E C O N O M I C S   B U L L E T I N

Cost reducing incentives in a mixed duopoly market

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Abstract

This note studies the cost−reducing incentives in a mixed duopoly market. The result shows that while a profit−maximizing private firm carries out the cost−reducing investment, a social welfare−maximizing firm does not have an incentive to reduce its costs as long as the market share of the private firm is sufficiently large.

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1 Introduction

In a mixed duopoly market, whether a public or a private firm has more incentives to reduce its costs is an essential concern. In line with common knowledge, a number of recent studies show that private firms produce at lower costs [see Megginson and Netter (2001)]. A stream of explanation for why a public firm has less incentive to lower its costs and for why it produces inefficiently has been offered by the previous literature. The main focus of these studies is on the informational and institutional aspects of the market. So-called X-inefficiency is most often seen for the public firm that has a great deal of market control and information [Peacock (1983)].

In contrast to the previous studies focusing on the X-inefficiency of a public firm, this present paper investigates a mixed duopoly model in which the public firm behaves under the appropriate objective (social welfare-maximizing objective), to examine the cost-reducing incentives of both the private and public firms. We show that while the profit-maximizing private firm carries out the cost-reducing investment, the social welfare-maximizing public firm may have no incentive to reduce its production costs, even though it can reduce its production costs without any additional investment costs. Keeping a high cost-structure works as an essential strategic device for the welfare-maximizing public firm in the mixed duopoly market.

In one of the recent articles, Matsumura and Matsushima (2004) investigate a mixed duopoly model with product differentiation represented by the Hotelling-type spatial. Under a situation where both public and private firms can endogenize their own production costs by introducing cost-reducing activities (managerial efforts or R&D investment), they show that strategic interactions between both firms yield a higher production cost to the public firm than to the private firm. Though the models differ widely from each other, Nett (1994), Willner (1994), Poyago-Theotoky (1998), and Nishimori and Ogawa (2002, 2004) provide an examination on the endogenous determination of cost structure. The main difference of our paper with the previous studies cited above is that we show that the welfare-maximizing public firm does not carry out the cost-reducing investment even if it suffers *no* additional cost for investment, and such an environment takes place when *the market share of the private firm is sufficiently large.*

2 Model

Let us consider a simple mixed duopoly market in which the issue of cost-reducing incentives for both firms can be addressed. There are two firms operating in a homogeneous good market with an inverse demand function given by

$$p = a - Q = a - (q_1 + q_2) \qquad a > 0, \tag{1}$$

where p is the market price and Q is the total output. q_i denotes the output of firm $i (= 1, 2)$. Firm 1 is a private firm that maximizes its own profits, while firm 2 is a public firm that maximizes the social welfare defined by a summation of the consumer surplus and both firms' profits.

The production technology of firm i is represented by the cost function, $C_i = c_i q_i$, where c_i is the marginal cost. We assume that private firm 1 is the more efficient firm. The cost differential between the two firms is described by $0 < c_1 < c_2 < a$[1] . Note that the values of these three parameters, a, c_1, c_2, are fixed throughout the analysis of our study.

The objective function of private firm 1 is the profit given by

$$\pi_1 = pq_1 - c_1 q_1. \tag{2}$$

Public firm 2 maximizes the social welfare defined by

$$SW = \frac{Q^2}{2} + (pq_1 - c_1 q_1) + (pq_2 - c_2 q_2), \tag{3}$$

where $Q^2/2$ is the consumer surplus and $pq_i - c_i q_i$ is the profit of firm i.

Given the inverse demand function (1), and the firms' cost functions with constant unit costs c_1 and c_2, the standard Cournot-Nash equilibrium can be straightforwardly derived. The equilibrium levels of the firms' output can be written as

$$q_1(c_1, c_2) = c_2 - c_1, \tag{4}$$
$$q_2(c_1, c_2) = a - 2c_2 + c_1. \tag{5}$$

Since a, c_1 and c_2 are fixed, we express $c_2 - c_1$ as $\bar{q}_1 (= c_2 - c_1)$ and $a - 2c_2 + c_1$ as $\bar{q}_2 (= a - 2c_2 + c_1)$ in the following analysis.

Using (1) and substituting (4) and (5) into (2) and (3), the corresponding equilibrium value of the profit of private firm 1 and the social welfare can be shown as follows:

$$\pi_1(c_1, c_2) = (c_2 - c_1)^2 = \bar{q}_1{}^2, \tag{6}$$
$$SW(c_1, c_2) = \frac{1}{2}(a - c_2)^2 + (c_2 - c_1)^2 = \frac{1}{2}(\bar{q}_1 + \bar{q}_2)^2 + \bar{q}_1{}^2. \tag{7}$$

[1] We see no reason to assume $c_2 \leq c_1$ since that would yield zero output for the private firm, particularly since our interest is in the study of active mixed duopolies.

private firm 1 / public firm 2	$e = 0$	$e > 0$
$e = 0$	$SW(c_1, c_2), \pi_1(c_1, c_2)$	$SW(c_1 - e, c_2), \pi_1(c_1 - e, c_2)$
$e > 0$	$SW(c_1, c_2 - e), \pi_1(c_1, c_2 - e)$	$SW(c_1 - e, c_2 - e), \pi_1(c_1 - e, c_2 - e)$

Table 1: Payoff Matrix

3 Equilibrium with Cost-reducing Innovation

We now consider the cost-reducing incentives of both firms. In this paper, we assume that both firms can reduce their production costs by investing a fixed cost F to introduce new technology. The firms will face the marginal cost $c_i - e$, where $e = 0$ if the firm does not introduce the new technology, while $e > 0$ if the firm introduces it. Let us assume $F = 0$ for convenience of analysis. Further, to assume that both firms produce positive output in all possible cost configurations, we restrict our analysis to the non-drastic innovation in which e satisfies

$$a - c_2 - e > c_2 - c_1 > e. \tag{8}$$

Here, we construct a two-stage game between the two firms. In the first stage, both firms simultaneously choose whether to introduce the new technology or not. In the second stage, given the decision on the introduction for the new technology, both firms simultaneously choose their output levels. This game is solved by backward induction.

In the first stage, there are four possible cases for the decision of the new technology introduction. Let us list the payoff of both firms in each case as a matrix table (Table 1). The decision on whether private firm 1 will introduce the new technology is determined by the following equations;

$$\pi_1(c_1 - e, c_2) - \pi_1(c_1, c_2) = e[2\bar{q}_1 + e] > 0, \tag{9}$$
$$\pi_1(c_1 - e, c_2 - e) - \pi_1(c_1, c_2 - e) = e[2\bar{q}_1 - e]. \tag{10}$$

(9) shows that given that public firm 2 decides not to introduce the new technology, private firm 1 will introduce it. Moreover, given the assumption that both firms produce positive output, i.e., (8), we have $\pi_1(c_1 - e, c_2 - e) -$

$\pi_1(c_1, c_2 - e) > 0$. The fact that (10) is positive shows that when public firm 2 decides to introduce the new technology, private firm 1 will introduce it as well. Thus private firm 1 always has an incentive to introduce the new technology to reduce its production costs. In other words, introducing the new technology is the dominant strategy for private firm 1[2] .

Similarly, public firm 2 determines whether to introduce the new technology by accounting for the following relationship;

$$
\begin{aligned}
SW(c_1, c_2 - e) - SW(c_1, c_2) &= e(\bar{q_2} - \bar{q_1} + 1.5e), \qquad (11)\\
SW(c_1 - e, c_2 - e) - SW(c_1 - e, c_2) &= e(\bar{q_2} - \bar{q_1} - 0.5e). \qquad (12)
\end{aligned}
$$

The signs of (11) and (12) are ambiguous, and depend on the relationship of both firms' output levels (market share) decided before the stage of decision-making for the investment. To derive the Nash equilibrium in the first stage, only (12) matters in the sequel, since the dominant strategy for private firm 1 is to introduce the new technology. When $\bar{q_1} > \bar{q_2} - 0.5e$, (12) is negative, which means public firm 2 will choose $e = 0$. When $\bar{q_1} < \bar{q_2} - 0.5e$, (12) is positive, which means public firm 2 will choose $e > 0$.

According to Table 1, and the relationship of (9)-(12), we find that the unique sub-game perfect equilibrium consists of private firm 1 choosing $e > 0$, and public firm 2 choosing $e = 0$ ($e > 0$) when the market share of private firm 1 is large (small). Summarizing the above discussion, we have the following main result of our study.

Result. In a mixed duopoly market where a private firm has lower production costs than the public firm, and both firms can reduce their production costs by investing a fixed cost to introduce new technology, the private firm has an incentive to introduce the new technology to reduce its production costs. However, even though the public firm can reduce its production costs without any additional investment costs, the public firm does not do so as long as the market share of the private firm is sufficiently large.

Intuitively, this result can be explained as follows[3] . Given the asymmetry of production costs and the assumption that the public firm is the less efficient one, the public firm acts as a "residual" provider of the good. When the private firm has a relatively small share of the market, it is in the interest of the social

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[2] It should be noted that, in the case where $F > 0$, if $e(2\bar{q_1} - e) < F < e(2\bar{q_1} + e)$, private firm 1 optimally introduces the new technology for cost-reducing only when public firm 2 does not introduce it.

[3] This intuitive explanation is basically offered by the anonymous referee.

welfare-maximizing public firm to engage in cost reduction. However, when the private firm has a relatively large market share, there is no point in the public firm adopting a cost reduction.

To put the explanation in more detail, let us provide an alternative expression that coincides with the above intuitive explanation[4] . Suppose that private firm 1 introduces the new technology and has production costs $c_1 - e$. If public firm 2 introduces the new technology to reduce its production costs as well, then the output of public firm 2 rises by $2e$, and the output of private firm 1 falls by e. Total output rises by e, which raises consumer surplus. On the other hand, e units of output that were produced at a cost of $c_1 - e$ are now produced at the higher cost, $c_2 - e$. This lowers social welfare. If c_2 is much greater than c_1, then the net effect may be a decrease in social welfare. Of course, private firm 1 has a large market share when c_2 is much greater than c_1. Thus the result holds[5] .

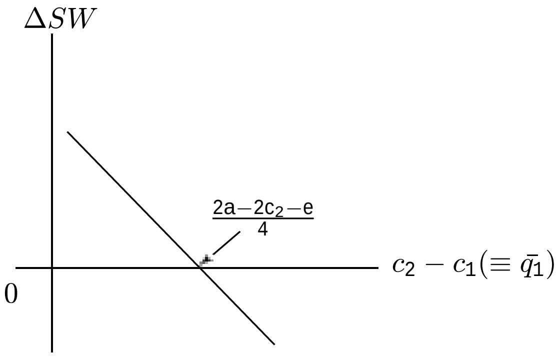

Figure 1.

4 Concluding Remarks

It seems plausible to expect that not only the private firm but also the public firm introducing new technology to reduce production costs will result in an improvement in social welfare. Being opposite to this expectation, the main result of our study is perhaps interesting. In a mixed duopoly market in which the private firm maximizes its own profit and the public firm maximizes the

[4] This alternative explanation is based on the comment of the associate editor, which enhances our explanation more illuminating.

[5] To see the effects on the social welfare resulting from the cost-reducing activities by public firm 2 clearly, we can rewrite (12) as $\Delta SW \equiv SW(c_1 - e, c_2 - e) - SW(c_1 - e, c_2) = e[a - 2(c_2 - c_1) - c_2 - 0.5e]$. According to this equation, we can sketch Figure 1 to illustrate that the social welfare rises (falls) when the differential between c_2 and c_1 is small (large), corresponding to market share of private firm 1 is small (large).

social welfare, the best choice for the public firm could be either to introduce or not to introduce the new technology for cost-reducing strategically, depending on the market share of theprivate firm.

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