E C O N O M I C S   B U L L E T I N

Spatial Cournot competition in a circular city with transport cost differentials

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Abstract

For an even number of firms with identical transport cost, spatial Cournot competition in a circular city generates a continuum of equilibria. We establish that any transport cost differential between the firms, however small, may eliminate all the equilibria, except the partial agglomeration equilibrium pattern characterized in Matsushima (2001).

I am grateful to an anonymous reviewer for extremely helpful comments. I also thank the Richard T. Farmer School of Business, Miami University for financial support during this project and Debashis Pal for helpful comments.
Citation: Gupta, Barnali, (2004) "Spatial Cournot competition in a circular city with transport cost differentials." *Economics Bulletin*, Vol. 4, No. 15 pp. 1−6
Submitted: October 11, 2004. **Accepted:** November 29, 2004.
URL: http://www.economicsbulletin.com/2004/volume4/EB−04D40010A.pdf

1. Introduction

Since Pal (1998), spatial Cournot competition along a circular market has gained increasing attention.[1] In recent years, there have been a number of applications with Cournot competition in a circular city. (See for example, Matsumura (2003), Matsushima and Matsumura (2003), Nariu and Flath (2004)). Although spatial Cournot competition has attractive and useful properties, it is known to have multiple equilibria, which impedes the applicability of this framework. The objective of this work is to find reasonable criteria that resolve the multiplicity of equilibria.

Matsushima (2001) is the first to demonstrate the multiple equilibria by establishing an equilibrium, which is qualitatively different from the *equidistant* equilibrium location pattern as characterized in Pal (1998).[2] He establishes that half of the firms agglomerating at one point and the other half agglomerating at the diametrically opposite point, can also be sustained as an equilibrium location pattern (*partial agglomeration*).[3] In recent work, Gupta et al (2004) show that there is a continuum of equilibria, such that with $n = 2m$ ($m \geq 1$) firms, if they locate at opposite ends of any set of m diameters of the circle, then the locations are sustainable as SPNE locations. Hence, the equilibria characterized by Pal and Matsushima are in fact special cases of the set of infinitely many equilibria.

Observe that the equilibrium locations discussed in all the aforementioned work are derived with the assumption of identical transport costs across firms. Indeed this is a standard assumption in this literature. One might argue that transport cost differentials across firms will not qualitatively change the equilibrium and are hence of minimal interest. This work will show that this is actually far from the truth. We allow firms to have different transport costs, and demonstrate that this leads to qualitatively different equilibrium location patterns. In fact, this paper will show that of the infinitely many equilibria along opposite ends of any m diameters of the circle, only the *partial agglomeration* equilibrium in Matsushima (2001) is robust to non-identical transport costs.

[1] See Anderson and Neven (1991) and Hamilton, Thisse and Weskamp (1989) for spatial Cournot competition in Hotelling's (1929) linear city.

[2] Pal (1998) shows that the firms locates equidistantly along the perimeter of the circular city.

[3] This result holds for an even number of firms. When the number of firms is $2n + 1$, $(n + 1)$ firms agglomerate at one point and n firms agglomerate at the diametrically opposite point.

2. The Model

We consider a spatial Cournot oligopoly serving a circular market with perimeter one. The consumers are distributed uniformly on the circle. The market demand at each point x on the circle is given by $p(x) = a - bQ(x)$, where $a, b > 0$ are constants and independent of x. $Q(x)$ is the aggregate quantity supplied and $p(x)$ is the market price at x. There are $n = 2m$ $(m \geq 1)$ firms who choose their locations on the circle. For simplicity, we assume $m = 2$. We conjecture, however, that the results hold qualitatively for all m. The points on the circle are identified with numbers in $[0, 1]$, the north most point being 0 and the values increasing in a clockwise direction. Thus, the north most point is considered both 0 and 1. The vector $\underline{\xi} = (\xi_1, \xi_2, \xi_3, \xi_4)$ denotes the locations of the 4 firms and the vector $\underline{\xi}^{-i} = (\xi_j, \xi_k, \xi_l)$ denotes the location of all firms except i $(1 \leq i \leq 4)$. The firms produce and sell a homogeneous output that they deliver to consumers. Arbitrage among consumers is assumed to be infeasible, enabling the firms to discriminate across consumers. The firms have identical production technology, with constant marginal and average cost (both normalized to zero). The transportation technologies, however, may differ among firms. Let $t_i > 0$, be the linear transport cost per unit distance for firm i $(1 \leq i \leq 4)$. The good can be transported only along the perimeter of the circle. Each firm serves a market point x incurring the lowest possible transport cost. We also assume that $a \geq 2 \max[t_1, t_2, t_3, t_4]$. This condition ensures that all firms will always serve the entire market. We study the subgame perfect Nash equilibria (SPNE) of a two-stage game, where firms choose their locations in stage one and compete in quantities in stage two.

3. Properties of a location equilibrium

Notation 1. $c_i(x) = \min[t_i |\xi_i - x|, t_i(1 - |\xi_i - x|)]$ *denotes Firm i's* $(1 \leq i \leq 4)$ *delivered marginal cost at* x*.*

Notation 2. *Let* $\widehat{\xi}$ *be the point diametrically opposite* ξ*. Then* $L(\xi)$ *denotes the half circle from* ξ *to* $\widehat{\xi}$ *(not including* $\widehat{\xi}$*) in the clockwise direction and* $R(\xi)$ *denotes the half circle from* ξ *to* $\widehat{\xi}$ *(not including* ξ*) in the counter-clockwise direction.*

Definition 1. *Competitors' aggregate cost median:* ξ *is a competitors' aggregate cost median for Firm i if and only if the aggregate delivered marginal cost of all other firms in* $L(\xi)$ *equals the aggregate delivered* *marginal cost of all other firms in* $R(\xi)$. *That is,* $\int_{x\in L(\xi)}\left[\sum_{j\neq i}c_j(x)\right]dx$ $=\int_{x\in R(\xi)}\left[\sum_{j\neq i}c_j(x)\right]dx$

Proposition 1. *At SPNE locations, each firm maximizes its profit only if it locates at its competitors' aggregate cost median.*

Proof. See proof of Proposition 1 in Gupta et al. (2004). They present the proof for identical transport cost. The result, however, holds for non-identical transport cost and the proof is similar. □

The search for SPNE locations, therefore, can be confined to a set of locations such that each firm locates at its competitors' aggregate cost median. If there are multiple vectors that satisfy the competitors' cost median property for each firm, the second order condition can be used to eliminate vectors that satisfy the competitors' aggregate cost median property, but are not a profit maximizing location for each firm, given the locations of the others. It can be checked that the second order condition requires $\sum_{j\neq i}c_j(\xi_i)\geq\sum_{j\neq i}c_j\left(\widehat{\xi_i}\right)$.

4. The SPNE locations

Proposition 2. *(Gupta et al, 2004) Suppose that* $t_1=t_2=t_3=t_4$, *and* $n=2m$ $(m\geq 1)$. *If the firms locate at opposite ends of any set of* m *diameters of the circle, then the firm locations are sustainable as SPNE locations.*

The above proposition holds for identical transport cost for all firms and results in a continuum of equilibria. However, we will establish that this result is critically dependent on identical transport costs. To do so, consider a marginal adjustment in the transport cost assumption, such that firms 1, 2, and 3 have identical transport cost t , while firm 4's transport cost is $t+\varepsilon$, where $\varepsilon>0$.

Proposition 3. *If* $t_1=t_2=t_3=t$, *but* $t_4=t+\varepsilon,\varepsilon>0$, *then the equilibrium locations described in Proposition 2 cannot be sustained as SPNE locations, except when the firms agglomerate at the opposite ends of the same diameter.*

Proof. Let firms 3 and 4 locate at 0 and $\frac{1}{2}$ respectively and let firm 2 locate at $\xi<\frac{1}{2}$. It can be checked that firm 1 will not locate at the opposite end of the diameter, across from firm 2, at $1-\xi$, since its first order condition given by the competitor′s aggregate cost median (Proposition 1), is violated. It can be checked that the competitors' aggregate cost median property is satisfied if the firms locate at the opposite ends of a diameter, with at least one firm at each end point. □

The intuition behind Proposition 3 is as follows. Suppose that there are two pairs of firms: (*i*) firms 1 and 2 locate at ξ_a and $\xi_a + \frac{1}{2}$ $\left(\xi_a \in \left[0, \frac{1}{2}\right]\right)$ respectively; (*ii*) firms 3 and 4 locate at ξ_b and $\xi_b + \frac{1}{2}$ $\left(\xi_b \in \left[0, \frac{1}{2}\right]\right)$ respectively. To satisfy the competitor's aggregate cost median, the inefficient firm (firm 4) must be located at one end of the diameter passing through ξ_a, because otherwise the pair of firms 3 and 4 distort the symmetry of the aggregate costs.

Proposition 4. *Suppose* $t_1 = t_2 = t_3 = t$, *but* $t_4 = t + \varepsilon, \varepsilon > 0$. *(i) Two firms located at* 0 *and two firms located at* $\frac{1}{2}$ *can be sustained as SPNE locations if and only if* $t \geq \varepsilon$. *(ii) Firms 1, 2, 3 located* 0 *and firm 4 located at* $\frac{1}{2}$ *can be sustained as SPNE locations if and only if* $t \leq \varepsilon$.

Proof. First note that the locations satisfy the competitors' cost median property (Proposition 1) by symmetry. The rest of the proof follows from the second order conditions $\left(\sum_{j \neq i} c_j\left(\xi_i\right) \geq \sum_{j \neq i} c_j\left(\widehat{\xi_i}\right)\right)$. Without loss of generality, let firms 1 and 2 locate at 0 and let firm 4 locate at $\frac{1}{2}$. Now consider firm 3. The aggregate competitors' cost at 0 equals $\frac{1}{2}t_4$ while at $\frac{1}{2}$ it equals $\frac{1}{2}(t_1 + t_2) = \frac{1}{2}(2t)$. For $t_4 > 2t$, firm 3 will locate at 0, whereas for $t_4 < 2t$, firm 3 will locate at $\frac{1}{2}$. □

Proposition 3 establishes that the result with infinitely many equilibria characterized in Gupta et al (2004) is sensitive to identical transport costs for all firms. For any nonidentical transport cost, however small the difference, the result may no longer hold. Only an equilibrium where the firms cluster at opposite ends of one diameter, as characterized in Matsushima (2001), remains robust to the assumption of nonidentical transport costs. However, we show that there does not have to be an equal number of firms at opposite ends of the diameter. If the transport cost differential is large enough, the least efficient firm is located by itself at one end of the diameter. This result that all the other firms cluster and do so away from the least efficient firm is quite counter-intuitive, as one might (incorrectly) imagine that locating close to the inefficient firm would be advantageous.[4]

[4]It is also interesting to note that a different pattern of location equilibrium may exist where the least efficient firm with transport cost t + ε locates at 0 and the other three firms locate at $\frac{1}{2}, \xi$ and $1 - \xi$ where $\frac{1}{2} > \xi > \frac{1}{4}$. That is, the three efficient firms may not cluster at one market point but will all locate along one half of the market perimeter.

5. Concluding Remarks

This paper addresses the issue of infinitely many equilibria associated with spatial Cournot competition in a circular city. We show that this result is sensitive to identical transport cost across all firms. If transport cost differentials are allowed, it is no longer valid. In fact, the only part of the original result to survive is that firms cluster (not necessarily in equal numbers) at opposite ends of a single diameter. This work makes the point that different transport cost across firms is not a non trivial issue and it qualitatively changes our understanding of the location patterns with Cournot competition in a circular city.

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