E C O N O M I C S B U L L E T I N

Robust Comparisons of Natural Resources Depletion Indices

Paul Makdissi
Département d'économique and GRAP, Université de Sherbrooke

Quentin Wodon
AFTPM, World Bank

Abstract

This note applies tools from the stochastic dominance literature on poverty to environmental data in order to test in a robust way whether over−consumption and thereby depletion of natural resources is increasing over time. The method is illustrated with country data on per capita CO2 emissions.

The opinions expressed here are those of the authors and need not represent those of the World Bank, its Executive Directors, or the countries they represent.

Citation: Makdissi, Paul and Quentin Wodon, (2004) "Robust Comparisons of Natural Resources Depletion Indices." *Economics Bulletin,* Vol. 9, No. 2 pp. 1−9
Submitted: April 7, 2004. **Accepted:** June 8, 2004.
URL: http://www.economicsbulletin.com/2004/volume9/EB−04I30002A.pdf

1 Introduction

Human production of greenhouse gases exceeds the capacity of the earth's natural sinks to absorb them (e.g., IPCC, 2001; Heil and Selden, 2001). Is the problem getting worse or better over time? This may depend on the standards used for measurement. In order to compare to what extent the problem of excess emissions is becoming worse or not, we adopt an approach similar to the technique of stochastic dominance used in the literature on poverty. In that literature, the focus is on the households who have a level of consumption or income below a given threshold (the poverty line). The technique of stochastic dominance is then used to assess whether poverty comparisons over time are robust to the choice of the poverty line and poverty measure (e.g., Atkinson, 1987; Zheng, 1999, 2000; Duclos and Makdissi, 2004). Our setting here is reversed: we make comparisons in terms of over-consumption by countries, i.e. consumption or emissions above a threshold which represents a natural resources sustainability line.

From a technical point of view, the main difference with the poverty literature is that we build "downward" instead of "upward" dominance conditions. These conditions can then be used to assess whether comparisons of over-consumption are robust to the choice of the natural resources sustainability line and the measures of natural resource depletion used to aggregate at the world level over-consumption by countries. Section 2 presents the approach. Section 3 provides empirical estimates in the case of per capita CO_2 emissions. Although we focus here on country-level CO_2 emissions, the method could be applied to other types of over-consumption by countries, as well as to over-consumption by firms or households in applications with appropriate data for these agents.

2 Natural Resource Depletion Indices

A natural resource has a renewal capacity of c units per agent and per period. If an agent's consumption is higher than c, it depletes the common environmental capital. That is, any consumption level higher than c is not sustainable if all agents have the same level of consumption. Since agents consuming more than c are "mortgaging" future consumption, we will say that they over-consume. Let $F(x)$ be the cumulative distribution function of consumption of the natural resource x. This distribution is defined over $[0, a]$. In order to aggregate the level of over-consumption of many agents, we use an additive index of natural resource depletion

$$S_F(c) = \int_c^a s(x, c)\, dF(x), \tag{1}$$

where $s(x, c)$ represents the contribution to total over-consumption of an individual with consumption x. We assume that $s(x, c) \geq 0$ for all x and that $s(x, c) = 0$ for all $x \leq c$. We also assume that $s(x, c)$ is continuous over $[0, a]$. A particular subclass of those additive indices corresponds to the FGT indices for poverty (Foster, Greer and Thorbecke, 1984). The equivalent of the FGT indices for over-consumption measurement are given by

$$S_F^{\alpha}(c) = \int_c^a \left(\frac{x-c}{c}\right)^{\alpha} dF(x), \tag{2}$$

where $\alpha \geq 0$ is a parameter representing inequality aversion with respect to over-consumption. If $\alpha = 0$, we have the over-consumption headcount which gives the proportion of the population who is over-consuming

$$S_F^{\mathbf{0}}(c) = 1 - F(c). \tag{3}$$

If $\alpha = 1$, the index represents the over-consumption gap which gives the average over-consumption in the entire population. For $\alpha > 1$, the index will be sensitive to the inequality in the distribution of over-consumption[1]. For example, $\alpha = 2$ corresponds to the over-consumption squared gap. Using those indices enables us to make comparisons of over-consumption between groups of agents (or countries) or between time periods.

However, there may be uncertainty regarding the exact value of c. If this is the case, a question arises as to wether or not over-consumption comparisons (between groups of agents or over time) are robust to changes in c. Furthermore, even if we can agree on an exact value for c, comparions of over-consumption may also depend on the particular functional form chosen for $s(x,c)$. While the FGT-inspired measures of over-consumption presented above have well known attractive properties, some observers might very well prefer to use other indices of over-consumption.

Stochastic dominance conditions enable us to test for the robustness of over-consumption comparisons to particular choices for both c and $s(x,c)$. In order to describe those conditions, we define two classes of over-consumption indices:

$$\Sigma^1 = \left\{ S(c) : \frac{\partial s(x,c)}{\partial x} \geq 0 \right\}, \tag{4}$$

and

$$\Sigma^2 = \left\{ S(c) : \frac{\partial s(x,c)}{\partial x} \geq 0 \text{ and } \frac{\partial^2 s(x,c)}{\partial x^2} \geq 0 \right\}. \tag{5}$$

The class Σ^1 includes all additive indices such that an increase of the consumption of one agent cannot induce a decrease in aggregate over-consumption. Indices of the class Σ^2 have an additional property: the social evaluation of over-consumption increases more than proportionally. That is, these indices obey the so-called Pigou-Dalton transfer principle, whereby in our context a reduction in consumption for an individual over-consuming more together with an equivalent increase in consumption

[1]For applications on inequality of CO_2 emissions, see Heil and Wodon (1997, 2000)

for an individual who over-consumes less will reduce the aggregate index of over-consumption. Indices of the class Σ^2 are therefore sensitive to the inequality in the distribution of over-consumption.

If there is a consensus on the fact that the over-consumption threshold c cannot be lower than some minimum value c^-, it is feasible to lay out the following first order stochastic dominance condition (for ease of exposition the proofs of the propositions are shown in appendix).

Proposition 1 *Take two consumption distributions F and G. Over-consumption decreases when we move from F to G for all over-consumption indices $S \in \Sigma^1$ and all over-consumption thresholds $c \in [c^-, a]$ if*

$$G(x) \geq F(x) \text{ for all } x \in \left[c^-, a\right]. \tag{D1}$$

This result is equivalent to the result obtained for poverty dominance curves in the poverty literature. As in the case of robust poverty comparisons, it is also possible to lay out a second order stochastic dominance condition for indices that are sensitive to inequality. In order to expose this condition we define a new function $\Psi_F(x) = \int_x^a F(u)\,du$.

Proposition 2 *Take two consumption distributions F and G. Over-consumption decreases when we move from F to G for all over-consumption indices $S \in \Sigma^2$ and all over-consumption thresholds $c \in [c^-, a]$ if*

$$\Psi_G(x) \geq \Psi_F(x) \text{ for all } x \in \left[c^-, a\right]. \tag{D2}$$

If the dominance test fails over $[c^-, a]$ (whether at the first or second order), it may still be possible to obtain robust over-consumption comparisons for a restricted range of over-consumption lines. These over-consumption thresholds c^1 and c^2 beyond which conditions D1 and D2 do not hold anymore would then be given by

$$c^1 = \inf\{c : G(x) \geq F(x), \quad x \in [c, a]\}, \tag{6}$$

and

$$c^2 = \inf\{c : \Psi_G(x) \geq \Psi_F(x), \quad x \in [c, a]\}. \tag{7}$$

As in the literature on poverty, it is feasible to increase the order of dominance in order to make robust comparisons of over-consumption over time or between groups, with the understanding that at higher orders of dominance, the corresponding aggregate indices of over-consumption have more restrictive properties (see Duclos and Makdissi, 2004). In general, robust comparisons typically can be obtained by either increasing the order of dominance, or accepting higher minimum over-consumption thresholds for the comparisons.

3 Application to CO_2 emissions

To illustrate the methodology, we consider CO_2 emissions. The idea is to test whether over-consumption, or more precisely in the application presented here over-emissions, have gone up over time. Countries are the units of observations, but each country is weighted by its population. Per capita CO_2 emissions are used to run the tests. The data come from the Carbon Dioxide Information Analysis Center of the U.S. Oak Ridge National Laboratory (the data were prepared by Marland et al., 2002). The emissions estimates include all major sources of anthropogenic carbon emissions, except those from land-use changes which are not sufficiently developed. We use a sub-sample of 152 countries for which we have emissions in both 1985 and 1998, and we compare emissions for these two years.

Figure 1 provides the test for first order dominance. In the poverty literature, the test is used to check whether for a range of poverty measures and poverty lines, one can say that there is more poverty in one year versus another. The only condition imposed on the poverty measures under first order dominance is that as income goes up, poverty should go down, or at least it should not increase. Here, we are checking whether we can say with confidence that there is more over-consumption in one period versus another, and the only condition imposed at the first order is that as consumption of the natural resource goes up, the index of over-consumption also goes up, or at least is not reduced.

In Figure 1, in order to better show where the emissions incidence curves for both years intersect, we have cut the horizontal axis at a value of 11 metric tons per capita, even though one country (Qatar, because of flaring) has a higher level of per capita emissions in 1998. The curves for 1985 and 1998 intersect at a level of emissions above approximately 3.3 metric tons, which is very high since only a dozen countries emit at higher levels in both years. These countries account for less than 7 percent of the sample's population (as indicated on the vertical axis which provides the cumulative density of the population with a level of consumption below the value indicated at the horizontal axis). The largest emitter in terms of population size in this group of a dozen countries is the United States.

The main implication of Figure 1 is that if we were considering a threshold for over-consumption below 3.3 metric tons per person per year, we would not be able to establish that over-consumption has gone up between 1985 and 1998, essentially because some of the countries (mostly in Europe) who emit quite a bit but less than 3.3 metric tons per person per year have made efforts over the period in review in order to curtail their emissions per capita. Thus, for natural resources sustainability lines below 3.3 metric tons, there is no first order dominance of the 1985 distribution over the 1998 distribution. We cannot say with much confidence that over-consumption has gone up over time for the most general class of natural resources depletion or over-consumption indices and for reasonable thresholds of sustainability.

Figure 2 provides the test for second order dominance. The curves in Figure 2 are obtained by taking the cumulative value of the integral under the curves in Figure 1. In the poverty literature, this test is related to measures of poverty such as the poverty gap which takes into account the distance separating the poor from the poverty line. In our context, the curves are related to the over-consumption gap, which captures by how much countries over-consume in per capita terms. In Figure 2, we have shown the curves only for emissions below 4 metric tons per capita. This is because we know that over that threshold, the curve for 1985 will necessarily be higher than that for 1998 (meaning that there is more over-consumption in 1998 than in 1995), since in Figure 1, for every level above 4 metric tons, the incidence curve for 1985 is higher than that for 1998. What Figure 2 shows is that even for sustainability thresholds below 4 metric tons, indeed for any values of the natural resource depletion line, there is more over-consumption in 1998 than in 1985 at the second order of dominance.

4 Conclusion

In this note, we have adapted and applied tools from the stochastic dominance literature on poverty to environmental data in order to show how one can test in a robust way whether over-consumption and thereby depletion of natural resources is increasing over time. The method was illustrated with country data on per capita CO_2 emissions. We found that at the first order of dominance, countries have unambiguously over-consumed (i.e., emitted CO_2) more in 1998 than in 1985 only for natural resource depletion lines above 3.3 metric tons per person per year, which is a very high threshold. However, at the second order of dominance, which takes into account by how much countries over-consume above the sustainability threshold, we have found that whatever the threshold used for the comparisons, there was more over-consumption in 1998 than in 1985.

References

[1] Atkinson, A.B. (1987), On the Measurement of Poverty, *Econometrica*, 55, 759-764.

[2] Duclos, J.Y. and P. Makdissi (2004), Restricted and Unrestricted Dominance for Welfare, Inequality and Poverty Orderings, *Journal of Public Economic Theory*, 6, 145-164.

[3] Foster, J.E., J. Greer et E. Thorbecke (1984), A Class of Decomposable Poverty Measures, *Econometrica*, 52, 761-776.

[4] Heil, M.T. and T.M. Selden (2001), Carbon Emissions and Economic Development: Future Trajectories Based on Historical Experience, *Environment and Development Economics*, 6, 63-83.

[5] Heil, M.T. and Q.T. Wodon (1997), Inequality in CO_2 Emissions Between Poor and Rich Countries, *Journal of Environment and Development*, 6, 426-452.

[6] Heil, M.T. and Q.T. Wodon (2000), Future Inequality in CO_2 Emissions and the Impact of Abatement Proposals, *Environmental and Resource Economics*, 17, 163-181.

[7] IPCC (2001), *Climate Change 2001: Synthesis Report. A Contribution of Working Groups I, II, and III to the Third Assessment Report of theIntegovernmental Panel on Climate Change* [Watson, R.T. and the Core Writing Team (eds.)], Cambridge University Press, Cambridge,United Kingdom, and New York, NY, USA, 398 pp.

[8] Marland, G., T. A. Boden, and R. J. Andres, 2002, *Global, Regional, and National Fossil Fuel CO2 Emissions*, available at http://cdiac.esd.ornl.gov/trends/trends.htm.

[9] Zheng, B. (1999), On the Power of Poverty Orderings, *Social Choice and Welfare*, 49, 317–327.

[10] Zheng, B. (2000), Poverty Orderings, Journal of Economic Surveys, 14, 427-466.

A Proof of proposition 1

If over-consumption decreases when we move from a distribution F to a distribution G then

$$\Delta S_{FG}(c) = \int_c^a s(x,c)\,dG(x) - \int_c^a s(x,c)\,dF(x) \leq 0. \tag{8}$$

In order to prove that testing (D1) is sufficient to know that equation (8) is non-positive, we first need to integrate equation (1) by parts:

$$\int_c^a s(x,c)\,dF(x) = s(x,c)\,F(x)|_c^a - \int_c^a \frac{\partial s(x,c)}{\partial x} F(x)\,dx. \tag{9}$$

Using this result, we can rewrite equation (8) as

$$\Delta S_{FG}(c) = s(x,c)\,[G(x) - F(x)]|_c^a - \int_c^a \frac{\partial s(x,c)}{\partial x}\,[G(x) - F(x)]\,dx. \tag{10}$$

Since by asumption, $s(c,c) = 0$ and since by definition of the domain, $F(x) = G(x) = 1$, the first term of the right hand side of the equation is equal to 0. We then have

$$\Delta S_{FG}(c) = \int_c^a \frac{\partial s(x,c)}{\partial x} [F(x) - G(x)] \, dx. \tag{11}$$

Using the definition of Σ^1, we know that $\partial s(x,c) / \partial x \geq 0$. It is then easy to see that if $G(x) \geq F(x)$ for all $x \in [c^-, a]$ then $\Delta S_{FG}(c) \leq 0$. ■

B Proof of proposition 2

First, by integrating by parts, we know that

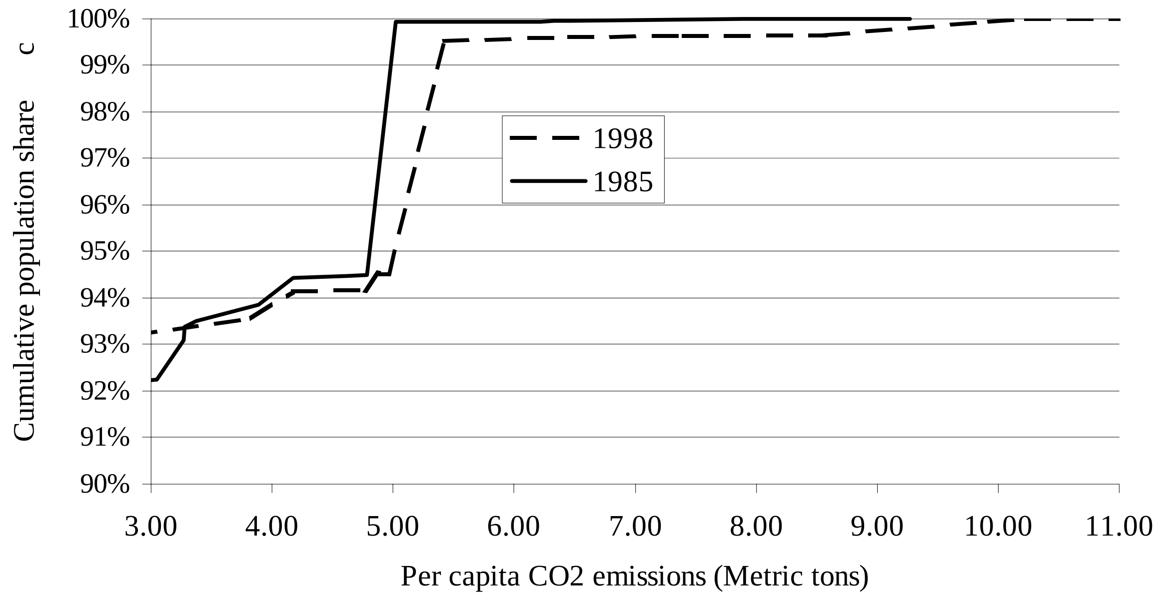
$$\int_c^a \frac{\partial s(x,c)}{\partial x} F(x) \, dx = - \frac{\partial s(x,c)}{\partial x} \Psi_F(x) \Big|_c^a + \int_c^a \frac{\partial^2 s(x,c)}{\partial x^2} F(x) \, dx. \tag{12}$$

By definition of the domain, $\Psi_F(a) = 0$. Our continuity asumption of $s(x,c)$ insures that $\partial s(c,c) / \partial x = 0$. The first term of the right hand side is then equal to 0. We then have

$$\Delta S_{FG}(c) = \int_c^a \frac{\partial^2 s(x,c)}{\partial x^2} [\Psi_F(x) - \Psi_G(x)] \, dx. \tag{13}$$

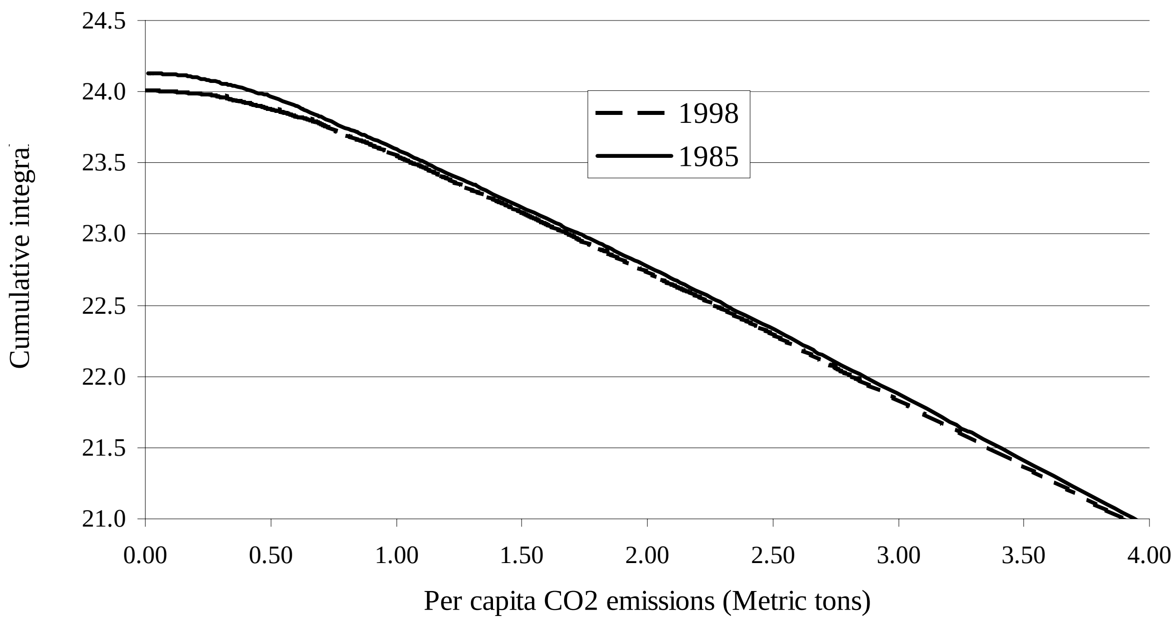
Using the definition of Σ^2, we know that $\partial^2 s(x,c) / \partial x^2 \geq 0$. It is then easy to see that if $\Psi_G(x) \geq \Psi_F(x)$ for all $x \in [c^-, a]$ then $\Delta S_{FG}(c) \leq 0$. ■

Figure 1: First Order Dominance, 1985-1998



Source: Authors' estimates using Oak Ridge National Laboratory data.

Figure 2: Second Order Dominance, 1985-1998



Source: Authors' estimates using Oak Ridge National Laboratory data.