduced game. However, the best response of voter 3 to $(\sigma_1, \sigma_2) = (1, 1)$ is $\sigma_3 = 0$, a contradiction. If $\sigma_3 \leq 2c/u$, then $0 \leq (1 - 2c/u)/(1 - \sigma_3) \leq 1$ and straightforward arithmetic shows that the unique equilibrium of the reduced game is

$$(\sigma_1, \sigma_2) = (\frac{1 - 2c/u}{1 - \sigma_3}, \frac{1 - 2c/u}{1 - \sigma_3}).$$

There is no equilibrium in which $\sigma_3 = 0$ since, in this case, the reduced game equilibrium is $(\sigma_1, \sigma_2) = (1 - 2c/u, 1 - 2c/u)$ and voter 3's best response is $\sigma_3 = 1$. Hence, voter 3's equilibrium strategy is completely mixed and equilibrium requires that voter 3 be indifferent between *P* and *N*. This means that

$$\frac{u(2c/u - \sigma_3)}{1 - \sigma_3} - c = \left[\frac{2c/u - \sigma_3}{1 - \sigma_3}\right]^2 \frac{u}{2}$$

and solving for σ_3 yields $\sigma_3 = 1 - \sqrt{1 - 2c/u}$ from which it follows that $\sigma_1 = \sigma_2 = \sqrt{1 - 2c/u}$. This completes the proof. *QED*

It is immediate to see that $(\sigma_1, \sigma_2, \sigma_3) \to (1, 1, 0)$ as $c \to 0$. In other words, if a participation cost c is sufficiently small, then voters who belong to the majority go to the meeting place with the probability of near one (that is, the rate of abstention is nearly zero), but a voter who belongs to the minority goes to the meeting place with the probability of near zero (that is, the rate of abstention is nearly one). It is also easy to see that $(\sigma_1, \sigma_2, \sigma_3) \to (0, 0, 1)$ as $c \to u/2$.

In equilibrium, the payoff of players 1 and 2 is $\sqrt{u(u - 2c)}$, which is a strictly decreasing function of the cost c. On the other hand, player 3 obtains $u - c - \sqrt{u(u - 2c)}$, which is strictly increasing with respect to the cost. This is because the indirect, strategic effect, by which players 1 and 2 are discouraged to go voting, is more beneficial for voter 3 than the direct effect of reducing her voting cost is.

4. Welfare

First, note that the maximum social welfare is $2u - c$, which is achieved by making only one player among players 1 and 2 participate in voting.[8]

[8] If the social planner can directly choose one alternative among the two, then the maximized social welfare is $2u$, though.

In the class of mixed strategies $(\sigma_1, \sigma_2, \sigma_3) = (p, p, 1-p)$, the expected social welfare is calculated as

$$
\begin{aligned}
& \sigma_1\sigma_2\sigma_3(2u-3c) + \sigma_1\sigma_2(1-\sigma_3)(2u-2c) \\
& +(1-\sigma_1)\sigma_2\sigma_3(3u/2-2c) + (1-\sigma_1)\sigma_2(1-\sigma_3)(2u-c) \\
& +\sigma_1(1-\sigma_2)\sigma_3(3u/2-2c) + \sigma_1(1-\sigma_2)(1-\sigma_3)(2u-c) \\
& +(1-\sigma_1)(1-\sigma_2)\sigma_3(u-c) + (1-\sigma_1)(1-\sigma_2)(1-\sigma_3)(3u/2) \\
= \; & (1+3p/2-p^3/2)u-(1+p)c.
\end{aligned}
$$

It is easy to see that $p = \sqrt{1-2c/(3u)}$ maximizes the social welfare, whereas in the unique equilibrium $p = \sqrt{1-2c/u}$, which is smaller than $\sqrt{1-2c/(3u)}$. So, in equilibrium, members in the majority group are too discouraged, and those in the minority group are too encouraged to participate in voting from the viewpoint of the social welfare. This is caused by the free-rider problem within the majority group. And, anticipating that, player 3, who belongs to the minority group, becomes aggressive, though the effect of the free-riding inclination is dominant so that in equilibrium players 1 and 2 abstain from voting with a greater probability.

References

Börgers, T. (2003) “Costly Voting” *American Economic Review*, Forthcoming.

Coate, S., and M. Conlin (2002) “Voter Turnout: Theory and Evidence from Texas Liquor Referenda” Mimeo.

Fedderson, T.J., and W. Pesendorfer (1996) “The swing Voter’s Curse” *American Economic Review* **86**, 408-24.

Fedderson, T.J., and W. Pesendorfer (1999) “Abstention in Elections with Asymmetric Information and Diverse Preferences” *American Political Science Review* **93**, 381-98.

Fisher, S.D., and D.P. Myatt (2002) “Strategic Voting Experiments” Mimeo.

Osborne, M., J. Rosenthal, and M. Turner (2000) “Meetings with Costly Participation” *American Economic Review* **90**, 927-43.

Palfrey, T., and H. Rosenthal (1983) “A Strategic Calculus of Voting” *Public Choice* **41**, 7-53.

Palfrey, T., and H. Rosenthal (1985) “Voter Participation and Strategic Uncertainty” *American Political Science Review* **79**, 62-78.

Turner, M., and Q. Weninger (2001) “Meetings with Costly Participation: An Empirical Analysis” Mimeo.