

## On Technology Transfer to an Asymmetric Cournot Duopoly

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### *Abstract*

This note studies the transfer of a cost-reducing innovation from an independent patent-holder to an asymmetric Cournot duopoly that has different unit costs of production. It is found that royalty licensing can be superior to fixed-fee licensing for the independent patent-holder.

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## 1 Introduction

Starting from the pioneering work of Arrow (1962), a large body of literature has developed studying the issue of technology transfer by means of licensing. Kamien (1992) gives an excellent review of the earlier literature. One of the most important results is that of Kamien and Tauman (1986). These authors found that an independent patent-holder always prefers fixed-fee licensing to royalty licensing. Empirically, it has been found that both royalty and fixed-fee licensing are prevalent.<sup>1</sup> By going beyond the basic model of Kamien and Tauman, several papers have recently found that royalty can be superior to fixed-fee for the patent-holder. Wang (1998) and Kamien and Tauman (2002) found this result by considering the patent-holder as an insider in the oligopolistic industry. Saracho (2002) found a similar result when the oligopolistic industry is manager-managed.<sup>2</sup>

This note builds on the original basic model of Kamien and Tauman in that there is an independent patent-holder with a cost-reducing innovation licensing her innovation to a Cournot oligopolistic industry. For simplicity, we focus on a Cournot duopoly with a linear demand. Our departure from the basic model is that the duopoly firms do not have equal unit costs, different from that assumed in the basic model by Kamien and Tauman. Namely, we study licensing to a duopoly with cost asymmetry. The main finding of this note is that when the pre-licensing cost difference between the duopolists is large enough and the patent-holder's innovation is small enough, royalty licensing can be superior to fixed-fee licensing for the patent-holder. Obviously, from an empirical point of view, unequal unit costs must be the rule rather than an exception. For the purpose of most theoretical studies, the equal cost assumption is simplifying and inconsequential to the model. The present note indicates that this assumption plays an important role in the study of licensing choices and relaxing it produces a result that vindicates the prevalence of both fixed-fee and royalty licensing in the real world.

## 2 Basic Model

Consider a Cournot duopoly producing a homogeneous product with the (inverse) market demand  $p = a - Q$ , where  $p$  denotes price and  $Q$  represents industry output. With their old technologies, firm 1 produces at constant unit production cost  $c_1$  and firm 2 produces at constant unit production cost  $c_2$ . It is assumed that  $0 < c_1 \leq c_2 < a$ , i.e., firm 1 is the more efficient firm. The cost-reducing innovation by an independent innovator creates a new technology that lowers any adopting firm's unit cost by the amount of  $\epsilon$ . Obviously,  $\epsilon \leq c_1$ .

With constant unit costs  $c_1$  and  $c_2$ , the Cournot-Nash equilibrium is straightforward to obtain. In equilibrium, the firms' quantities are

$$q_1(c_1, c_2) = \frac{a - 2c_1 + c_2}{3}, \quad q_2(c_1, c_2) = \frac{a - 2c_2 + c_1}{3}, \quad (1)$$

their profits are

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<sup>1</sup> According to Rostoker (1984), royalty alone was used thirty-nine percent of the time, fixed fee alone thirteen percent, and royalty plus fixed fee forty-six percent, among the firms surveyed. Similar evidence has been reported by Macho-Stadler et al (1996), Jensen and Thursby (2001) and others.

<sup>2</sup> Wang (1998) is an extension of Marjit (1990) who studied technology transfer within a Cournot duopoly. Kamien and Tauman (2002) extended Wang (1998) to a Cournot oligopoly.

$$p_1(c_1, c_2) = \frac{(a - 2c_1 + c_2)^2}{9}, \quad p_2(c_1, c_2) = \frac{(a - 2c_2 + c_1)^2}{9}, \quad (2)$$

and total industry output is

$$Q(c_1, c_2) = \frac{2a - c_1 - c_2}{3}. \quad (3)$$

For simplicity and without loss of generality in terms of making the main point of this note, we assume that both firms produce positive output levels in all possible cost configurations. This assumption entails that the less efficient firm 2 produces a positive output even when the more efficient firm 1 is the only adopter of the cost-reducing innovation. Namely,  $q_2(c_1 - e, c_2) > 0$  or equivalently<sup>3</sup>

$$e < a - 2c_2 + c_1. \quad (4)$$

A licensing game consists of three stages. In the first stage, the independent patent-holder sets a fixed licensing fee or a per unit output royalty rate. In the second stage, the duopolists decide simultaneously whether to accept the offer from the patent-holder. In the last stage, the two firms engage in a noncooperative competition in quantities. The patent-holder sets a fixed licensing fee or royalty rate to maximize her total licensing revenue.

### 3 Fixed-fee Licensing

We consider first licensing by means of a fixed fee only. Under fixed-fee licensing, the patent-holder licenses her cost-reducing technology to either firm at a fixed fee  $F$  which is invariant of the quantity the licensee will produce using the new technology. The maximum license fee a firm is willing to accept is the one that makes it indifferent between licensing and not licensing the new technology. If firm 2 does not license, the maximum fee firm 1 is willing to accept is equal to

$$p_1(c_1 - e, c_2) - p_1(c_1, c_2) = \frac{4}{9}(a - 2c_1 + c_2 + e)e. \quad (5)$$

If firm 2 licenses, the maximum fee firm 1 is willing to accept is equal to

$$p_1(c_1 - e, c_2 - e) - p_1(c_1, c_2 - e) = \frac{4}{9}(a - 2c_1 + c_2)e. \quad (6)$$

For firm 2, it is willing to pay

$$p_2(c_1, c_2 - e) - p_2(c_1, c_2) = \frac{4}{9}(a - 2c_2 + c_1 + e)e. \quad (7)$$

if firm 1 is not a licensee and

$$p_2(c_1 - e, c_2 - e) - p_2(c_1 - e, c_2) = \frac{4}{9}(a - 2c_2 + c_1)e. \quad (8)$$

if firm 1 is a licensee. Obviously, both firms are willing to pay more if the other firm is not a licensee, as indicated by the fact is (5) is larger than (6) and (7) is larger than (8). Since (5) is larger than (7), if the patent-holder sells only one license she will charge a fee equal to (5), the amount firm 1 is willing to pay. Since (8) is smaller than (6), the patent-holder has to set a fee equal to (8) in order to sell two licenses.<sup>4</sup>

Based on the above results, we have the following proposition.

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<sup>3</sup> When  $c_1 = c_2 = c$ , (4) becomes  $e < a - c$ . Hence, this assumption essentially means that we consider non-drastring innovations.

<sup>4</sup> For simplicity, we assume that a firm will buy a license when it is indifferent between buying and not buying.

**Proposition 1.** Under fixed-fee licensing, the patent-holder will license her innovation to firm 1 only for  $\varepsilon > a + 4c_1 - 5c_2$  and she will license to both firms for  $\varepsilon < a + 4c_1 - 5c_2$ .

Proof: The patent-holder's licensing revenue is equal to (5) if she sells only one license (to firm 1) by charging a fixed fee equal to (5). Denote her revenue from selling one license by

$$M_L = \frac{4}{9}(a - 2c_1 + c_2 + \varepsilon)\varepsilon. \quad (9)$$

The patent-holder's licensing revenue is equal to two times (8) if she sells two licenses by charging a fixed fee equal to (8). Denote her revenue from selling two licenses by

$$M_{LL} = \frac{8}{9}(a - 2c_2 + c_1)\varepsilon. \quad (10)$$

The conclusion of the proposition is immediate by comparing  $M_L$  and  $M_{LL}$ .

This proposition indicates that, under fixed-fee licensing, large innovations will be licensed to the more efficient firm (firm 1) and small innovations will be licensed to both firms.<sup>5</sup>

#### 4 Royalty Licensing

Consider now licensing by means of a royalty only. Under a royalty licensing method, the patent-holder sets a fixed royalty rate  $r$  and the amount of royalty each firm pays will depend on the output level it will produce using the new technology. By adopting the new technology, firm 1's unit production cost becomes  $c_1 - \varepsilon + r$  and firm 2's unit production cost becomes  $c_2 - \varepsilon + r$ . Obviously, if  $r \leq \varepsilon$  both firms will choose to license and if  $r > \varepsilon$  neither firm will license.

Utilizing (3), for  $r \leq \varepsilon$  the patent-holder's licensing revenue is

$$rQ(c_1 - \varepsilon + r, c_2 - \varepsilon + r) = \frac{r(2a - c_1 - c_2 + 2\varepsilon - 2r)}{3}. \quad (11)$$

Maximizing (11) with respect to  $r$  subject to the constraint that  $r \leq \varepsilon$  implies

$$r = \begin{cases} \varepsilon & \text{if } \varepsilon \leq \frac{2a - c_1 - c_2}{2}; \\ \frac{2a - c_1 - c_2 + 2\varepsilon}{4} & \text{if } \varepsilon > \frac{2a - c_1 - c_2}{2}. \end{cases}$$

It is straightforward to verify that the assumption (4) implies that  $\varepsilon \leq (2a - c_1 - c_2)/2$ . Hence, the patent-holder's optimal royalty rate is  $r = \varepsilon$ . Substituting this into (11) gives the patent-holder's licensing revenue under royalty licensing, given as

$$R = \frac{(2a - c_1 - c_2)\varepsilon}{3}. \quad (12)$$

#### 5 Fee vs. Royalty

We evaluate next the superiority of fixed fee licensing versus royalty licensing. It is summarized

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<sup>5</sup> In particular, if  $c_1 = c_2 = c$  then the known result that all non-drastic innovations are licensed to both firms and drastic innovations are licensed to only one firm is implied.

in the following proposition.<sup>6</sup>

**Proposition 2.** For the patent-holder, royalty licensing is superior to fixed-fee licensing if and only if  $\varepsilon < (2a + 5c_1 - 7c_2)/4$  and  $c_2 > (2a + 11c_1)/13$ .

Proof. If  $\varepsilon < (2a + 5c_1 - 7c_2)/4$  then from (9) and (12),  $R > M_L$ . If  $c_2 > (2a + 11c_1)/13$  then from (10) and (12),  $R > M_{LL}$ . Hence, if  $\varepsilon < (2a + 5c_1 - 7c_2)/4$  and  $c_2 > (2a + 11c_1)/13$  then royalty licensing is superior to licensing via a fixed fee either to firm 1 only or to both firms. This proves the ‘if’ part of the proposition.

For the ‘only if’ part, consider first the case:  $\varepsilon > a + 4c_1 - 5c_2$ . By Proposition 1, in this case the patent-holder will license only to firm 1 under fixed-fee licensing. Thus, if royalty is superior to fixed-fee we must have  $R > M_L > M_{LL}$ . From (9) and (12),  $R > M_L$  implies  $\varepsilon < (2a + 5c_1 - 7c_2)/4$ . From (10) and (12),  $R > M_{LL}$  implies  $c_2 > (2a + 11c_1)/13$ . Consider next the case:  $\varepsilon < a + 4c_1 - 5c_2$ . By Proposition 1, in this case the patent-holder will license to both firms under fixed-fee licensing. Thus, if royalty is superior to fixed-fee licensing we must have  $R > M_{LL} > M_L$ . Again  $R > M_L$  and  $R > M_{LL}$  imply, respectively,  $\varepsilon < (2a + 5c_1 - 7c_2)/4$  and  $c_2 > (2a + 11c_1)/13$ . This completes the proof for the necessary part of the proposition.

The condition  $\varepsilon < (2a + 5c_1 - 7c_2)/4$  implies that for a small enough innovation royalty can be superior for the patent-holder. The condition  $c_2 > (2a + 11c_1)/13$  can be rewritten as  $c_2 - c_1 > 2(a - c_2)/11$ , indicating that for royalty to be better for the patent-holder the cost difference between the efficient firm and the inefficient firm has to be large enough relative to the difference between the maximum price consumers are willing to pay and the inefficient firm’s unit cost. Obviously, the last condition is impossible to hold when the firms are equally efficient, implying the known result that in a symmetric market fixed-fee licensing is preferred to royalty licensing by the patent-holder.

The intuition for the result in Proposition 2 is fairly straightforward. Under royalty licensing with symmetric costs ( $c_1 = c_2$ ) as in the basic model by Kamien and Tauman (1986) or asymmetric costs ( $c_1 < c_2$ ) as in the present model, firms do not gain in efficiency post-licensing and the total industry output is unchanged post-licensing. As a result, the patent-holder’s total licensing revenue is proportional to the size of her innovation as shown by (12). Under fixed-fee licensing, the licensing firm(s) become more efficient, as a result the patent-holder can exploit this gain in their efficiency by reaping a licensing revenue via fixed fee. This total revenue is always larger than that obtainable under royalty licensing when the firms are equally efficient, as shown in Kamien and Tauman (1986). However, the patent-holder’s power in appropriating this efficiency gain is mitigated when the firms are not equally efficient. This mitigation takes one of two forms. When licensing to both firms, the fee has to be at a level the less efficient firm is willing to accept. When the pre-licensing cost difference between

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<sup>6</sup> Based on (3), total industry output is always higher under fixed-fee licensing than under royalty licensing since at least one firm produces at a lower unit cost post-licensing under fixed-fee licensing while both firms’ unit costs stay at the same level post-licensing under royalty licensing. It is implied immediately that consumers always prefer fixed-fee to royalty licensing, as in the basic model by Kamien and Tauman (1986).

the two firms is sufficiently large, this renders fixed-fee licensing less preferable to royalty licensing as demonstrated above by comparing (10) and (12). When licensing to only one firm, the potential benefit from licensing to the other firm is totally lost. This loss is more significant (relative to total licensing revenue) the smaller is the innovation. When the innovation is sufficiently small, licensing to one firm via a fixed fee becomes less preferable to licensing by royalty as demonstrated above by comparing (9) and (12).

Figure 1 depicts a graphic illustration for the comparison between fixed-fee licensing and royalty licensing. In this graph,  $a = 1$  and the value of  $\varepsilon$  is fixed. The unit square box contains all possible cost configurations. The assumption  $c_1 \leq c_2$  removes all points below the forty-five degree line and the assumption  $a - 2c_2 + c_1 > 0$  removes all points above the AB line, where point A corresponds to  $(c_1, c_2) = (0, 0.5)$  and point B corresponds to  $(c_1, c_2) = (1, 1)$ . Hence, the triangle OAB represents all cost configurations of concern. At points C, D and E, respectively,  $(c_1, c_2) = (0, 2/13)$ ,  $(0, (2 - 4\varepsilon)/7)$  and  $(1 - 13\varepsilon/3, 1 - 1\varepsilon/3)$ . Based on Proposition 2, for cost configurations below the DE line royalty is better than licensing to one firm via a fixed fee, and for cost configurations above the CB line royalty is better than licensing to both firms via a fixed fee. Hence, the triangle CDE corresponds to all cost configurations such that royalty licensing is preferred to fixed-fee licensing by the patent-holder. Obviously, this triangle is non-vacuous if point D is higher than point C (i.e.,  $\varepsilon < 3/13$ ).

## 6 Concluding Remarks

This note has studied and compared licensing by means of a fixed fee and licensing by means of a royalty to a Cournot duopoly by an independent patent-holder with a cost-reducing innovation. It is found that licensing by means of a royalty may be superior to licensing by means of a fixed fee from the view point of the patent-holder when the duopolists have different unit costs of production.

We have maintained the assumption that the cost-reducing innovation lowers any adopting firm's marginal cost by a fixed amount. An alternative assumption, that is perhaps at least as natural as the one we adopted, is the case where the innovation lowers any adopting firm's marginal cost by a fixed proportion. The mechanics of the model under this alternative assumption is mostly the same as the one presented in the preceding sections with the following three noticeable facts. First, the main conclusion in this note that both royalty licensing and fixed-fee licensing can be optimal for the independent patent-holder when firms have different levels of marginal cost continues to hold. Second, there are three possibilities under fixed-fee licensing: licensing to the more efficient firm 1 only, the less efficient firm 2 only, and to both firms. Third, under royalty licensing, either the less efficient firm is licensed or both firms are licensed. The main reason that accounts for the additional possibilities under either fixed-fee or royalty licensing is that with proportional reduction in marginal costs, the less efficient firm always gets a larger reduction in marginal cost from the innovation than does the more efficient firm. As a result, the less efficient firm is willing to accept a higher per unit output royalty and in the case of fixed-fee licensing, it may reap a higher gain in profit than does the more efficient firm.

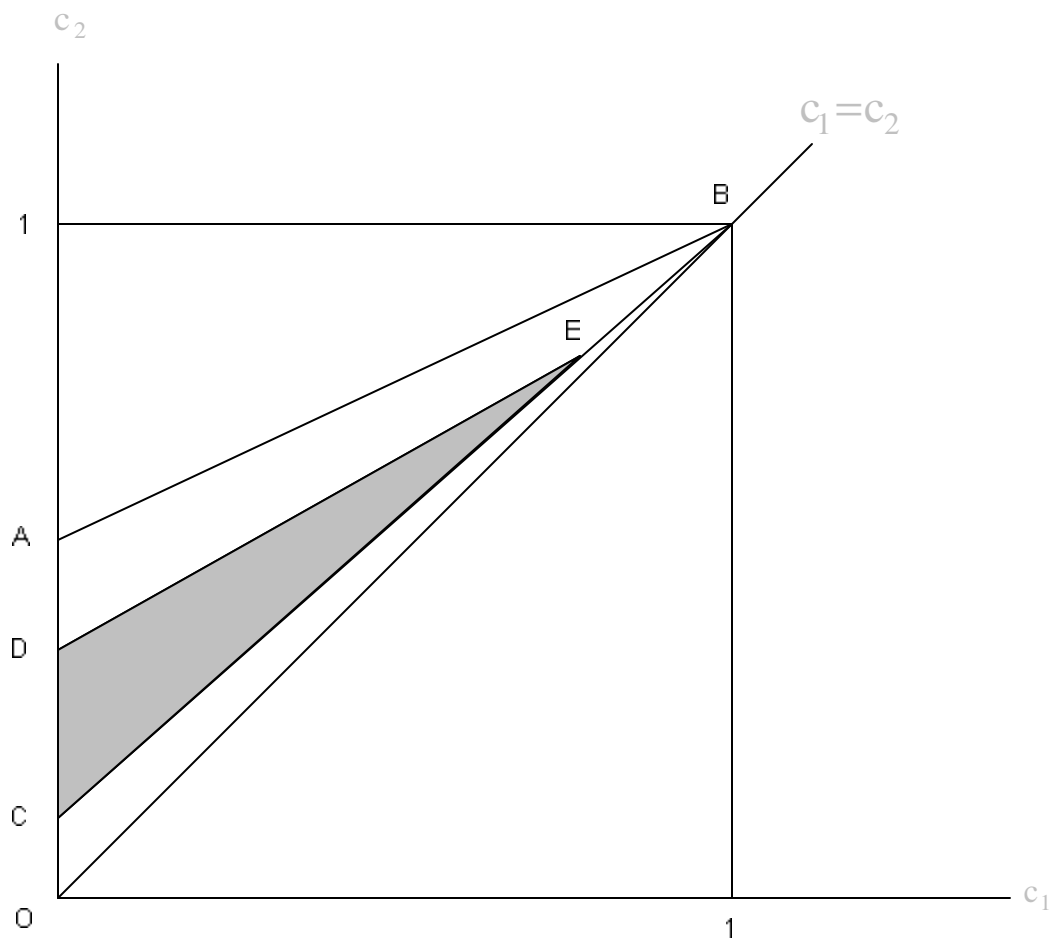


Figure 1  
 Triangle (shaded) in which royalty is preferred to  
 fixed-fee licensing given a fixed epsilon

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